Esercizio riscaldamento tutoraggio Fisica II

[Prob. 3 Elettrostatica, Griffith] ??

$\vec{E} = k r^3 \hat{u}_r$

Simmetria sferica $\vec{E}$ $\Rightarrow$ Simmetria sferica sorgenti

a) $\rho = \rho(r)$?

Gauss: $\oint(\vec{E}) = \frac{Q}{\varepsilon_0} \leftrightarrow \int_S d\sigma\, \vec{E} \cdot \hat{n} = \frac{1}{\varepsilon_0} \int_V dv\, \rho \leftrightarrow \int_V dv\, \vec{\nabla} \cdot \vec{E} = \frac{1}{\varepsilon_0} \int_V dv\, \rho \longrightarrow$

$\longrightarrow \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

$\vec{E} = E_r \hat{u}_r + E_\vartheta \hat{u}_\vartheta + E_\phi \hat{u}_\phi$

In sferiche: $\vec{\nabla} \cdot \vec{E} = \frac{1}{r^2} \frac{\partial (r^2 E_r)}{\partial r} + \cancel{\frac{1}{r \sin\vartheta} \frac{\partial}{\partial \vartheta}(A_\vartheta \sin\vartheta)}^{\,0} + \cancel{\frac{1}{r \sin\vartheta} \frac{\partial A_\phi}{\partial \phi}}^{\,0}$

· Per teorema di Gauss:

$\rho(r) = \varepsilon_0 \vec{\nabla} \cdot \vec{E}$

$\boxed{\rho(r)} = \varepsilon_0 \vec{\nabla} \cdot [E(r) \hat{u}_r] = \varepsilon_0 \frac{1}{r^2} \frac{\partial}{\partial r} [k r^3 \cdot r^2] =$

$= \varepsilon_0 \cdot \frac{1}{\cancel{r^2}} k \cdot 5 r^{\cancel{4}\,2} \boxed{= 5 k \varepsilon_0 r^2}$

b) Modo 1: $Q(R) = \int_V dV \rho = \int_0^R dr\, r^2 \int_0^\pi d\vartheta \sin\vartheta \int_0^{2\pi} d\varphi\, \overset{5K\varepsilon_0 r^2}{\rho(r,\vartheta,\varphi)} = \int_0^R dr\, r^2\, 5K\varepsilon_0 r^2 \overbrace{\int_0^\pi d\vartheta \sin\vartheta}^{2} \overbrace{\int_0^{2\pi} d\varphi}^{2\pi} =$

$$= 4\pi \cdot 5K\varepsilon_0 \int_0^R dr\, r^4 = 20K\pi\varepsilon_0 \frac{R^5}{5} = 4K\pi\varepsilon_0 R^5$$

Modo 2: Gauss? $Q(R) = \varepsilon_0 \oint_S d\sigma\, \underbrace{\vec{E} \cdot \hat{n}}_{=E_r(R)} \overset{\text{SFERICHE } (\hat{n} = \hat{u}_r)}{\overset{\uparrow}{=}} R^2 \varepsilon_0 \underbrace{\int_0^\pi d\vartheta \sin\vartheta}_{2} \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} E_r(R) = 4\pi R^2 \varepsilon_0 K R^3 =$

$$= 4\pi \varepsilon_0 K R^5$$

Oss: chiaramente potevamo usare Gauss in maniera + intelligente:

$\vec{E}$ é RADIALE e dipende solo da R → é costante su una superficie sferica → Flusso = campo · Area

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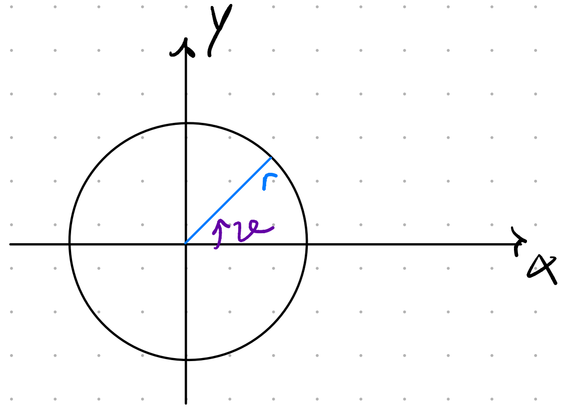
IDEFIX 1

1.0 Area di cerchio raggio R



$$\begin{cases} x = r\cos\vartheta \\ y = r\sin\vartheta \end{cases}$$

$$\begin{cases} r = \sqrt{x^2+y^2} \\ \vartheta = \arctan\left(\frac{y}{x}\right) \end{cases}$$

POLARI

$$A = \int_0^R dr\, r \overbrace{\int_0^{2\pi} d\vartheta}^{2\pi} \cdot 1 = \cancel{2}\pi \frac{r^2}{\cancel{2}} \Big|_0^R = \pi R^2$$