Ottimizziamo il pre-ricovero




Il pre-ricovero

- È un insieme di esami/visite da effettuare pochi giorni prima di un intervento programmato
- Può comprendere: prelievo di sangue, elettrocardiogramma, radiografia, visita cardiologica, visita anestesiologica, ecc.
- Una precisa pianificazione degli appuntamenti è importante per utilizare al meglio le sale operatorie

Il pre-ricovero all'Ospedale Cisanello di Pisa

- Risorse, personale e stanze per esami/visite sono condivise tra diverse specialità
- Ogni paziente deve completare il pre-ricovero entro una giornata
- Sono noti quali esami/visite deve effettuare ogni paziente, i tempi necessari per effettuarli ed i turni degli anestesisti
- Alcune attività devono essere effettuate contemporaneamente da un gruppo di pazienti (es. esami del sangue in laboratorio)
- A causa della pandemia di COVID-19, solo 20 persone possono essere presenti all'interno dell'edificio dedicato al pre-ricovero

Problema

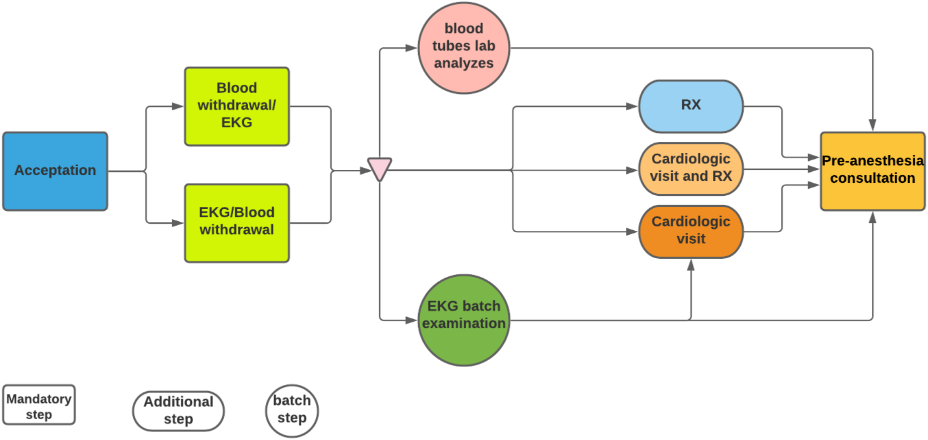
Ogni giorno vogliamo assegnare ad ogni paziente l'ora di ingresso in ospedale e programmare ogni fase del suo pre-ricovero in modo da

- massimizzare il numero di pazienti visitati nella giornata
- minimizzare il tempo trascorso in ospedale dai pazienti

tenendo conto

- dell'ordine in cui fare esami/visite
- della gestione dei lotti di provette da analizzare in laboratorio
- delle stanze a disposizione per esami/visite
- dell'orario di lavoro del personale
- del distanziamento sociale

Ordine di esami e visite



Dati

- Pre-ricovero dalle 7:00 alle 19:00
- 3 stanze per prelievo ed ECG (max 2 prelievi per stanza, max 1 ECG per stanza), 1 per il cardiologo, 4 per gli anestesisti
- radiografie sono effettuate in un edificio diverso
- ogni lotto di provette contiene tra 8 e 12 provette
- max 20 pazienti presenti contemporaneamente nell'edificio
- tempi: accettazione 5 min, prelievo 5 min, ECG 15 min, visita cardiologica 20 min, radiografia 45 min, visita anestesiologica 30 min, analisi laboratorio 70 min, referto ECG 5 min
- Turni degli anestesisti:

Monday				Tuesday				Wednesday				Thursday				Friday			
Room n. 5	Room n. 6	Room n. 7	Room n. 8	Room n. 5	Room n. 6	Room n. 7	Room n. 8	Room n. 5	Room n. 6	Room n. 7	Room n. 8	Room n. 5	Room n. 6	Room n. 7	Room n. 8	Stanza n. 5	Stanza n. 6	Stanza n. 7	Stanza n. 8
8:00 10 14:00	8:00 10 14:00	8:00 10 14:00	8:00 9 14:00	8:00 9 14:00	8:00 10 14:00	8:00 10 14:00	8:00 7 14:00	8:00 10 14:00	8:00 10 14:00	8:00 9 14:00	8:00 10 14:00	8:00 9 14:00	8:00 10 14:00	8:00 10 14:00	14:00 5 19:00	8:00 6 14:00	8:00 10 14:00	8:00 10 14:00	8:00 10 14:00
14:00 10 19:00	14:00 7 19:00	14:00 8 19:00	14:00 10 19:00	14:00 10 19:00	14:00 8 19:00	14:00 8 19:00	14:00 7 19:00	14:00 8 19:00	14:00 8 19:00	14:00 10 19:00	14:00 10 19:00	14:00 7 19:00	14:00 7 19:00	14:00 8 19:00	14:00 9 19:00	14:00 10 19:00	14:00 7 19:00	14:00 8 19:00	14:00 9 19:00

Soluzione

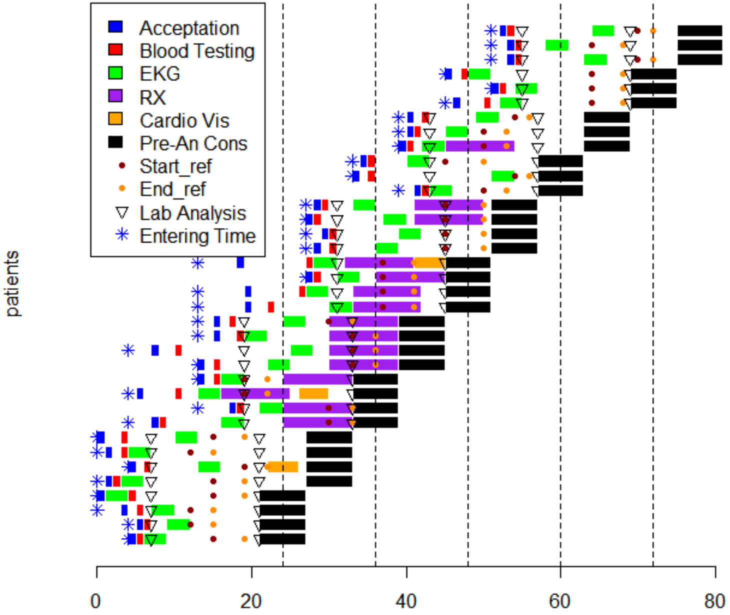
Abbiamo formulato un modello di ottimizzazione e sviluppato un algoritmo risolutivo per risolvere il problema

dettagli matematici?

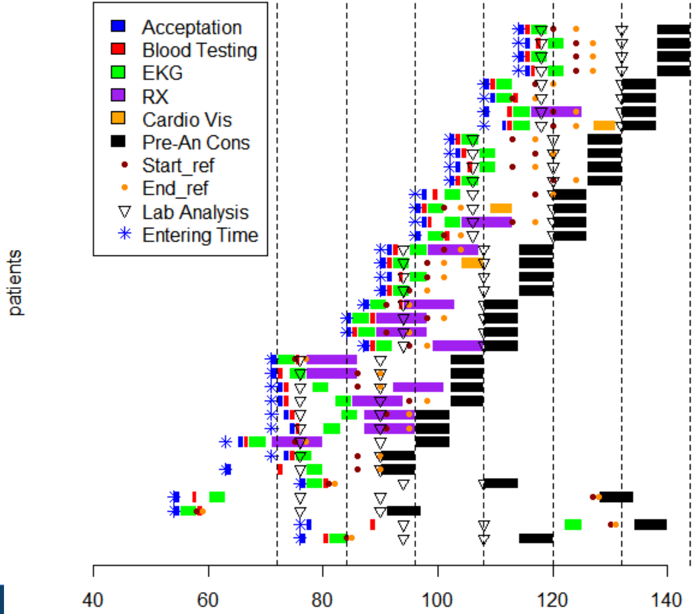
Risultati di una settimana tipica in ospedale

Giorno	Pazienti visitati	Tempo permanenza medio in ospedale
Lunedì	77	2h 23min
Martedì	75	2h 28min
Mercoledì	78	2h 29min
Giovedì	70	2h 31min
Venerdì	74	2h 28min

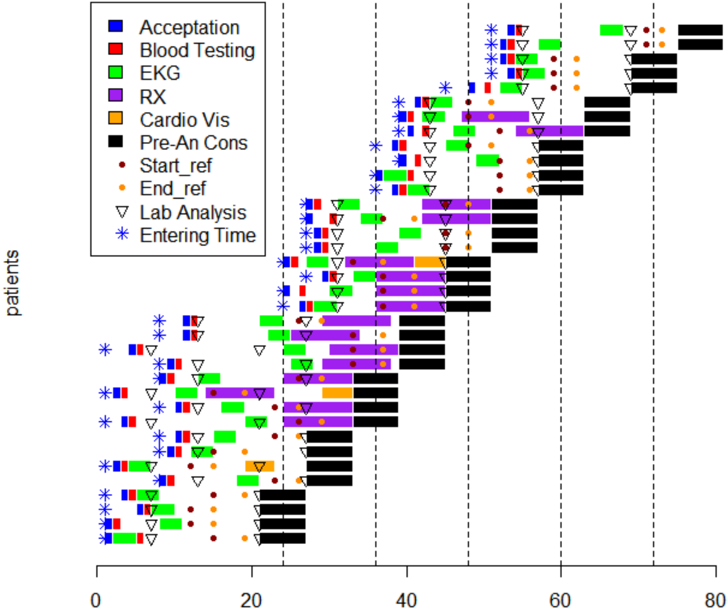
Lunedì mattina



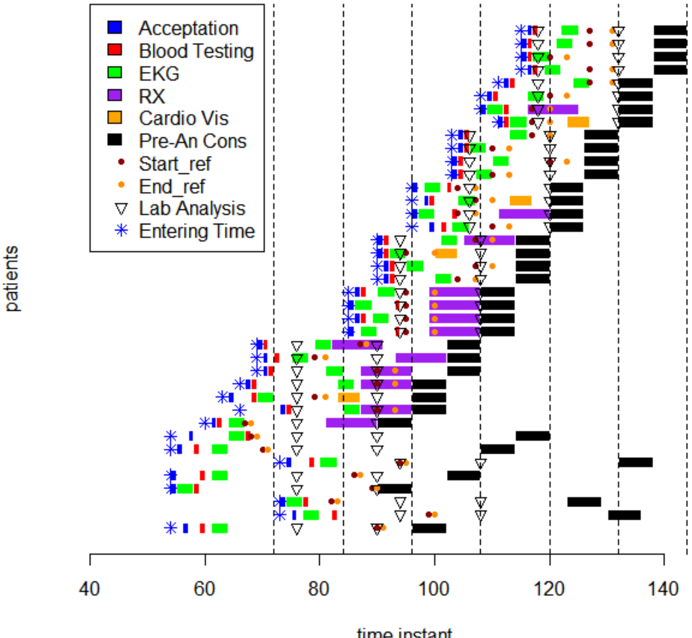
Lunedì pomeriggio



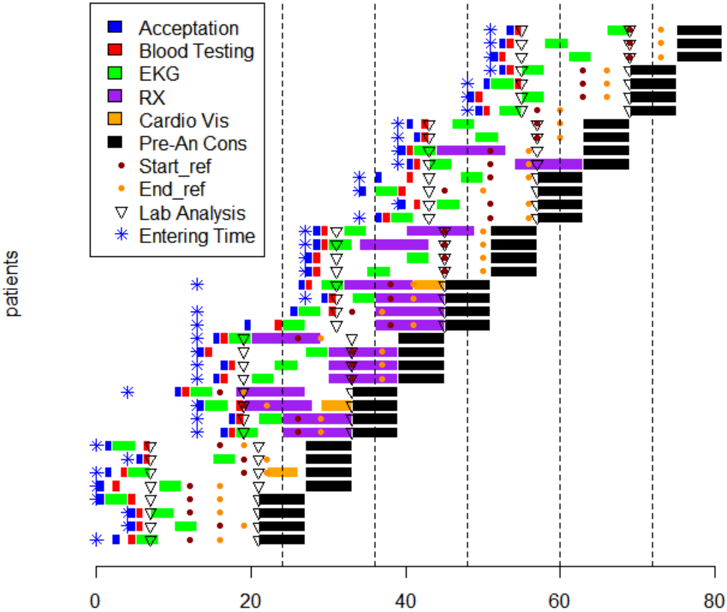
Martedì mattina



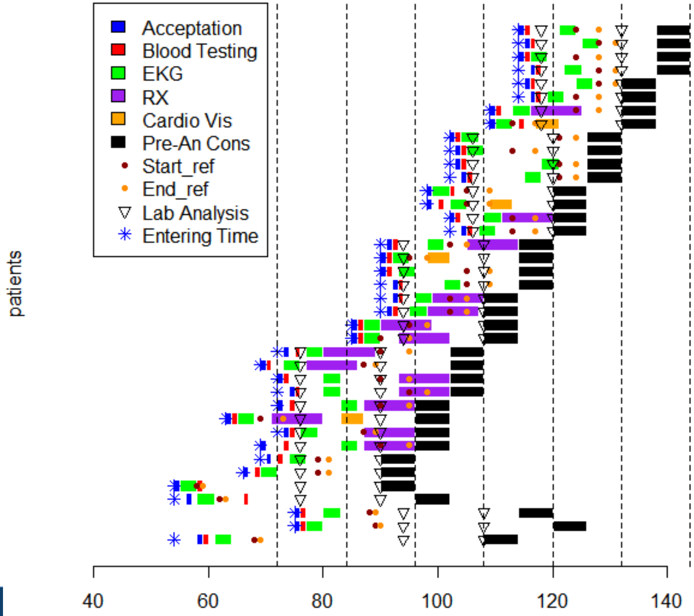
Martedì pomeriggio



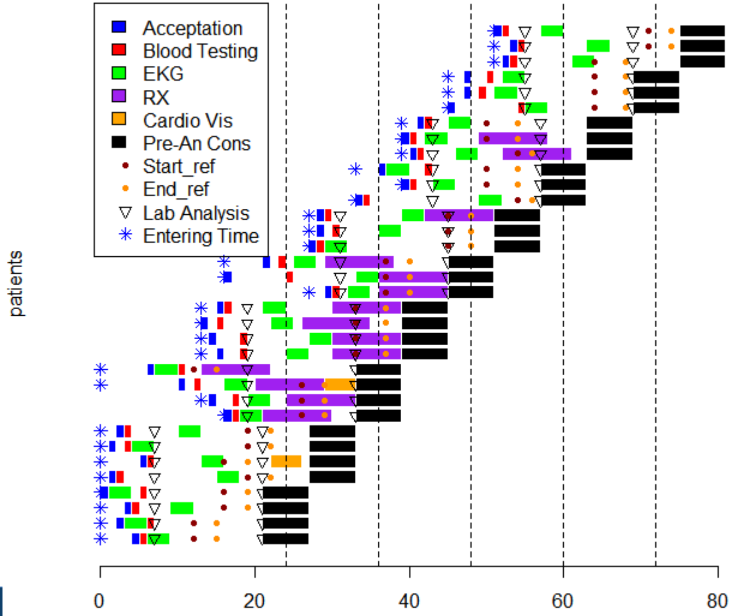
Mercoledì mattina



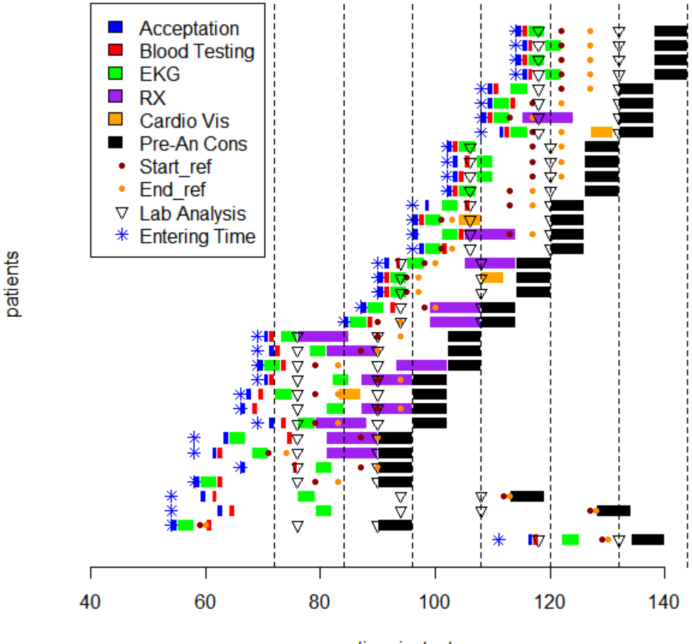
Mercoledì pomeriggio



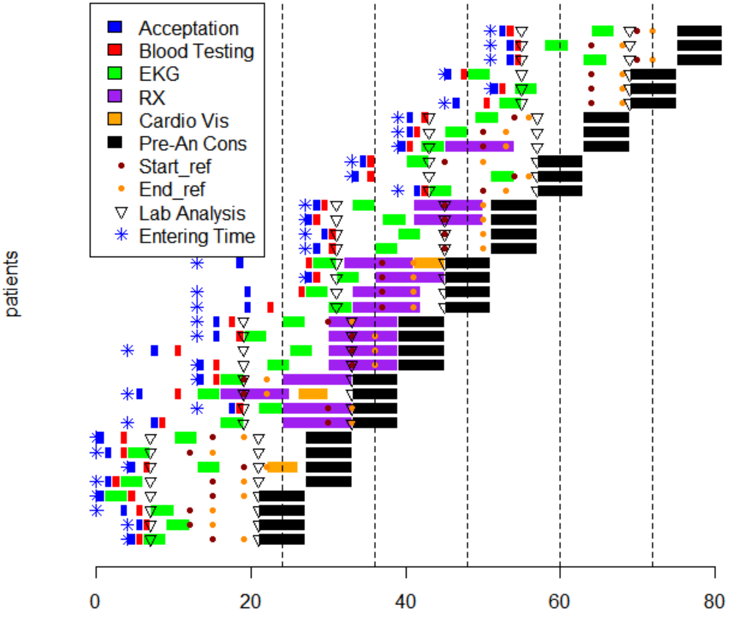
Giovedì mattina



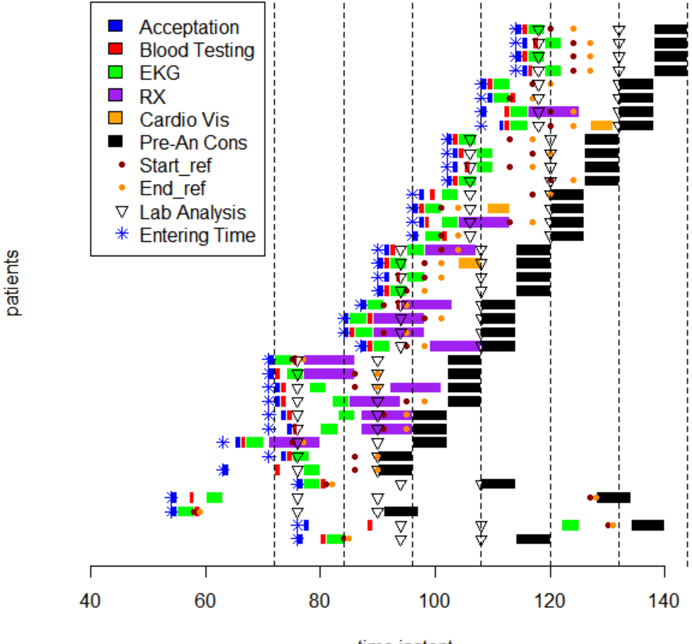
Giovedì pomeriggio



Venerdì mattina



Venerdì pomeriggio



Modello di ottimizzazione

Sets	
I	patients
S	anesthesiologists
T	time slots
B_{tubes}	blood sample batches
B_{ecg}	EKG batches
B_{pat}	entering patients batches
C_f	time slots in which the cardiologist does not work in the morning
A_f	time slots in which the anesthesiologists does not work in the morning
A_M	time slots that compose the morning shift of the anesthesiologists
A_E	time slots that compose the afternoon shift of the anesthesiologists

Modello di ottimizzazione

Parameters	
E_{ik}	binary, equal to 1 if patient i need examination k
p_s^{AN}	binary, equal to 1 if anesthesiologist s is available
m_s^{AN}	binary, equal to 1 if anesthesiologist s works on the morning shift
ev_s^{AN}	binary, equal to 1 if anesthesiologist s works on the afternoon shift
$Lpaz_{min}$	minimum number of patients for each entering batch
$Lpaz_{max}$	maximum number of patients for each entering batch
LP_{min}	minimum number of samples for each blood sample batch
LP_{max}	maximum number of samples for each blood sample batch
Le_{min}	minimum number of samples for each EKG batch
Le_{max}	maximum number of samples for each EKG batch
cap_{AC}	capacity of the reception
p_{max}	maximum number of patients that can stay inside the structure
n_E	maximum number of EKG examination at any time
n_P	maximum number of blood withdrawals at any time
t_{AC}	time needed for the reception of a patient
t_P	time needed for the blood withdrawal examination
t_E	time needed for the EKG examination
t_L	time needed for the lab analysis of a blood sample batch
t_{CR}	time of the cardiologist needed to report an EKG result
t_{CV}	time of the cardiologist needed to visit a patient
t_R	time needed for the RX examination
t_{AN}	time of the anesthesiologist needed to make a visit
$t_{M,L}$	maximum instant in which blood withdrawal must be completed
$t_{M,R}$	maximum instant in which RX can be done
$t_{M,C}$	maximum instant in which the cardiologist can work
δ	integer, minimum interval difference between two consecutive batches of entering times

Modello di ottimizzazione

Variables	
$\alpha_{i\tau}$	binary, equal to 1 if patient i enters at time τ
$\omega_{i\tau}$	binary, equal to 1 if patient i exits at time τ
$x_{i\tau}^{AC}$	binary, equal to 1 if patient i at time τ begins the reception step
$x_{i\tau}^{R}$	binary, equal to 1 if patient i at time τ begins the RX exam
$x_{i\tau}^{CV}$	binary, equal to 1 if patient i at time τ begins the cardiologic visit
$x_{i\tau}^{AN}$	binary, equal to 1 if patient i at time τ begins the pre-anesthesia consultation
$x_{i\tau}^{P}$	binary, equal to 1 if patient i start at time τ the blood withdrawal
$x_{i\tau}^{E}$	binary, equal to 1 if patient i start at time τ the EKG
λ_{il}^{P}	binary, equal to 1 if patient i is assigned to the batch l of blood samples
λ_{iL}^{IN}	binary, equal to 1 if patient i is assigned to the batch l in the entering step
λ_{il}^{E}	binary, equal to 1 if patient i is assigned to batch l of EKG grouping
t_{l}^{IN}	entering time of batch l of patients
$x_{l\tau}^{iP}$	binary, equal to 1 if batch of blood samples l goes to the laboratory at time τ
$CR_{l\tau}$	binary, equal to 1 if the cardiologist start to work on batch l of EKG at time τ
a_{l}^{IN}	binary, equal to 1 if batch l of entering patients is active
a_{l}^{E}	binary, equal to 1 if batch l of EKG is active
a_{l}^{P}	binary, equal to 1 if batch l of blood samples is active
y_{i}^{pe}	binary, equal to 1 if blood withdrawal is done before EKG for patient i
y_{i}^{rc}	binary, equal to 1 if RX is done before cardiological visit for patient i
y_{i}^{vcle}	binary, equal to 1 if the cardiologist first visit patient i then examines batch l
$\rho_{s\tau}$	binary, equal to 1 if the anesthesiologist s is working at time τ

Modello di ottimizzazione

$$\max \quad \sum_{i \in I} \sum_{l \in B_{pat}} \lambda_{il}^{IN} \tag{1}$$

subject to:

$$a_l^{IN} \leq a_{l-1}^{IN} \qquad \forall l \in B_{pat} \setminus \{1\} \tag{2}$$

$$a_l^{P} \leq a_{l-1}^{P} \qquad \forall l \in B_{tubes} \setminus \{1\} \tag{3}$$

$$a_l^{E} \leq a_{l-1}^{E} \qquad \forall l \in B_{ecg} \setminus \{1\} \tag{4}$$

$$\sum_{i \in I} \lambda_{il}^{IN} \leq Lpaz_{max} a_l^{IN} \qquad \forall l \in B_{pat} \tag{5}$$

$$\sum_{i \in I} \lambda_{il}^{IN} \leq Lpaz_{max} a_l^{IN} \qquad \forall l \in B_{pat} \tag{6}$$

$$Lpaz_{min} a_l^{IN} \leq \sum_{i \in I} \lambda_{il}^{IN} \qquad \forall l \in B_{pat} \tag{7}$$

$$\sum_{i \in I} \lambda_{il}^{E} \leq Le_{max} a_l^{E} \qquad \forall l \in B_{ecg} \tag{8}$$

Modello di ottimizzazione

$$Le_{min} a_l^E \leq \sum_{i \in I} \lambda_{il}^E \qquad \forall l \in B_{ecg} \tag{9}$$

$$\sum_{i \in I} \lambda_{il}^P \leq LP_{max} a_l^P \qquad \forall l \in B_{tubes} \tag{10}$$

$$LP_{min} a_l^P \leq \sum_{i \in I} \lambda_{il}^P \qquad \forall l \in B_{tubes} \tag{11}$$

$$t_1^{IN} = 1 \tag{12}$$

$$t_{l-1}^{IN} + \delta \leq t_l^{IN} \qquad \forall l \in B_{pat} \setminus \{1\} \tag{13}$$

$$t_l^{IN} + M(\lambda_{il}^{IN} - 1) \leq \sum_{\tau \in T} \tau \, \alpha_{i\tau} \qquad \forall i \in I,\ l \in B_{pat} \tag{14}$$

$$\sum_{\tau \in T} \tau \, \alpha_{i\tau} \leq t_l^{IN} + M(1 - \lambda_{il}^{IN}) \qquad \forall i \in I,\ l \in B_{pat} \tag{15}$$

$$\sum_{\tau \in T} \alpha_{i\tau} = \sum_{l \in B_{\mathrm{pat}}} \lambda_{il}^{IN} \qquad \forall i \in I \tag{16}$$

Modello di ottimizzazione

$$\sum_{l \in B_{\text{pat}}} \lambda_{il}^{IN} \leq 1 \qquad \forall i \in I \tag{17}$$

$$\sum_{\tau \in T} x_{i\tau}^{P} = E_{i1} \sum_{l \in B_{\text{pat}}} \lambda_{il}^{IN} \qquad \forall i \in I \tag{18}$$

$$\sum_{\tau \in T} x_{i\tau}^{E} = E_{i2} \sum_{l \in B_{\text{pat}}} \lambda_{il}^{IN} \qquad \forall i \in I \tag{19}$$

$$\sum_{\tau \in T} x_{i\tau}^{R} = E_{i3} \sum_{l \in B_{\text{pat}}} \lambda_{il}^{IN} \qquad \forall i \in I \tag{20}$$

$$\sum_{\tau \in T} x_{i\tau}^{CV} = E_{i4} \sum_{l \in B_{\text{pat}}} \lambda_{il}^{IN} \qquad \forall i \in I \tag{21}$$

$$\sum_{\tau \in T} x_{i\tau}^{AN} = E_{i5} \sum_{l \in B_{\text{pat}}} \lambda_{il}^{IN} \qquad \forall i \in I \tag{22}$$

$$\sum_{\tau \in T} \omega_{i\tau} = \sum_{\tau \in T} \alpha_{i\tau} \qquad \forall i \in I \tag{23}$$

Modello di ottimizzazione

$$\sum_{i \in I} x_{i\tau}^{AC} \leq cap_{\mathrm{AC}} \qquad \forall \tau \in T \tag{24}$$

$$\sum_{\tau \in T} \tau\, \alpha_{i\tau} \leq \sum_{\tau \in T} \tau\, x_{i\tau}^{AC} \qquad \forall i \in I \tag{25}$$

$$\sum_{\tau \in T} (\tau + t_{AC})\, x_{i\tau}^{AC} \leq \sum_{\tau \in T} \tau\, x_{i\tau}^{P} \qquad \forall i \in I : E_{i1} = 1 \tag{26}$$

$$\sum_{\tau \in T} (\tau + t_{AC})\, x_{i\tau}^{AC} \leq \sum_{\tau \in T} \tau\, x_{i\tau}^{E} \qquad \forall i \in I : E_{i2} = 1 \tag{27}$$

$$\sum_{\tau \in T} (\tau + t_{AC})\, x_{i\tau}^{AC} \leq \sum_{\tau \in T} \tau\, x_{i\tau}^{R} \qquad \forall i \in I : E_{i3} = 1 \tag{28}$$

$$\sum_{\tau \in T} (\tau + t_{AC})\, x_{i\tau}^{AC} \leq \sum_{\tau \in T} \tau\, x_{i\tau}^{CV} \qquad \forall i \in I : E_{i4} = 1 \tag{29}$$

$$\sum_{\tau \in T} (\tau + t_P)\, x_{i\tau}^{P} \leq \sum_{\tau \in T} \tau\, x_{i\tau}^{E} + M(1 - y_i^{pe}) \qquad \forall i \in I : E_{i1} = E_{i2} = 1 \tag{30}$$

Modello di ottimizzazione

$$\sum_{\tau \in T} (\tau + t_E)\, x^E_{i\tau} \leq \sum_{\tau \in T} \tau\, x^P_{i\tau} + M y^{pe}_i \qquad \forall i \in I : E_{i1} = E_{i2} = 1 \tag{31}$$

$$\sum_{\theta = max(1, \tau - t_P + 1)}^{\tau} \sum_{i \in I} x^P_{i\theta} \leq n_P \qquad \forall \tau \in T \tag{32}$$

$$\sum_{\theta = max(1, \tau - t_E + 1)}^{\tau} \sum_{i \in I} x^E_{i\theta} \leq n_E \qquad \forall \tau \in T \tag{33}$$

$$\sum_{l \in B_{\text{tubes}}} \lambda^P_{il} = E_{i1} \sum_{l \in B_{\text{pat}}} \lambda^{IN}_{il} \qquad \forall i \in I \tag{34}$$

$$\sum_{\tau \in T} x^{iP}_{l\tau} = 1 \qquad \forall l \in B_{tubes} \tag{35}$$

$$\sum_{\tau \in T} \tau\, x^{iP}_{l-1\tau} \leq \sum_{\tau \in T} \tau\, x^{iP}_{l\tau} \qquad \forall l \in B_{tubes} \setminus \{1\} \tag{36}$$

$$\sum_{\tau \in T} (\tau + t_P)\, x^P_{i\tau} \leq \sum_{\tau \in T} \tau\, x^{iP}_{i\tau} + M(1 - \lambda^P_{il}) \quad \forall l \in B_{tubes}, i \in I : E_{i1} = 1 \tag{37}$$

Modello di ottimizzazione

$$\sum_{\tau \in T} (\tau + t_P)\, x^P_{i\tau} \leq t_{M,L} \qquad \forall i \in I : E_{i1} = 1 \tag{38}$$

$$\sum_{l \in B_{\text{ecg}}} \lambda^E_{il} = E_{i2} \sum_{l \in B_{\text{pat}}} \lambda^{IN}_{il} \qquad \forall i \in I \tag{39}$$

$$\sum_{\tau \in T} (\tau + t_E)\, x^E_{i\tau} \leq \sum_{\tau \in T} \tau\, CR_{l\tau} + M(1 - \lambda^E_{il}) \quad \forall l \in B_{ecg}, i \in I : E_{i2} = 1 \tag{40}$$

$$\sum_{\tau \in T} CR_{l\tau} = 1 \qquad \forall l \in B_{ecg} \tag{41}$$

$$\sum_{l \in B_{ecg}} CR_{l\tau} = 0 \qquad \forall t \in C_f \tag{42}$$

$$\sum_{\tau \in T} \tau\, CR_{l-1\tau} + t_{CR} \left(\sum_{i \in I} \lambda^E_{il-1}\right) \leq \sum_{\tau \in T} \tau\, CR_{l\tau} \qquad \forall l \in B_{ecg} \tag{43}$$

Modello di ottimizzazione

$$\sum_{\tau \in T} (\tau + t_{CV})\, x_{i\tau}^{CV} + M(y_{il}^{vcle} - 1) \leq \sum_{\tau \in T} \tau\; CR_{l\tau} \quad \forall l \in B_{ecg}, i \in l : E_{i4} = 1 \tag{44}$$

$$\sum_{\tau \in T} \tau\; CR_{l\tau} + t_{CR} \sum_{k \in l} \lambda_{kl}^{E} - M y_{il}^{vcle} \leq \sum_{\tau \in T} \tau\; x_{i\tau}^{CV} \tag{45}$$

$$\forall l \in B_{ecg},\; i \in l : E_{i4} = 1$$

$$\sum_{\tau \in T} \tau\; CR_{l\tau} + t_{CR} \sum_{k \in l} \lambda_{kl}^{E} + M(\lambda_{il}^{E} - 1) \leq \sum_{\tau \in T} \tau\; x_{i\tau}^{CV} \tag{46}$$

$$\forall l \in B_{ecg},\; i \in l : E_{i2} = E_{i4} = 1$$

$$\sum_{\theta = max(1, \tau - t_{CV} + 1)}^{\tau} \sum_{i \in I} x_{i\theta}^{CV} \leq 1 \qquad \forall \tau \in T \tag{47}$$

$$\sum_{\tau \in T} (\tau + t_{CV})\, x_{i\tau}^{CV} \leq t_{M,C} \qquad \forall i \in I : E_{i4} = 1 \tag{48}$$

Modello di ottimizzazione

$$\sum_{\tau \in T} (\tau \, CR_{l\tau}) + t_{CR}(\sum_{k \in l} \lambda^{E}_{kl}) \leq t_{M,C} \qquad \forall l \in B_{ecg} \tag{49}$$

$$\sum_{\tau \in T} (\tau + t_P) \, x^{P}_{i\tau} \leq \sum_{\tau \in T} \tau \, x^{R}_{i\tau} \qquad \forall i \in I : E_{i1} = E_{i3} = 1 \tag{50}$$

$$\sum_{\tau \in T} (\tau + t_E) \, x^{E}_{i\tau} \leq \sum_{\tau \in T} \tau \, x^{R}_{i\tau} \qquad \forall i \in I : E_{i2} = E_{i3} = 1 \tag{51}$$

$$\sum_{\tau \in T} (\tau + t_R) \, x^{R}_{i\tau} \leq \sum_{\tau \in T} \tau \, x^{CV}_{i\tau} + M(1 - y^{rc}_{i}) \qquad \forall i \in I : E_{i3} = E_{i4} = 1 \tag{52}$$

$$\sum_{\tau \in T} (\tau + t_{CV}) \, x^{CV}_{i\tau} \leq \sum_{\tau \in T} \tau \, x^{R}_{i\tau} + My^{rc}_{i} \qquad \forall i \in I : E_{i3} = E_{i4} = 1 \tag{53}$$

$$\sum_{\tau \in T} (\tau + t_L) x^{iP}_{l\tau} \leq \sum_{\tau \in T} \tau \, x^{AN}_{i\tau} + M(1 - \lambda^{P}_{il}) \qquad \forall l \in B_{tubes}, i \in l : E_{i1} = E_{i5} = 1 \tag{54}$$

$$\sum_{\tau \in T} \tau \, CR_{l\tau} + t_{CR} \, (\sum_{k \in l} \lambda^{E}_{kl}) \leq \sum_{\tau \in T} \tau \, x^{AN}_{i\tau} + M(1 - \lambda^{E}_{il}) \tag{55}$$

$$\forall l \in B_{ecg}, i \in l : E_{i2} = E_{i5} = 1$$

Modello di ottimizzazione

$$\sum_{\tau \in T} (\tau + t_R)\, x^R_{i\tau} \leq \sum_{\tau \in T} \tau\, x^{AN}_{i\tau} \qquad \forall i \in I : E_{i3} = E_{i5} = 1 \tag{56}$$

$$\sum_{\tau \in T} (\tau + t_{CV})\, x^{CV}_{i\tau} \leq \sum_{\tau \in T} \tau\, x^{AN}_{i\tau} \qquad \forall i \in I : E_{i4} = E_{i5} = 1 \tag{57}$$

$$\rho_{s\tau} \leq p^{AN}_s \qquad \forall \tau \in T, s \in S \tag{58}$$

$$\sum_{\theta = max(1, \tau - t_{AN} + 1)}^{\tau} \sum_{i \in I} A_{is}\, x^{AN}_{i\theta} = \rho_{s\tau} \qquad \forall \tau \in T, s \in S \tag{59}$$

$$\rho_{s\tau} = 0 \qquad \forall s \in S, \tau \in A_f \tag{60}$$

$$\rho_{s\tau} \leq m^{AN}_s \qquad \forall s \in S, \tau \in A_M \tag{61}$$

$$\rho_{s\tau} \leq ev^{AN}_s \qquad \forall s \in S, \tau \in A_E \tag{62}$$

$$\sum_{\tau \in T} (\tau + t_P)\, x^P_{i\tau} \leq \sum_{\tau \in T} \tau\, \omega_{i\tau} \qquad \forall i \in I : E_{i1} = 1 \tag{63}$$

$$\sum_{\tau \in T} (\tau + t_E)\, x^E_{i\tau} \leq \sum_{\tau \in T} \tau\, \omega_{i\tau} \qquad \forall i \in I : E_{i2} = 1 \tag{64}$$

Modello di ottimizzazione

$$\sum_{\tau \in T} (\tau + t_R)\, x_{i\tau}^R \leq \sum_{\tau \in T} \tau\, \omega_{i\tau} \qquad \forall i \in I : E_{i3} = 1 \tag{65}$$

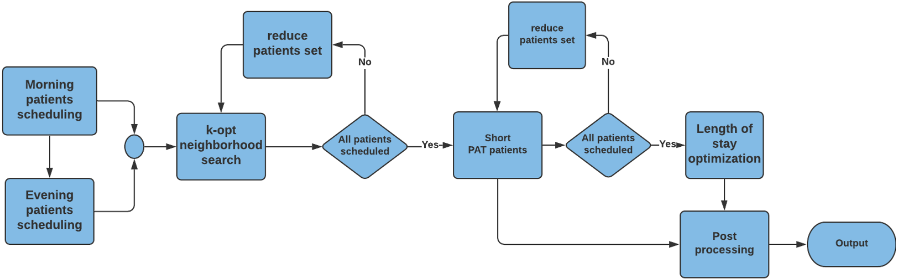
$$\sum_{\tau \in T} (\tau + t_{CV})\, x_{i\tau}^{CV} \leq \sum_{\tau \in T} \tau\, \omega_{i\tau} \qquad \forall i \in I : E_{i4} = 1 \tag{66}$$

$$\sum_{\tau \in T} (\tau + \sum_{s \in S} A_{is} t_{AN})\, x_{i\tau}^{AN} \leq \sum_{\tau \in T} \tau\, \omega_{i\tau} \qquad \forall i \in I : E_{i5} = 1 \tag{67}$$

$$\sum_{\theta=1}^{\tau} \sum_{i \in I} (\alpha_{i\theta} - \omega_{i\theta}) \leq p_{max} \qquad \forall \tau \in T \tag{68}$$

$$\sum_{\tau \in T} (\tau + t_R)\, x_{i\tau}^R \leq t_{M,R} \qquad \forall i \in I : E_{i3} = 1 \tag{69}$$

Algoritmo risolutivo



torna ai risultati