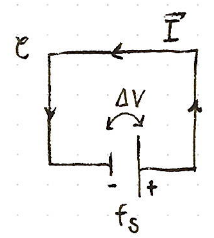
Induzione elettromagnetica

· Ripasso

- Legge di Ohm: $\vec{J} = \sigma \vec{f}$ — σ: conducibilità, $\vec{f}$: forza per unità carica (eq. moto con attrito → deriva)

- Conduttori ideali: $\sigma \approx +\infty$ $\quad \vec{J} \neq 0$ con $\vec{f} \approx 0$ $\quad (f \approx 0, E \approx 0)$

- Forza elettromotrice:



$\vec{f} = \vec{f}_s + \vec{E}$

1. $\mathcal{E} \equiv \oint_{\mathcal{C}} \vec{f}\cdot d\vec{l} = \oint_{\mathcal{C}} (\vec{f}_s + \underbrace{\overset{-\nabla V}{\vec{E}}}_{\to 0})\cdot d\vec{l} = \oint_{\mathcal{C}} \vec{f}_s \cdot d\vec{l}$ circuitazione di f_s

2. $\Delta V = V_+ - V_- = -\int_-^+ \vec{E}\cdot d\vec{l}$

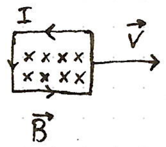
se generatore ideale $f \approx 0$ all'interno, cioè $\vec{E} = -\vec{f}_s$

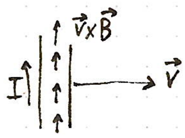
quindi $\Delta V = \int_-^+ \vec{f}_s \cdot d\vec{l} = \oint_{\mathcal{C}} \vec{f}_s \cdot d\vec{l} = \mathcal{E}$. lavoro per unità carica

"f_s ~ pompa
E ~ gravità"

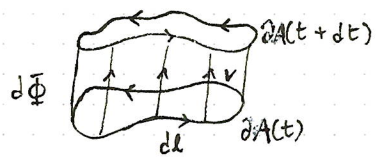
$f_s = -E$, lavoro contro E dentro il generatore, lavoro di E fuori, muove le cariche.

- Fem motorica: $\mathcal{E} = \oint_{\mathcal{C}} \underbrace{(\overset{\to 0}{E} + v \times B)}_{f} \cdot dl$





- Legge del flusso:

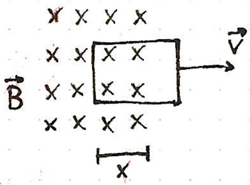


$\Phi(t) = \int_{A(t)} \vec{B}\cdot d\vec{A}$ $\qquad d\Phi = \Phi(t+dt) - \Phi(t) = \int_{\partial A(t)} \vec{B}\cdot d\vec{A}$

$d\vec{A} = \vec{v}dt \times d\vec{l}$ $\qquad \frac{d\Phi}{dt} = \oint_{\mathcal{C}(t)} \vec{B}\cdot(\vec{v}\times d\vec{l}) = \oint_{\mathcal{C}(t)} d\vec{l}\cdot(\vec{B}\times\vec{v}) = -\oint_{\mathcal{C}(t)} \vec{f}\cdot d\vec{l}$

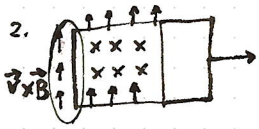
$$\boxed{\mathcal{E} = -\frac{d\Phi}{dt}}$$

Es.



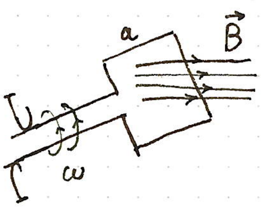
1. $\Phi(t) = x(t)\,l\,B$ $\qquad \frac{d\Phi}{dt} = -v(t)\,l\,B$ $\Rightarrow$ $\mathcal{E} = v(t)\,l\,B$

2.

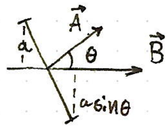


$\mathcal{E} = \int_0^l d\vec{l}\cdot\vec{v}\times\vec{B} = v(t)\,l\,B$

1· alternatore [es. 7.10, electrodynamics]



1.



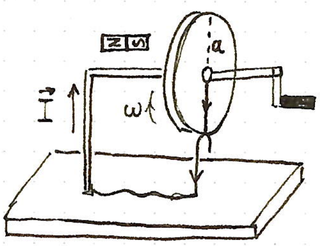
$\Phi(t) = a^2 \sin\theta(t)\, B$ $\qquad \frac{d\Phi}{dt} = a^2 B\cos\theta(t)\frac{d\theta}{dt}$

$= a^2 B\cos(\omega t)\,\omega$

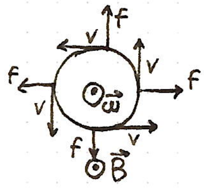
$\mathcal{E} = a^2 B\cos(\omega t)\,\omega$ $\qquad I = \frac{\mathcal{E}}{R}$

2. con $\mathcal{E} = \oint (v\times B)\, dl$ ... buona fortuna

2. Generatore omopolare, "disco di Faraday" [ex. 7.4 electrodynamics]



1. $\mathcal{E} = \oint \vec{f}\cdot d\vec{l}$
   $\hookrightarrow \parallel \vec{J}$

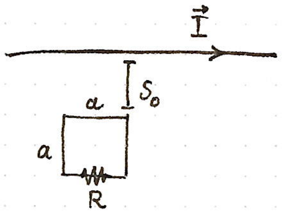


$$\mathcal{E} = \int_0^a (\omega s\hat{\varphi})\times\hat{z}B\cdot d\vec{s} = \omega B\int_0^a \hat{s}s\cdot d\vec{s} = \frac{1}{2}\omega B a^2$$

$$I = \frac{\omega B a^2}{2R}$$

2. $\mathcal{E} = -\frac{d}{dt}\int_{A?} B\cdot dA \;\dots$

3. Corrente indotta da un filo infinito [prob. 7.8 electrodynamics]



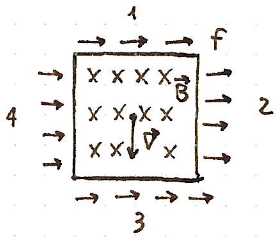
1. calcolare flusso attraverso la spira
2. calcolare la fem, corrente se $\vec{v} = v_s\hat{s}$, $v = v_z\hat{z}$
3. calcolare la forza sulla spira

1. $$\Phi = \oint \vec{B}\cdot d\vec{A} = \int_0^a dz\int_{s_0}^{s_0+a} ds\left(\frac{\mu_0 I}{2\pi s}\hat{\varphi}\right)\cdot\hat{\varphi} = a\frac{\mu_0 I}{2\pi}\log s\Big|_{s_0}^{s_0+a} = a\frac{\mu_0 I}{2\pi}\log\left(1+\frac{a}{s_0}\right)$$

2. $$\frac{d\Phi}{dt} = a\frac{\mu_0 I}{2\pi}\frac{1}{1+\frac{a}{s_0}}\left(\frac{-a}{s_0^2}\right)\frac{ds_0}{dt} = -\frac{\mu_0 I}{2\pi}\frac{(a/s_0)^2}{1+(a/s_0)}v_s \Rightarrow \mathcal{E} = \frac{\mu_0 I}{2\pi}\frac{(a/s_0)^2}{1+a/s_0}v_s,\quad I' = \frac{\mu_0 I}{2\pi}\frac{(a/s_0)^2}{1+a/s_0}\frac{v_s}{R}$$

se $\vec{v} = v_s\hat{s}$ $\mathcal{E}\neq 0$, se $\vec{v} = v_z\hat{z}$ $\mathcal{E} = 0$

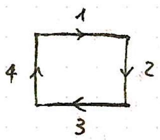
direzione?



$\mathcal{E} = \int_{\mathcal{C}}\vec{f}\cdot d\vec{l}$:

$$\left.\begin{array}{l}\int_{\mathcal{C}_2} = -\int_{\mathcal{C}_4}\\ \int_{\mathcal{C}_1} > \int_{\mathcal{C}_3} \quad B\propto\frac{1}{s}\end{array}\right\} \Rightarrow \begin{array}{l}\vec{I}_1' = I'\hat{z}\\ \vec{I}_3' = -I'\hat{z}\end{array} \quad \circlearrowright$$

3. $$\vec{F} = \oint dl\,\vec{I}'\times\vec{B}$$



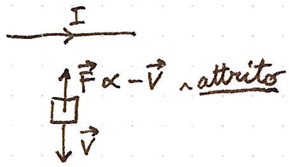
$\int_{\mathcal{C}_2} = -\int_{\mathcal{C}_4}$

$$\vec{F}_1 = \int_0^a dz\,I'\hat{z}\times\hat{\varphi}\,B(s_0) = -\hat{s}\,aI'B(s_0)$$

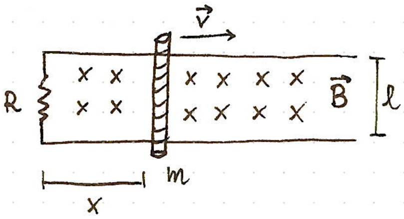
$$\vec{F}_3 = \int_0^a dz\,I'\hat{z}\times\hat{\varphi}\,B(s_0+a) = +\hat{s}aI'B(s_0+a)$$

$$\Rightarrow \vec{F} = -\hat{s}\,a\frac{I'I\mu_0}{2\pi}\left(\frac{1}{s_0}-\frac{1}{s_0+a}\right) = -\hat{s}\,\frac{\mu_0 II'}{2\pi}\frac{(a/s_0)^2}{(1+a/s_0)}$$

$$= -\hat{s}\left(\frac{\mu_0 I}{2\pi}\right)^2\frac{(a/s_0)^4}{(1+a/s_0)^2}\frac{v_s}{R}$$



4. <u>Bacchetta su rotaie</u> [prob. 7.7 electrodynamics]



1. calcolare corrente e direzione
2. calcolare forza sulla bacchetta
3. calcolare $v(t)$ in assenza di altre forze
4. verificare che energia dissipata = cinetica iniziale

1. $\Phi(t) = x(t)\,l B \qquad \mathcal{E}(t) = -\frac{d\Phi}{dt} = -v(t)\,l B \qquad I(t) = \frac{\mathcal{E}}{R} = \frac{v(t)\,l B}{R}$

2. $\vec{F}_m = \int dl\,\vec{I}\times\vec{B} = \int_0^l dl\,(I\,\hat{y})\times(-B\,\hat{z}) = -I l B\,\hat{x} = -v(t)\frac{l^2B^2}{R}\,\hat{x} \qquad \underline{\vec{F}_m \propto -\vec{v}}$

3. $m\frac{d\vec{v}}{dt} = \vec{F}_m \qquad m\frac{dv}{dt} = -v\frac{l^2B^2}{R} \Rightarrow \underline{v(t) = v_0\,e^{-\frac{l^2B^2}{mR}t}} \qquad \tau = \frac{mR}{l^2B^2}$

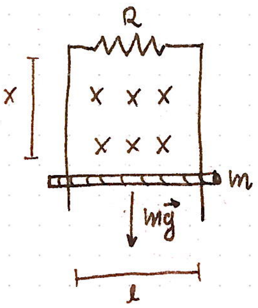
[con $\begin{cases} m = 1\,kg \\ R = 50\,\Omega \\ l = 30\,cm \\ B = 1\,T \end{cases}$ $\quad \tau = 555\,s$, piuttosto debole]

4. $T_0 = \frac{1}{2}mv_0^2 \quad T_1 = 0$

$P_{diss} = VI = \frac{V^2}{R} = \frac{\mathcal{E}^2}{R} = \frac{l^2B^2v(t)^2}{R} = \frac{l^2B^2v_0^2}{R}e^{-\frac{2t}{\tau}}$

$E_{diss} = \int_0^{+\infty} P_{diss}\,dt = \underbrace{\frac{l^2B^2}{mR}}_{\frac{1}{\tau}} v_0^2\,m\, e^{-\frac{2t}{\tau}}\left(-\frac{\tau}{2}\right)\Big|_0^{+\infty} = \frac{1}{2}mv_0^2 = T_0$

5. <u>Bacchetta su rotaia in caduta</u> [2° prova 24-02-22]



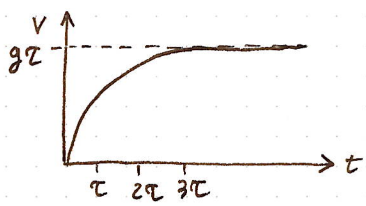
• $m\frac{d\vec{v}}{dt} = \vec{F}_m + m\vec{g} \qquad (\vec{F}_m \propto -\vec{v}) \qquad m\frac{dv}{dt} = -\frac{l^2B^2}{R}v + mg$

$\frac{dv}{dt} = \left(-\frac{v}{\tau} + g\right) \rightarrow \frac{dv}{(g - v/\tau)} = dt \rightarrow -\tau\log\left(\frac{g - \frac{v}{\tau}}{c}\right) = t$

$g - \frac{v}{\tau} = c\,e^{-\frac{t}{\tau}} \rightarrow v(t) = g\tau\left(1 - \frac{c}{g\tau}e^{-t/\tau}\right)$

vogliamo $v(0) = 0 \Rightarrow c = g\tau$ e $\underline{v(t) = g\tau\left(1 - e^{-t/\tau}\right)}$

• Per $t \gg \tau \quad v \approx \underline{g\tau}$ velocità terminale



• se $\frac{dv}{dt} = 0 \quad v = g\tau \quad \vec{F}_m = -\vec{F}_p \quad P_m = P_{diss} = \frac{V^2}{R} = P_p = mgv = mg^2\tau$ (caso stazionario)