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‘Mathematical Law’

SUMMARY: 1. Premise. – 2. Meaning and concept of the syntagma ‘mathematical law’. – 3. ‘Mathematical law’ and the positive legal system. – 4. ‘Digital legistics’ and ‘digital law’. – 5. Final summary.

1. *Premise*

Observing present-day legistics and endeavouring to hypothesize as to its future evolution, the first thought that comes to mind is that such an evolution can only be digital¹. This perspective raises various questions: are legal provisions (which constitute the objects of current legistics) endowed with ‘mathematical truths’ or in any case classifiable by means of the conceptual types peculiar to ‘mathematical truths’? Can they, therefore, be produced (as well as interpreted and applied) by ‘mathematical words’, that is, by means of mathematical methods, concepts and symbols (not numbers)? Can a ‘mathematical’ type of law thus be derived from the present positive legal system?

If a law of this kind were distinguishable, could it form the “base analysis” of a futuristic ‘digital legistics’ that could be used for the creation (and application) of a science-fiction ‘digital law’?

These doubts point indirectly to the object of the analysis by means of which I shall try, here, to project the present into the possible future.

In fact, turning to the first group of questions, I shall attempt to find and demonstrate some signs of the ‘mathematical law’ that can be extracted empirically from current provisions and thus lay the foundations on which to develop a predictive evaluation concerning the last and most uncertain of the doubts listed above.

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¹ Slightly integrated and modified text of the paper on “*The evolution of fiscal law-making - La légistique fiscale en progression*” presented at the conference held in Bologna on 13 November 2015 on the theme “*Modernizing tax - Modernizer l’impôt*”.

There is no room here for even a mention of the gigantic and innumerable limits to an attempt of this kind. Nor, obviously, is it possible to give an account of the vast and profound literature which has, in the past (though in technological contexts very different from the present-day ones), examined the relationships between law, numbers and mathematics. Nevertheless, it may perhaps be a consolation to observe that the very limited objective pursued is to show, by way of exemplification, some of the relations existing between current law and mathematical truths, words, methods, symbols and concepts. Moreover, the research will be carried out without examining the legal concepts of general theory involved, because their highly debatable nature would require more ample space. Thus, to make the research possible, such concepts are taken for granted, without any dispute, within the confines and contents delineated in previous research by the author (and so written in 'single commas') and are utilized as variables or object entities (susceptible to substitution by entities with different confines and/or contents) whose role is (simply) to contribute to showing the functioning of the hermeneutical method. The aim is to demonstrate, or at least to justify, the capacity (in an abstract sense) of the method itself to observe both the real 'mathematical nature of law' which exists and its consequent aptitude to manage in a rigorous manner concepts that are far removed from mathematics inasmuch as extremely debatable, or in other words, 'endlessly controvertible'.

2. Meaning and concept of the syntagma 'mathematical law'. Reasons for the use of this oxymoron

In this perspective it is possible to understand the sense of the "spiral combination" (= 'combining') of terms commonly thought to be opposites; in effect, in the hypothesis proposed, the syntagma 'mathematical law' expresses an existing reality (actually in continuous evolution) which however has no name simply because by convention it is not admitted.

In fact, it is commonly held that the debatable nature of law is incompatible with the non-debatable nature of mathematics and that mathematical logic cannot be identified with the logic of law. To the extent that, although the relations between law, numbers and mathematics have long been investigated, it seems that, at least recently, the meaning and the concept of 'mathematical law' have not been defined.

On the other hand, if one proposes to speak of something one is looking for, it is first necessary to reflect on the meaning and/or concept which,

at least in approximate terms, distinguishes the thing one intends speaking about.

With regard to the meaning, the sense that can be derived from the syntagma 'mathematical law' is, in the first instance, that of 'law that can be produced or applied by mathematical truths, words (not numbers), methods, concepts and symbols'.

Then on the conceptual level, 'mathematical law' can be detected primarily in 'laws with truth' (like, for example, taxation law, penal law, bankruptcy law, legal obligations in private law, etc), that is, in those sectors of law which imply an (apophantic) relation between the legal provision and its (given) parameter of verification and which imply, therefore, a continuous and natural 'combining' by the 'man of law' (legislator, citizen, official or judge) of an 'ethical' type with the 'scientific' type of truths incorporated in the legal provisions.

3. *'Mathematical law' and the positive legal system*

The observations in the following pages and, above all, the many combinations and the many logical and analogical 'combinings' presented in the following tables seem to suggest that 'mathematical law' is a part, characteristic and an innate quality of the positive legal system but it has to be found and extracted from the single legal provisions by means of specific hermeneutical techniques.

One of these techniques, that which is proposed here, poses as a premise, or rather, 'combines' before all else, 'mathematical truths' and the 'truths of law'. To use a metaphor, it appears that what happens to 'mathematical law' is the same as what happens to a part or a characteristic or a quality present in a mineral, the possession of which can be gained only by the use of specific techniques of extraction which recognize a priori the 'natural truths' present in that mineral. Paraphrasing and correlating 'mathematical truths' and 'natural truths', one could say that 'mathematical law' : legal provision = ability to memorize data : silicon.

If one adopts this point of view, it becomes simple to note that, in effect, just as mathematics has its truths, so law has its truths. But are the types of 'mathematical truths' similar to the types of 'law truths'? In other words, are there analogical determinants between the two typologies of truths?

Observing current law, it would appear that determinants of this kind exist and that in fact they function at both a material and procedural level.

In effect, on the material level, it is evident that, as in mathematics, the bulk of current law is 'with truth' (and not 'without truth'). This is because the change which has taken place in the social, economic and legal environment seems long to have undermined the validity of Hobbes' general premise to law, namely that "it is authority and not truth that makes law". Indeed, contrary to Hobbes' affirmation, today laws must be true because they have to sustain confrontation with external parameters (constitutional, Communitarian, international...); to the extent that they are true, that is, valid, if their variables move in the set of solutions (or environment) delimited by the given parameter and false, that is, invalid, if their variables do not move in that set (or environment).

Going on to the procedural level, it seems possible to ascertain that the 'man of law' (legislator or citizen or official or judge), in interpreting or applying a legal provision 'with truth' or proving the truth of a fact, in effect performs a reasoning aimed at finding a/the solution/s that makes/make a statement contained in the text of the provision true (like a mathematical relation). This way of reasoning is therefore also methodologically analogous (if not identical) to that of the 'mathematical person' trying to find x (for example in equations or inequalities).

In short, there are analogical determinants, both material and procedural, which imply (should there be a defectiveness of legislation and compatibility) the applicability to law, in analogical terms, of mathematical truths, words, methods, concepts and symbols.

Now, hopefully to communicate more clearly and rapidly both the analogical correlations which exist between the types of 'mathematical truths', the types of 'philosophical-theoretical truths' and the types of 'legal (normative or cognitive) truths', and the hermeneutical consequences which derive from them in distinguishing and defining the characteristic traits of 'mathematical law', some tables are presented to illustrate, albeit with all the limits intrinsic to the cold immediacy of tables, some of the numerous 'combinings' that characterize 'mathematical law' (and which cannot now be treated in depth).

As can be seen in *Table 1*, the starting point of the argument is the exhaustive definition of the conceptual types of 'mathematical truths' expressed by four mathematical symbols: 'identity truth' represented by the symbol ($\equiv$); 'punctual truth' represented by the symbol ($\cdot$); 'interval truth' represented by the symbol ($—$); 'impossible truth' (in one set but possible in another) represented by the symbol ($\emptyset$). The first ($\equiv$) is an absolute truth (or ideal) which, if included in a legal provision, does not require a conjectural activity of research for the solution. It is that

Table 1

Mathematical conceptual types	Conceptual types of 'mathematical truths'	Solutions
($\equiv$)	'identity truth'	The problem of seeking a solution does not exist (always true)
($\cdot$)	'punctual truth'	Only one
($-$)	'interval truth'	One, more than one, infinite
($\emptyset$)	'impossible truth in one set and not impossible in a different set'	None One, more than one, infinite

which, from this point of view, is present in the things that everyone already knows before even thinking about them. The second ($\cdot$) is a truth deduced through conjecture from a relation to which in practice, or by convention, there pre-exists only one solution. The third ($-$) is also a conjectural truth, however it is a truth that is in part deduced and in part selected among the multiple or infinite possible solutions comprised in a given interval, all just because internal or equal at the extremes, even if different from each other. Finally the fourth ($\emptyset$) is the truth which it is impossible to find at least in one set (in which there are no predetermined solutions) but which may be found in a different set; in this case, for the limited conceptual ends that interest law, the truth found in a different set can be called a 'substitutive truth' which, it should be underlined, from the conceptual point of view may be of the 'equivalent' type, if a proportional equality exists, or of the 'alternative' type if there is no proportional equality.

So, having distinguished the exhaustive types of 'mathematical truths', and the types of mathematical symbols that express the concepts involved, Table 2 undertakes to render explicit both the kinds of sets containing the solutions that make the legal enunciations comprised in the provisions true, and the kinds of subjects that can accede to these sets. The first three types of mathematical truth, ($\equiv$), ($\cdot$) and ($-$), which correspond respectively, in philosophical-theoretical terms, to "identity truth", "correspondence truth" and "coherence truth", all refer to the set of unilaterality to which all types of 'men of law' (legislator, citizen, official or judge) can obviously accede. In fact there are truths that can be arrived at unilaterally by any 'man of law' who reasons with himself without committing errors. When, on the contrary, it is a question of substitutive truth

Table 2

Mathematical conceptual	Mathematical truths'	'Set'	'Man of Law'	'Philosophical- theoretical legal truths'
($\equiv$)	'identity'	'unilaterality'	'all'	«identity»
($\cdot$)	'punctual'	'unilaterality'	'all'	«correspondence»
($-$)	'interval'	'unilaterality'	'all'	«coherence»
($\emptyset$)	'substitutive'	'bilaterality' 'impartiality'	parties judge	«consensus» 'equity'

(because the set of unilaterality is empty – $\emptyset$ –), the reference sets become respectively that of bilaterality (to which “consensus truth” refers and to which the ‘man of law’- legislator, citizen or official – can accede) or that of impartiality (to which “ethical-equity truth” refers and to which the judge-‘man of law’ can accede).

There follows, in conceptual continuity, *Table 3*, which indicates the types of legal provisions that refer to ‘legal truths’ considered in their analogical relationship to ‘mathematical truths’. This table, after giving some examples of legal facts or legal concepts on which each of the four exhaustive types of legal provision may respectively be founded, also illustrates, with regard to the public ‘man of law’, the types of *forma mentis*, or *ratio decidendi*, implied respectively by each type of legal provision, corresponding to the different types of legal regime that each type of *forma mentis* demands to combine with. Legal provisions with the ($\equiv$) code, that is, ‘absolutely incontrovertible’, imply ‘absolute unavailability (zero-discretion)»; legal provisions with the ($\cdot$) code, that is ‘in practice incontrovertible’ (where ‘in practice’ means that any roundings off are irrelevant to law) imply ‘punctual unavailability (intractability)»; the other legal provisions with the ($-$) code, that is, ‘controvertible within certain limits’ (because given by the legislator) imply ‘interval unavailability’ (that is, the discretionary power to establish a solution between limits given by law); lastly, the legal provisions with the ($\emptyset$) code, that is, ‘endlessly controvertible’, imply ‘reverse unavailability’ from which follows the ‘obligatory’ research within the set of consensus of a ‘solution that satisfies both parties as regards the principle of the «lesser evil»’ or, if no such solution is found in that set, the ‘obligatory’ deliberation of an impartial judge in accordance with ‘ethical equity’ (that is, in accordance with the principle ‘of proportional minimization, for both parties, of the legally just evil’).

Table 3

Mathematical conceptual types	Types of 'legal provisions with truth'	Example of legal fact	Example of legal concept	Type of legal reasoning
(=)	'absolutely incontrovertible'	result of the sum of two numbers	51% = majority	absolute unavailability (= zero discretionality)
(·)	'in practice incontrovertible'	calculation of the area of a piece of land	degree of kinship	punctual unavailability (= zero discretionality)
(—)	'controvertible within certain limits'	evaluation of final inventories on the basis of legal intervalled values	calculation of a sanction between min and max established by law	interval unavailability (discretionality)
(Ø)	'endlessly controvertible'	fair market value or serious breach		reverse unavailability (= zero discretionality in one set and discretionality in a different set)

To conclude, *Table 4*, while interrupting the conceptual continuity with the previous tables because of the need for brevity, combines mathematical conceptual symbols both with the types of 'legal truths' and the types of normative and epistemological connections, that is, connections that are efficient, respectively, in the future and in the past. These connections are fundamentally of two kinds, one 'scientific' ('scientific probability') and the other 'ethical' ('ethical normality'). The type of connection that can be used for purposes of implementation, if instrumental in determining the type of justice, certainty, efficiency and effectiveness peculiar to each legal provision, depends however on the type of legal uncertainty that the application of each provision implies. This uncertainty can be divided first in binary terms into subjective and objective typologies, and then divided again into the types corresponding to the mathematical types outlined above on the basis of the kind of legal reasoning ontologically suitable to overcoming the type of uncertainty that occurs in each instance. If the legal provision contains the (=) code, the subjective uncertainty that

Table 4

Mathematical conceptual types	'Legal truths'	Normative and epistemological type	Type of legal uncertainty	Type of superability	Type of legal efficacy
(=)	"identity"	'scientific'	'subjective' determinable 'never negotiable'	'always'	descriptive
(·)	"correspondence"	'scientific'	'subjective'	'always ascertainable' 'never negotiable'	descriptive
(—)	"coherence"	'ethical'	'objective'	'always ascertainable' 'never negotiable'	constitutive
(Ø)	"consensus" 'ethical equity'	'ethical' 'ethical'	'objective'	'never determinable or ascertainable' 'always negotiable'	constitutive

characterizes it can be overcome via 'mere knowledge' (that is, via settlement) which has mere declaratory efficacy; in the case of 'laws with truth', this prevents assessment, mediation, conciliation or deliberation and excludes constitutive efficacy. If, on the contrary, the legal provision contains the (·) code, the legal uncertainty, although still subjective, can be overcome via a precise assessment having descriptive (or declaratory) efficacy: this, in 'laws with truth', prevents settlement, mediation, conciliation or deliberation and excludes constitutive efficacy; if, then, the legal provision contains the (—) code, the legal uncertainty is objective but can be overcome via an alternate assessment with constitutive efficacy; in 'laws with truth', this again prevents settlement, mediation, conciliation or pure deliberation and excludes descriptive (declaratory) efficacy. If, finally, the legal provision contains the (Ø) code, the legal uncertainty is objective and can only be overcome via (consensual) negotiation/mediation/conciliation or (authoritative) deliberation, all having constitutive efficacy; this ontologically prevents settlement and assessment and excludes descriptive (declaratory) efficacy.

In conclusion, even a summary and superficial exposition like the one conducted so far would seem to demonstrate the existence of 'mathematical law' in the legal system, understood in the sense defined above, both as regards meaning and concept. Therefore, the doubts expressed in the

first group of questions at the beginning of this discussion would appear to be positively surmountable.

It is time now to turn to the last of the questions posed at the onset, objectively the most difficult to face in terms of probability, or at least of possibility.

4. *'Digital legistics' or 'digital law'*

It is necessary to imagine, first of all, a science fiction reality in which a futuristic 'digital legistics' is used to create a 'digital law' discovered and/or elaborated by a 'man of law' endowed with 'legislatorial software' able to generate and write 'digital laws' destined to be interpreted and applied (perhaps at the option of the person to whom they are addressed) by a 'man of law' endowed with 'implementary software' which is functionally complementary to the 'legislatorial software'.

For that matter, observing the reality in which we live, it can be seen that some publishers of magazines and newspapers already use software which, with just a few factual elements furnished by the "author", produces digital texts (that is, articles) that are then published. Hence, if we already have software able to relate or illustrate by itself (without going over or under a given number of characters) the simple, single facts introduced into the computer by a journalist, what stops us from thinking that all this may expand fairly quickly to include the utterances (laws, acts, regulations and sentences) of the 'man of law'?

Probably nothing immediately evident.

In fact, a similarly futuristic expansion of digital technology in the creation and application of law cannot be held unthinkable on the assumption that a technological development capable of obtaining a result of this kind is unimaginable. In effect, in the abstract at least, it is not possible to pose limits to a technological evolution which surprises humanity every day in a very concrete way.

Nor can such a digital expansion be considered in contrast with the dogmas of "the exclusive nature of law made of words" or of "the absolute primacy of words over numbers"; in fact, on the one hand, those dogmas, dated, evanescent and never demonstrated, especially if observed in the light of the numerous studies carried out on the relationships between mathematics and law, today appear in a very different perspective, if not completely inconsistent. On the other, the dogmas in question seem in any case extraneous to the theme of a futuristic 'digital law' because

evolution in a digital direction would concern, as will be argued, 'mathematical truths' and 'mathematical words' (not numbers); hence, only mathematical methods and concepts, expressed however in words (or with conceptual symbols different from numbers).

Lastly, the unthinkable nature of a 'digital legistics' of this kind appears unjustifiable even accepting the argument that the 'man of law' (whose spiritual component is necessarily present) can never be composed only of reason; in effect, the mathematical method does not impede, and on the contrary, as will be argued, can fully explain, the 'combining' of a reasoning (scientific) human disposition with a spiritual (ethical) human disposition, a 'combining' that the current 'mathematical' law implies.

If anything, it seems possible to detect a serious and not virtual obstacle to the development of 'digital law' in the very absence, at the present moment, of a shared general theory/theories of law reconstructed with mathematical truths, words, methods, concepts and symbols.

In effect, putting aside the often authoritative approaches that have focussed on specific aspects of the law-mathematics relationship, it seems opportune to continue to pay attention to the innately mathematical nature of the general theory of law and subsequently to the principles which, emanating from it, regulate the sectors of law (or at least those 'with truth'); as was attempted, within limits, in the previous pages.

If therefore one observes the 'mathematical law' that can be derived, at least at the level of general theory, from the combinations and from the 'combinings' between variables and the legal and mathematical entities described in the tables (albeit summary and incomplete) included to the present text, then perhaps one can reply positively also to the last interrogative posed provocatively at the beginning.

In effect, the mathematical truths, words (not numbers), methods, concepts and symbols present in law (or at least in its general theory) and which, in their organic entirety, constitute 'mathematical law', seem to provide a possible "base analysis" which could feasibly be used to plan a futuristic 'digital legistics' whose software could be used by the 'man of law' to produce or apply an even more futuristic 'digital law'.

5. *Final summary*

To summarize this brief exposition, it seems evident that part (if not all) of modern law, is, like mathematics, 'with truth', both in its contents and methods.

In fact, as far as contents are concerned, the (typologies of) truths of law (or rather of legal reasoning) are very similar to the typologies of mathematics truths. There seem to be no incompatibilities or antithetical juxtapositions to impede the extraction of the innately 'mathematical nature' of current law.

As to methods, the form of mathematical reasoning is able to provide a rigorous and complete explanation of the law that exists and which, among other things, is frequently the fruit of a 'combining' of binary components both natural, such as ethics and science, spirit and reason, normality and probability, and artificial, such as justice and certainty, cognizance and decision, assessment and conciliation, declaratory efficacy and constitutive efficacy, and so on.

All this enables us to accede to defining profiles (an indispensable part of every base analysis necessary for the production of any software).

In this perspective, 'mathematical law' is above all 'law with truth' because, as has been seen, it is intrinsic (or peculiar) to sectors or areas of law that present this specificity (taxation law, penal law, bankruptcy law, labour law, legal sources of obligation in private law, etc).

Secondly, 'mathematical law' is current law (inasmuch as it is "extractable" from current legal provisions).

In the third place, mathematical law is such because it can be obtained by applying the mathematical genetic code which, from the inside of every legal provision communicates (rectius: orders) to the outside both the type of mathematical concept (or rather the type of truth) that the single legal provision contains and the type and times of the interpretative, probative or applicative method that is legally peculiar (or proportionate) to it.

To be more precise, each legal provision incorporates from its incipience a mathematical genetic code of its own ascribable to one of the four exhaustive types: $(\equiv)$, $(\cdot)$, $(—)$ and $(\emptyset)$.

This mathematical code (metaphorically similar to a cell's DNA) identifies the mathematical concept that characterizes the provision and imposes the 'method of use' of the provision from the qualitative and diachronic points of view.

From the qualitative point of view, the type of mathematical code that the provision incorporates characterizes the type of 'rule for its use': this regulation for use is the source of the type of unavailability according to the method of legal reasoning that corresponds to it, so that, if the type of legal reasoning used in practice corresponds to the type indicated by the rule of use, the reasoning is not invalid *a priori*, whereas if it does not correspond, the reasoning is invalid *a priori* (because mathematically

disproportionate and so unlawful). Thus, for example, 'ethical provision' implies 'ethical reasoning' (and all that, in terms of legal regime, such a 'combining' infers); in the same way, 'scientific provision' implies 'scientific reasoning' (and all that, in terms of legal regime, such a 'combining' infers).

From the diachronic point of view, the aforesaid mathematical code obliges the 'man of law', before all else, to concretely distinguish the two phases into which, in the abstract, the legal reasoning must ontologically be divided: the first to identify the type of mathematical code present in the legal provision; the second to adopt (on the basis of a 'mathematical combining') the kind of reasoning (or *forma mentis* or *ratio decidendi*) corresponding to that type of mathematical code. The rules and the principles regarding interpretation and proof, which are embodied in the legal system, follow (and do not precede) these two phases.

All this accounts for the enormous applicative potential of 'mathematical law' because, through its codes, it can characterize the infinite branches of law that extend from the trunk of the general theory to the fruit of the agreement or the sentence.

Its innate binding efficacy implies the identification of the rules that consent the coexistence of the two (in this sense 'binary') 'fundamental constituents of current law', namely 'ethics' (that is, spirit) and 'science' (that is, reason).

The efficacy that derives, then, from these 'constituents' also characterizes, among other things, the identification of legislative objectives (justice or certainty); legislative functions (assessment or conciliation); materials (proportionality, rationality, solidarity, etc); instruments (interpretation, proof, decision, etc); vehicles (laws, acts, regulations, sentences, etc).

To the point, clearly, of also determining sanctioning profiles.

In effect, the violation of the rules of 'mathematical law', or rather, first of all, the original violation of the mathematical code ('rule for its use') that the legal provision incorporates, implies both the illegality of the activity (performed by the 'man of law') and the illegitimacy of the law, act, regulation or sentence resulting from such activity; in both cases, this occurs because of the violation of an imperative rule of method (inalienable), regardless of merit.

Thus, the sanctions that 'mathematical law' imposes as a consequence of the illegality of actions and the illegitimacy of the result bear witness to the non-offensive character of the function (= legislative reason) of the mathematical code (= 'rule for use') present in the legal provisions.

In fact, the legislative reason that justifies the mathematical code, which is capable of assuming at any given time one of the above-mentioned four

types of 'mathematical truth', is that it is the truth of abstract law, and not that of the single person or of the authority, that is responsible for the truth of, and in, concrete law.

This same legislative reason, which appears to constitute the indissoluble nucleus of 'mathematical law', would seem, at the moment, able to contribute to the planning of a futuristic 'digital logistics' at the service of an equally futuristic 'man of law', and to give rise to a 'digital law', in which, even in a science-fiction perspective, the harmonious coexistence within the legal system as a whole and within the single 'man of law' of the two binary and immortal elements of law, 'ethics' and 'science', must however necessarily remain constant and eternal.

Abstract

The mathematical truths, words (not numbers), methods, concepts and symbols that law, or at least its general theory, today incorporates would seem to constitute a whole ('mathematical law') that could become a possible "base analysis", the eventual starting point for the construction of a futuristic 'digital logistics' (= 'digital rules of legislative drafting'). The software pertaining to this type of logistics could be used by the 'man of law' to produce (by means of 'legislatorial software') or to apply (by means of 'applicative software') an even more futuristic 'digital law'.

