An Introduction to Partial Identification and Causal Bounds

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Introduction

Our goal is to make critical decisions (e.g., *which drug is best?*) in complex medical settings.

The Problem: Why purely data-driven models could fail?

- **Insufficient Data:** Clinical data is often sparse, low-quality, expensive to collect, and may not represent the real-world population.
- **Opaque Mechanisms:** The true biological mechanisms of a disease are incredibly complex and full of *unobserved* factors (e.g., genetics, lifestyle, co-morbidity).

Our Goal: A More Realistic Approach

- Use assumptions that are transparent and realistic for the healthcare setting.
- Formally incorporate all the information we have.

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Part 1 - Partial identification

Average Treatment Effect (ATE)

A central goal in causal inference is to quantify the effect of an intervention or treatment on an outcome.

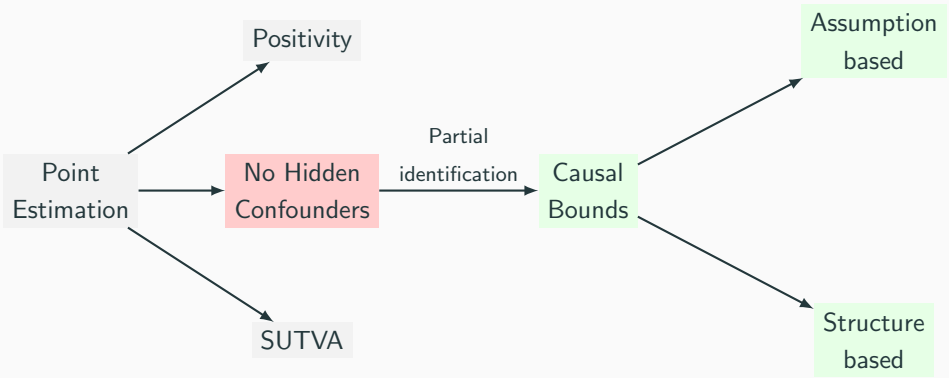
The **Average Treatment Effect (ATE)** measures the expected difference in outcomes if all individuals in a population were treated versus not treated:

$$\tau = \mathbb{E}[Y(1)] - \mathbb{E}[Y(0)]$$

where:

- $T \in \{0, 1\}$ is the treatment variable,
- $Y(1)$ and $Y(0)$ are the potential outcomes under treatment and control.

Causal Effect Map

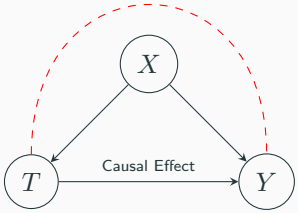


No Hidden Confounders

Assumption

Is the claim that we have successfully measured and "adjusted for" all common causes (X) that influence both the treatment (T) and the outcome (Y)

$$Y(0), Y(1) \perp\!\!\!\perp T \mid X$$



Assumption fail

This assumption fails when there is at least one hidden confounder (U)—a variable that also affects both T and Y, but which we did not (or could not) measure.

$$Y(0), Y(1) \not\perp\!\!\!\perp T|X$$

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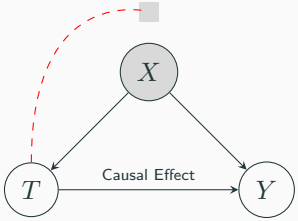

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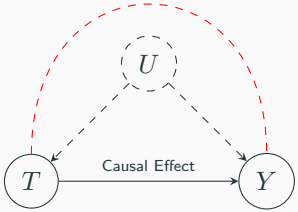

Moving from Points to Bounds

If we relax the "No Hidden Confounders" assumption, we can no longer identify a single point value for the CATE. Instead, we identify a **range of possible values**, known as bounds.

$$CATE \in [CATE^L, CATE^U]$$

The width of this interval shows us the *price* we pay (in ambiguity) for not making strong, untestable assumptions.

Part 1.1 - Assumption based

Partial Identification: The Worst-Case Scenario

What if we make no assumptions about the confounder?

Concept (Manski, 1990) [Man90]

- Instead of making a strong, untestable assumption (like "no unobserved confounding" or "ignorability"), we make *only* a minimal, verifiable assumption.
- **Minimal Assumption:** The outcome Y is bounded.
  - For this presentation, we assume Y is a binary outcome: $Y \in [0, 1]$.
- **Method:** We use the observed data and these logical bounds to calculate the widest possible range for the true CATE.
- This range represents all possible values of the CATE that are logically consistent with the data, no matter how the hidden confounder U behaves.

Bounding $\mathbb{E}[Y(1)|x]$ (Effect of Treatment)

$$\mathbb{E}[Y(1)|x] = \mathbb{E}[Y(1)|T = 1, x]P(T = 1|x) + \mathbb{E}[Y(1)|T = 0, x]P(T = 0|x)$$

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Observed

Counterfactual

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Observed Counterfactual

Counterfactual Term: we substitute its logical bounds: $[0, 1]$.

- $\mathbb{E}^L[Y(1)|x] = \mathbb{E}[Y|T=1,x]P(T=1|x) + (0) \cdot P(T=0|x)$
- $\mathbb{E}^U[Y(1)|x] = \mathbb{E}[Y|T=1,x]P(T=1|x) + (1) \cdot P(T=0|x)$

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Observed — Counterfactual

Bounding $\mathbb{E}[Y(0)|x]$ (Effect of Control)

$$\mathbb{E}[Y(0)|x] = \mathbb{E}[Y(0)|T=1,x]P(T=1|x) + \mathbb{E}[Y(0)|T=0,x]P(T=0|x)$$

Similarly, we substitute $[0,1]$ for the unknown $\mathbb{E}[Y(0)|T=1,x]$.

- $\mathbb{E}^L[Y(0)|x] = (0)\cdot P(T=1|x) + \mathbb{E}[Y|T=0,x]P(T=0|x)$
- $\mathbb{E}^U[Y(0)|x] = (1)\cdot P(T=1|x) + \mathbb{E}[Y|T=0,x]P(T=0|x)$

Worst-Case Scenario: Computation & Bounds

Bounding $\mathbb{E}[Y(1)|x]$ (Effect of Treatment)

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Bounding the CATE

- $CATE^L(x) = \mathbb{E}^L[Y(1)|x] - \mathbb{E}^U[Y(0)|x]$
- $CATE^U(x) = \mathbb{E}^U[Y(1)|x] - \mathbb{E}^L[Y(0)|x]$

Monotone Treatment Response (MTR)

Concept (Manski, 1997) [Man97]

Monotone Treatment Response (MTR) Assumption

The **Monotone Treatment Response (MTR)** assumption states that the treatment can only help or have no effect; it can never harm.

For every individual unit i:

$$Y_i(1) \geq Y_i(0)$$

The individual treatment effect τ_i is always non-negative ($\tau_i \geq 0$).

Plausibility

- This is not a statistical assumption, but one based on **domain knowledge**.
- *Example:* A job training program is unlikely to make someone *less* employable.
- This single assumption dramatically shrinks the bounds.

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MTR Scenario: Computation & Bounds 1/4

MTR $Y(1) \geq Y(0)$ gives us new information on the counterfactuals:

- $\mathbb{E}[Y(1)|T=0,x] \geq \mathbb{E}[Y(0)|T=0,x] = \mathbb{E}[Y|T=0,x]$
- $\mathbb{E}[Y(0)|T=1,x] \leq \mathbb{E}[Y(1)|T=1,x] = \mathbb{E}[Y|T=1,x]$

MTR Scenario: Computation & Bounds 2/4

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Bounding $E[Y(1)|x]$ **Bounding** $E[Y(0)|x]$

$$\mathbb{E}[Y(1)|x] = \mathbb{E}[Y(1)|T=1,x]P(T=1|x) + \mathbb{E}[Y(1)|T=0,x]P(T=0|x)$$

The new lower bound for the counterfactual is $\mathbb{E}[Y|T=0,x]$ (not 0).

- $\mathbb{E}^L_{MTR}[Y(1)|x] = \mathbb{E}[Y|T=1,x]P(T=1|x) + \mathbb{E}[Y|T=0,x]P(T=0|x)$
- $\mathbb{E}^U_{MTR}[Y(1)|x] = \mathbb{E}[Y|T=1,x]P(T=1|x) + (1)\cdot P(T=0|x)$

MTR Scenario: Computation & Bounds 3/4

MTR $Y(1) \geq Y(0)$ gives us new information on the counterfactuals:

- $\mathbb{E}[Y(1)|T=0,x] \geq \mathbb{E}[Y(0)|T=0,x] = \mathbb{E}[Y|T=0,x]$
- $\mathbb{E}[Y(0)|T=1,x] \leq \mathbb{E}[Y(1)|T=1,x] = \mathbb{E}[Y|T=1,x]$

Bounding $E[Y(1)|x]$ **Bounding** $E[Y(0)|x]$

$$\mathbb{E}[Y(0)|x] = \mathbb{E}[Y(0)|T=1,x]P(T=1|x) + \mathbb{E}[Y(0)|T=0,x]P(T=0|x)$$

The new upper bound for the counterfactual is $E[Y|T=1,x]$ (not 1).

- $\mathbb{E}^{L}_{MTR}[Y(0)|x] = (0) \cdot P(T=1|x) + \mathbb{E}[Y|T=0,x]P(T=0|x)$
- $\mathbb{E}^{U}_{MTR}[Y(0)|x] = \mathbb{E}[Y|T=1,x]P(T=1|x) + \mathbb{E}[Y|T=0,x]P(T=0|x)$

Bounding the CATE

- $CATE^{L}_{MTR}(x) = \mathbb{E}^{L}_{MTR}[Y(1)|x] - \mathbb{E}^{U}_{MTR}[Y(0)|x] = E[Y|x] - E[Y|x] = \mathbf{0}$
- $CATE^{U}_{MTR}(x) = \mathbb{E}^{U}_{MTR}[Y(1)|x] - \mathbb{E}^{L}_{MTR}[Y(0)|x]$

Result: The CATE is bounded in the interval $[0, CATE^{U}_{MTR}(x)]$.

Comparison of Identification Strategies

Strategy	Assumption(s)	Result
Point Identification	**No hidden confounders:** $Y(t) \perp T \mid X$	**Point Estimate:** $CATE(x))$
Worst-Case (Partial ID)	**Bounded Outcome:** $Y \in [0, 1]$	**Wide Interval:** $[CATE^L(x), CATE^U(x)]$ (Correct, but often uninformative)
MTR (Partial ID)	**Bounded Outcome** + **MTR:** $Y(1) \geq Y(0)$	**Narrowed Interval:** $[0, CATE^U_{MTR}(x)]$ (Rules out negative effects)

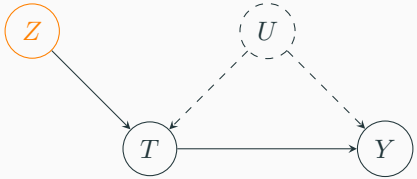
Part 1.2 - Structure based

Instrumental Variable (IV) 1/2

Instrumental Variable

An Instrument Z is a variable that can "isolate" the unconfounded part of the treatment T.

1. **Relevance:** The instrument Z must have a causal effect on the treatment T.
2. **Exclusion:** Z only affects the outcome Y *through* the treatment T. It has no direct path to Y.
3. **Independence (Ignorability):** Z is independent of all unobserved confounders U.



Instrumental Variable (IV) 2/2

Assumption: Linear outcome

$$Y := \delta_t T + \alpha_u U$$

$$\mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0] \neq \delta_t$$

$$\mathbb{E}[Y|Z=1] - \mathbb{E}[Y|Z=0] = \alpha_z \delta_t$$

$$\mathbb{E}[T|Z=1] - \mathbb{E}[T|Z=0] = \alpha_z$$

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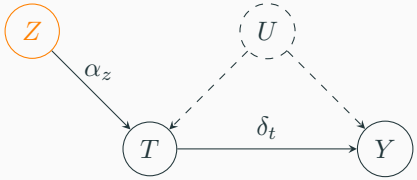

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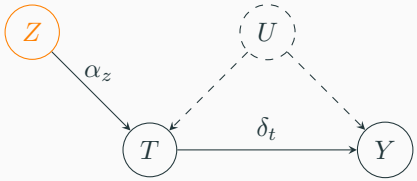

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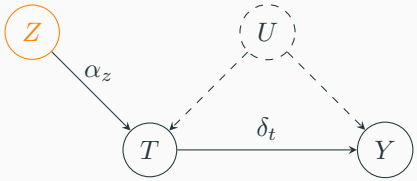

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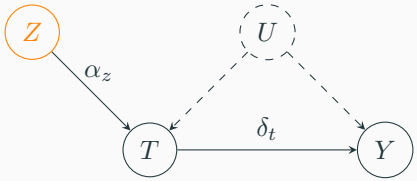

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No Non-parametric Identification

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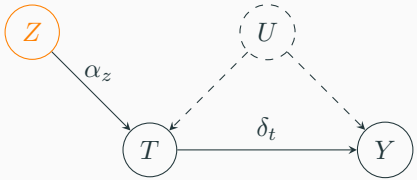

"Normal" Mediator (Front-Door)

Core Assumptions:

1. M "blocks" the entire effect of T on Y. (No direct $T \to Y$ path).
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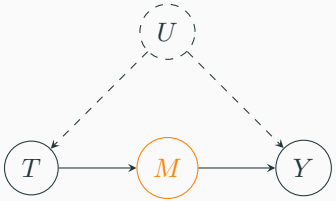

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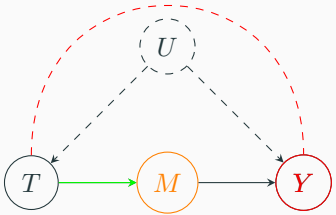

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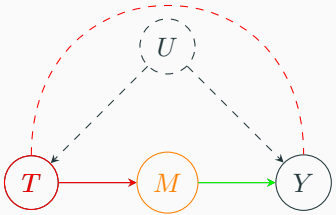

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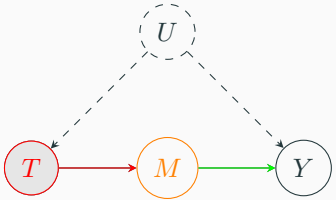

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Stochastic Causal Programming

It turns causal bounding into a **constrained optimization problem** [Pad+23].

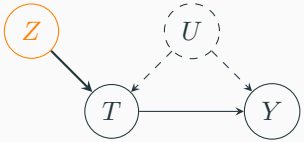
The Causal Mathematical Program:

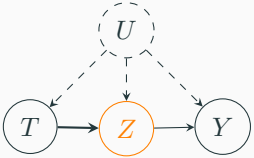
$$\begin{aligned} \min / \max \quad & o(S) = \mathbb{E}[Y|do(T=t)] \\ \text{subject to} \quad & dist(p_S, \hat{p}) \leq \epsilon, \\ & \text{structural constraints} \end{aligned}$$

1. **The Data Constraints**: any "possible" model S must generate a distribution p_S that is "close" to our observed data $\hat{p}$.
2. **Structural Constraints**:The model S must obey our causal graph (e.g., in IV, Z does not directly cause Y).

The Simplified SCM

Standard SCM (Hard Problem)





Simplified SCM (The Assumption)

We *assume* noise is separable. Confounding is just a correlation between two "private" noises.

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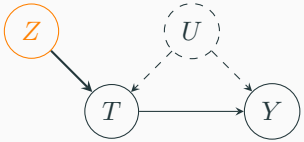
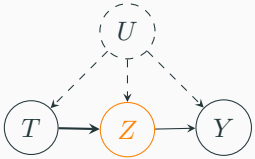
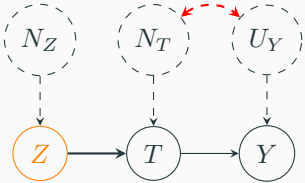
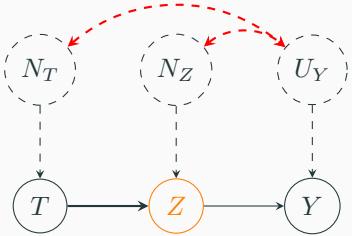




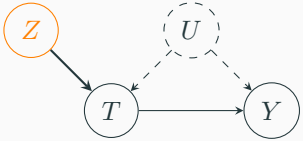
Simplified SCM (The Assumption)

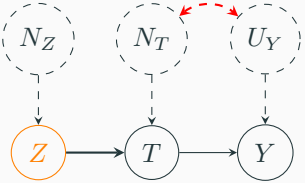




We *assume* noise is separable. Confounding is just a correlation between two "private" noises.

Example of the simplified SCM





- Z: Price of food in patient area.
- T: Daily Calorie Intake.
- Y: Heart Disease.
- N_T: Personal metabolism. This is a biological factor.
- U_Y: Genetic predisposition for heart disease.

The Problem with Bounds: How to Make them Useful?

The Core Problem

Without strong assumptions, these bounds are often mathematically correct but too wide to be informative.

We can leverage **domain expertise**. But experts don't think in precise equations. They use qualitative, linguistic rules:

> *"If the patient's inflammation (U) is high, the drug (T) is less likely to be prescribed."*

The New Challenge: How do we formally translate this vague, qualitative knowledge into quantitative constraints for our SCP? Part of the answer is **Fuzzy modeling**

Fuzzy Logic [Zad88]

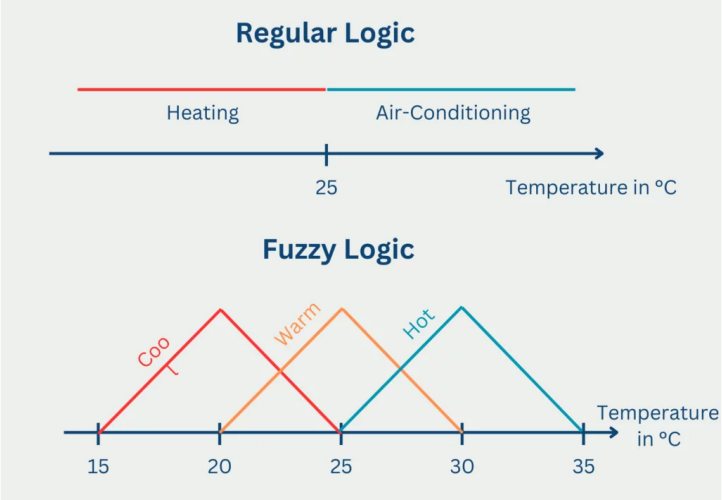


Figure from https://databasecamp.de/

Fuzzy Model 1/4 [Dub80]

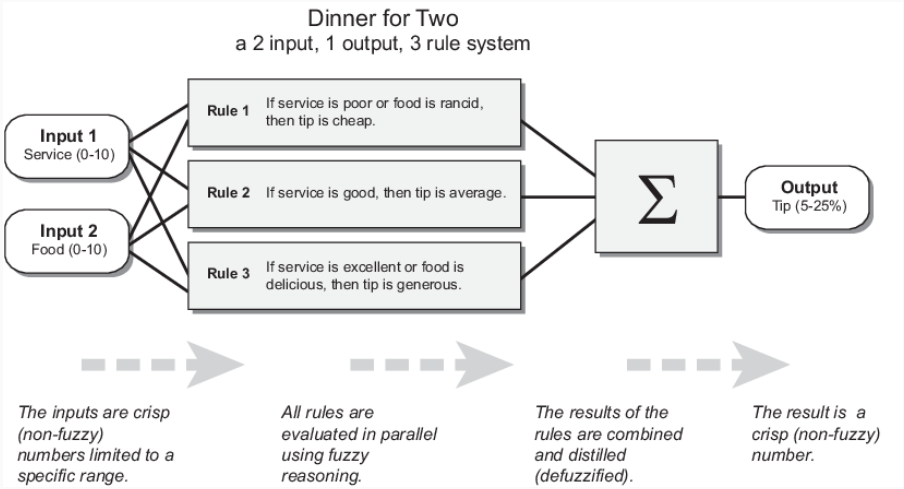
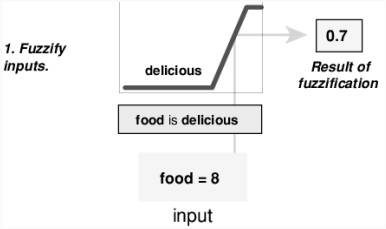
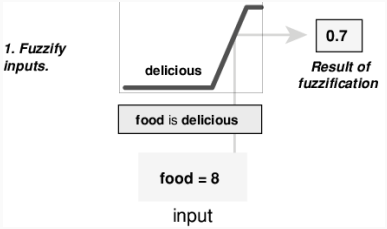


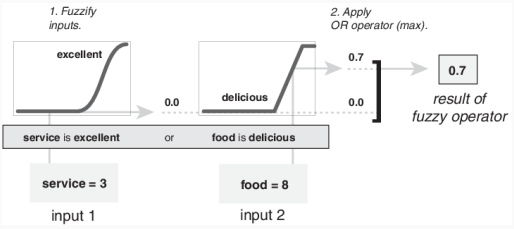
Figure from https://it.mathworks.com/

Fuzzy Model 2/4

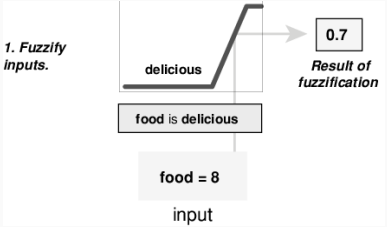


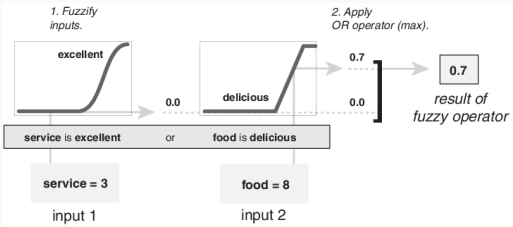
Fuzzy Model 2/4

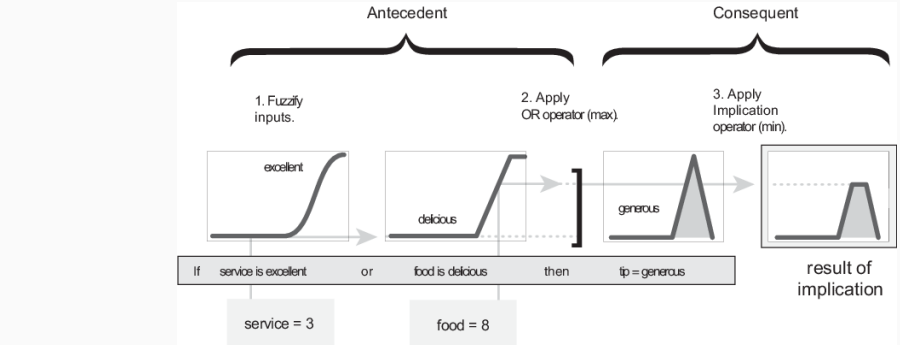




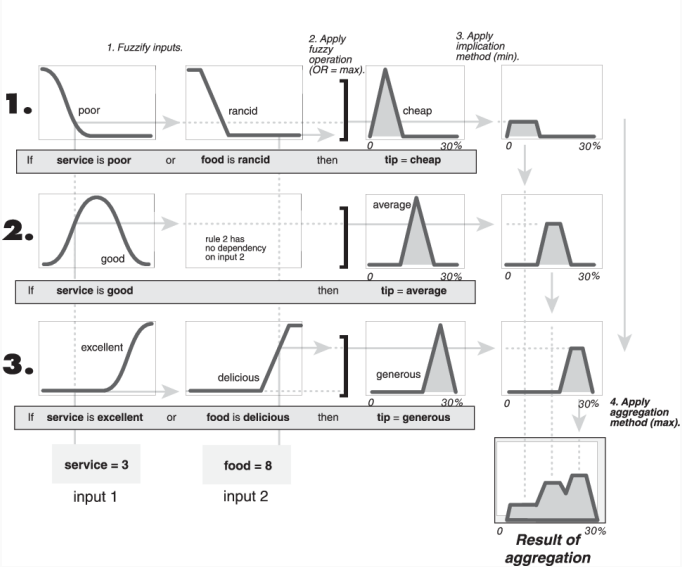
Fuzzy Model 2/4







Fuzzy model 3/4



Fuzzy model 4/4

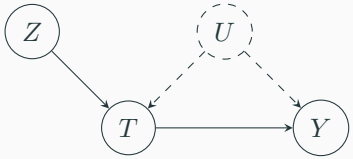
Fuzzy models are uniquely suited for encoding the **domain knowledge** of human experts.

Based on Natural Language:
Experts don't think in precise equations. They use qualitative, linguistic rules. Fuzzy logic's IF-THEN rules directly map to this human-like reasoning.

Handles and Imprecise Knowledge:
Human expertise is full of terms like *"about"*, *"near"*, *"low"*, *"very fast"*. Membership functions provide a formal way to define these vague concepts, which crisp logic cannot.

Incorporate expert knoweldge into causal inference

$$\begin{aligned} \min / \max \quad & o(S) = \mathbb{E}[Y|do(T=t)] \\ \text{subject to} \quad & dist(p_S, \hat{p}) \leq \epsilon, \\ & \text{structural constraints,} \end{aligned}$$



We use expert knowledge to **constrain** the "space of possible models" we are searching over.

Main aspect:

- Computational time: what is the computational cost of adding new constraints?
- Knowledge bias: what happens if our knowledge is incorrect?
- How to transform qualitative knowledge into quantitative knowledge?

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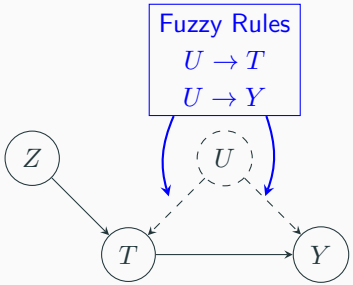

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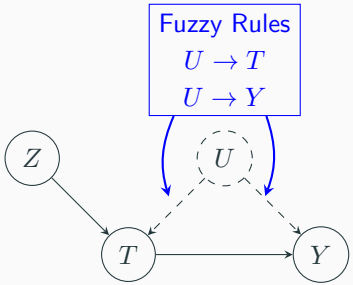

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Part 2 - POMDP & Myasthenia Gravis

Myasthenia Gravis (MG)

- Myasthenia gravis (MG) [Dre+21] is a **neurological autoimmune disorder** characterized by muscle weakness and fatigue.
- Abnormal **autoantibodies** that **attack muscle contraction** proteins.
- **Biological drugs** like Ravulizumab are a final fallback to ease MG symptoms, especially for patients **who do not respond** to standard treatments.

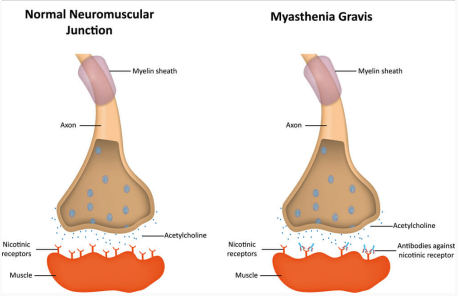


Figure 1: *

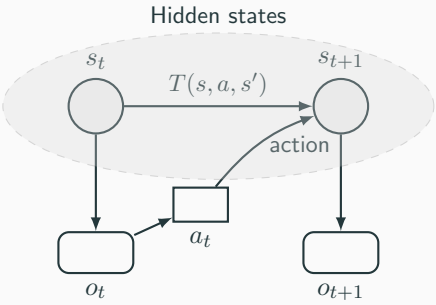
Normal versus Myasthenia Gravis neuromuscular junction.

Source: *Myasthenia Gravis: Etiology*, The Rare Disease Advisor.

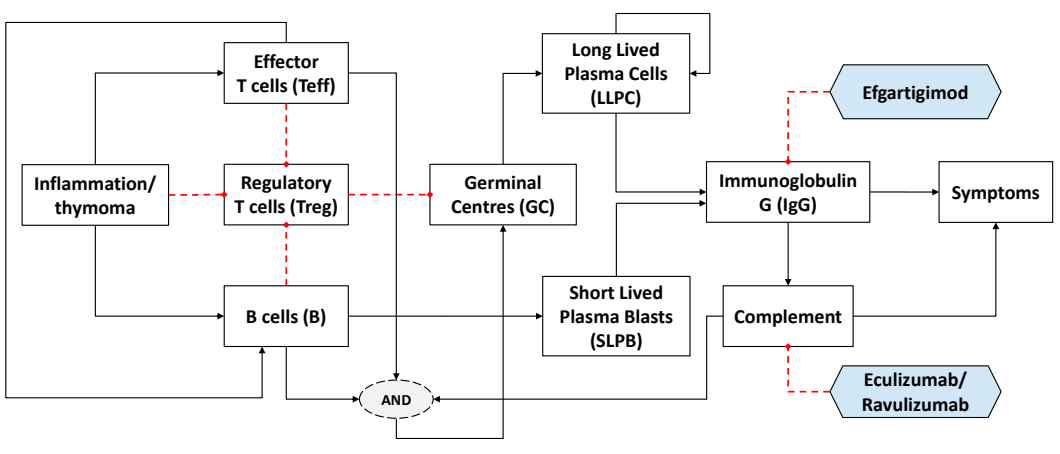
Partially Observable Markov Decision Processes

A **POMDP** [CKL94] models **sequential decision-making under uncertainty**, where the true state of the system (e.g., a patient condition) is **partially hidden**.

- **States (S):** latent clinical conditions (e.g., *Mild*, *Severe*)
- **Actions (A):** possible interventions (e.g., *Treat*, *Wait*)
- **Transition model (T):** $T(s, a, s') = P(s_{t+1} = s' \mid s_t = s, a_t = a)$
- **Observation model (O):** $O(o_t|s_t) = P(o_t \mid s_t)$

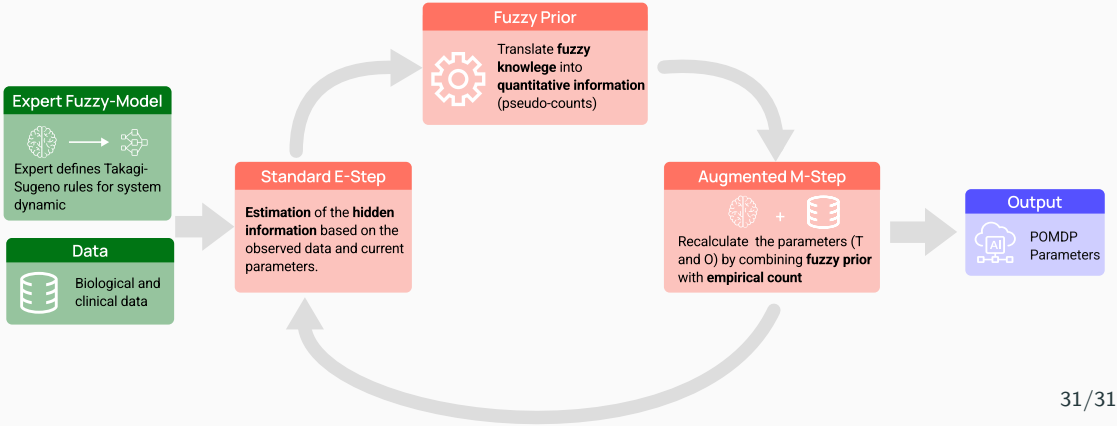


Fuzzy model of Myasthenia Gravis (MG)



Fuzzy-MAP EM workflow

We developed a formal way to **learn the parameters** of the POMDP using the **data** (*standard Expectation-Maximization*) and the **expert knolwedge** coming from the fuzzy model.



Thank You for your Attention!

MADLab

The Models and Algorithms for Data & Text Mining Laboratory is a research lab at University of Milano-Bicocca focused on Causal Networks, Bayesian Networks and Continuous-Time Bayesian Networks applied to Healthcare and Medicine.

More at: https://mad.disco.unimib.it/

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