University of Milano-Bicocca

School of *Natural Sciences*

STUDENTS' GUIDE

MATHEMATICS

UNDERGRADUATE PROGRAM
GRADUATE PROGRAM

ACADEMIC YEAR 2019-2020

Contents

INTRODUCTION

"Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality. (...) Without doubt, all mathematical development has its psychological roots in more or less practical requirements. But, once started under the pressure of necessary applications, it inevitably gains momentum in itself and transcends the confines of immediate utility."

(R. Courant and H. Robbins, What is Mathematics?)

Progress in Mathematics usually takes place starting from practical problems, then passing to abstraction, and producing, sometimes even after centuries, fascinating and unexpected applications. Programs in Mathematics intend to equip students with the theoretical tools and with the experience useful for the design and management of models and mathematical methods in physics, chemistry, computer science, engineering, life and environmental sciences, medicine, economic and social sciences. Graduates in Mathematics are appreciated for their flexibility and ability to learn techniques and disciplines required for professions that need a mixed knowledge of mathematical and other kind of tools. At the University of Milano-Bicocca the following programs are offered:

1. The Laurea in Mathematics program (equivalent to Bachelor of Science). The program is open to students holding a high school diploma.
2. The Laurea Magistrale in Matematica program (equivalent to Master of Science). The program is open to students holding a Bachelor degree.
3. The Dottorato di Ricerca in Matematica (Ph.D. in Mathematics) The program can be accessed by students holding a Master degree.

The quality of teaching is ensured by the presence of the Department of Mathematics and Applications and a section of the National Institute of High Mathematics (INdAM), which organize seminars and conferences. Some scholarships are offered for the Bachelor's and Ph.D. Degrees.

Programs' description

The Bachelor of Science Degree in Mathematics aims to provide students with an adequate mastery of scientific methods and contents, as well as give specific professional skills. More specifically it aims to provide the student with a solid grounding in mathematics, with an adequate knowledge of the scientific method and give specific vocational skills in the areas of application of mathematics, thus providing the basis for the interpretation, both qualitatively and quantitatively, of economic, financial, biological and engineering phenomena. The program activities comprise a broad spectrum of basic and methodological disciplines in the areas of Mathematics, Physics and Informatics, as well as activities needed to prepare for the final exam. The skills of graduates in Mathematics (bachelor's degree) can be applied in research institutions and, in general, in companies for whose activities the modeling of physical, natural, economical, financial models is relevant. The particular methodological training will open the graduate (bachelor's degree) in Mathematics to the opportunity to pursue careers in several fields, not only in science and technology.

The Master of Science Degree in Mathematics is intended to be a reference for research and teaching in the field of mathematics. The Master of Science in Mathematics has four specific goals:

- provide advanced knowledge in core areas of Mathematics in the perspective of their use in both theoretical and applicative specialistic areas; such knowledge will provide a solid platform for the possible start of theoretical or applied research activities (eg, Ph.D., advanced level master degrees) or for the teaching of higher mathematics and the scientific communication at a specialist level;

- provide advanced methods and tools for the modeling and the mathematical formalization of complex problems that arise in the experimental sciences, engineering, economics and in other fields of application;
- provide methods and tools for the numerical and analytical solution of existing models; to this end, an essential and complementary role is assumed by advanced laboratory activities;
- refine the students' competence in the use of recent and sophisticated IT tools, of interest in mathematics.

It is expected that at the end of the program, graduates in Mathematics (Master of Science Degree):

- have acquired a sound knowledge of the language and techniques of a broad spectrum of topics in modern mathematics;
- are able to apply the above said techniques and contents to the analysis of complex problems;
- have acquired the ability to perform autonomously roles of high responsibility in working groups integrated into theoretical or applied research, or in teaching and scientific communication of high qualification;
- are able, in the course of the previous activities, to properly use, both in writing and orally, a language of the European Community;
- are able to present the results of their research justifying the methods and strategies adopted to a broad spectrum of partners.

Graduates in Mathematics (Master of Science Degrees) will therefore have a preparation aimed at roles of high responsibility in advanced scientific research projects, in the building and the computational development of mathematical models of several types, in different settings: applicative, scientific, environmental, healthcare, industrial, financial services, public administration, communication of mathematics and science.

UNDERGRADUATE PROGRAM (L-35)

Program outline

First year

Course name	ECTS
ALGEBRA I	8
FOREIGN LANGUAGE	3
GEOMETRY I	8
LABORATORY OF MATHEMATICS AND INFORMATICS	6
LINEAR ALGEBRA AND GEOMETRY	8
MATHEMATICAL ANALYSIS I	12
PHYSICS I	12

Second year

Course name	ECTS
ALGEBRA II	8
ALGORITHMS AND PROGRAMMING	6
DYNAMICAL SYSTEMS AND CLASSICAL MECHANICS	12
GEOMETRY II	8
INTRODUCTION TO NUMERICAL ANALYSIS	12
MATHEMATICAL ANALYSIS II	12
MEASURE THEORY	4

Third year

Course name	ECTS
PROBABILITY THEORY	12
PHYSICS II	8
THREE COURSES to be chosen among the ones listed in table A; specifically: - 2 courses in the following academic disciplines: MAT/02-MAT/03-MAT/05 - 1 course in the following academic disciplines: MAT/06-MAT/07-MAT/08, totalling 18 ECTS	18
FREE CHOICE COURSES among the courses in table A or among the other courses offered in the University (subject to approval of the curriculum committee), totaling 18 ECTS	18
MATHEMATICAL WORD PROCESSING (ICT)	1
FINAL EXAMINATION	4

Table A	ECTS	Academic Discipline
ALGEBRA III	6	MAT/02
COMPLEX ANALYSIS	6	MAT/05
GEOMETRY III	6	MAT/03
MATHEMATICAL ANALYSIS III	6	MAT/05
MATHEMATICAL PHYSICS	6	MAT/07
MATHEMATICAL STATISTICS	6	MAT/06
NUMERICAL ANALYSIS	6	MAT/08

UNDERGRADUATE COURSES DESCRIPTION

ALGEBRA I (2019/2020)

Teachers: Francesco Matucci, Andrea Previtali

Aims

Introduce students to methods and basic contents of algebra. Fundamental structures like sets, groups and rings will be studied. Time permitting some rudiments of programming languages as Magma, Gap and Mathematica will be imparted.

At the end of lectures students will be requested to solve both routine exercises and more sophisticated problems explotiong the theoretical contents of the course.

Contents

Sets, Relations, Operations; Modular Arithmetic; Elements of Group and Ring Theory; Polynomials;

Detailed program

A) Sets, Relations, operations: Axiom of choice; order relations (Zorn's Lemma); equivalence relations; homomorphism theorems for sets; congruences.

B) Arithmetic properties of the set of integers Z; modular arithmetics; residue classes.

C) Basics of Group Theory; subgroups, subgroup generated by a subset; cyclic groups; cosets; Lagrange's Theorem; congrunces in a group; normal subgroups; group homomorphisms and quotient groups; main theorems on homomorphisms; automorphisms; direct and semidirect products; symmetric and alternating groups; permutation groups; group actions (G-sets); regular representation; Cayley's Theorem; conjugation action; orbits: examles and applications; Sylow's Theorems.

D) Basics on Ring Theory: domains, division rings, fields; ring homomorphisms: ideals, quotint rings, elementary theory of homomorphisms; Chinese remainder Theorem; divisibility in a domain; embeddings of domains into fields; prime and maximal ideals; Euclidean, principal ideal and unique factorization domains; Gaussian integers.

E) Polynomial algebras: polynomials in one variable over a field; decomposition into irreducible factors.

Prerequisites

Basic notions of high school algebra and analysis

Teaching form

Lessons: 6 credits

Tutorials: 2 credits

Textbook and teaching resource

Textbook: Latex written Notes and Tablet taken lectures are available on this platform.

Suggested readings:

- Aschbacher, Finite Group Theory 2nd ed, Cambridge University Press, Cambridge, 2000.
- Childs, A Concrete Introduction to Higher Algebra 3rd ed, Undergraduate Texts in Mathematics, Springer, New York, 2009.
- Jacobson, Basic Algebra I, Freeman & Co, 1985
- Machi, Gruppi, Springer-Verlag, 2007

Exercise Books:

- Alzati, Bianchi, Cariboni, Esercizi di matematica discreta, Pearson, 2012
- Chirivi', del Corso, Dvornichich, Esercizi scelti di algebra Vols. 1 e 2, Springer, 2017

Semester

Second semester

Assessment method

In order to access the written assessment one has to pass a computer assisted exam.

This requires inscription to the WIMS platform.

There 12 tests are available (one for each week of lectures). They will be gradually activated.

Their resolutions will allow you to tune in with the course contents. Moreover the first part of the exam will consist of a few

exercises selected among those of all tests.

At the end of the lectures a bonus of xx will be assigned to a yy score according to following table:

- xx=2 if 27<yy<=30;
- xx=1.5 if 22<yy<=27;
- xx=1 if 18<=yy<22.

The bonus will be valid until March.

Office hours

By appointment via e-mail

ALGEBRA II (2019/2020)

Teacher: Thomas Stefan Weigel

Aims

On the basis of the knowledge acquired in the Algebra I course, the course is aimed to a) illustrate further topics in the theory of rings and fields; b) develop the theory of finitely generated modules over principal ideal domains, with applications to abelian groups and linear algebra.

Achievements of a successful attendance of the course include

- Knowledge: The knowledge and the understanding of the principle definitions, theorems and results in the theory of rings and their modules, as well as in field theory.
- Capability: The capacity to apply this abstract knowledge to concrete problems in algebra.

Contents

Rings and their modules, and fields

Detailed program

Topics in ring theory. Polynomial extensions. Polynomials in several variables. Noetherian domains. Hilbert's basis theorem.

Localization.

Extensions of rings and fields. Algebraic and transcendental extensions. The splitting field of a polynomial. Finite fields.

Modules over a ring and linear algebra. Free modules: bases. Rank, universal property. Torsion. Modules over principal ideal domains: finitely generated modules; equivalence of matrices and reduction to normal form. Structure theorem for finitely generated modules. Torsion modules and primary decomposition. Invariant factors, elementary divisors. Applications to abelian groups and matrices: Structure theorem for finitely generated abelian groups. Canonical forms of matrices: the companion matrix, rational canonical form, Jordan canonical form.

Prerequisites

Prerequisites: The contents of the courses *Linear algebra and Geometry* and *Algebra I*.

Teaching form

6 credits (ECTS) of lecturing, 2 credits (ECTS) of exercise classes

Textbook and teaching resource

N. Jacobson, Basic Algebra I, Freeman & Co, 1985.

Additional References:

S. Bosch, Algebra, Springer-Verlag, 2003.

B. Hartley & T. Hawkes. Rings, modules and linear algebra, Chapman & Hall 1970

Semester

1st semester

Assessment method

Examination: A **written exam** of ca. 90 minutes (non multiple choice) and an **oral examination** of ca. 20 minutes on the content of the course. Both exams contribute ca. 50 percent to the final mark. Passing the written exam (answering ca. 40 percent of all the questions correctly) is mandatory for being admitted to the oral examination.

In the first call the written exam will be devided into two partial exams. (The first before Christmas, the second after the completion of the course (around first of February)). Students are advised to participate at the first two partial exams in order to practise and to get accustomed to the type of questions they have to answer. Failure in these exams will not have any impact on the final mark.

The questions will concern definitions, examples, counterexamples, exposition and application of Theorems as well as their proofs.

From the second call onward there will be just one written exam covering all the material of the course.

Office hours

On appointment

ALGEBRA III (2019/2020)

Teacher: Pablo Spiga

Aims

One of the central subjects in field theory is the study of finite extensions, in particular finite Galois extensions. The main objective of the course will be to introduce the concepts necessary to formulate the Fundamental Theorem in Galois Theory and to analyse its consequences.

During the times of Galois many mathematicians were still working on problems raised by greek mathematicians. One problem of this type is the trisection of an arbitrary angle by ruler and compass. Using Galois theory one may show easily that this is indeed impossible.

Contents

Field extensions, the algebraic closure of a field, the fundamental theorem in Galois theory, applications.

Detailed program

1. finite field extensions,
2. algebraic closure,
3. splitting field
4. normal and separable extensions,
5. fundamental theorem of Galois theory,
6. galois group and soluble groups,
7. soluble extensions,
8. cyclotomic extensions,
9. solution of polynomial equations by radicals,
10. finite fields,
11. constructions with straight edge and ruler,
12. applications.

Prerequisites

Algebra I & Algebra II

Teaching form

Lectures, 6 CFU (ECTS)

Textbook and teaching resource

Basic algebra I,N.Jacobson

Semester

First semester

Assessment method

Oral examination on the content of the course.

Office hours

On appointment

ALGORITHMS AND PROGRAMMING (2019/2020)

Teacher: Fabio Sartori

Aims

To design and implement software systems integrating different problem solving methods. This course aims at introducing the basic knowledge of software systems from the object-oriented paradigm perspective. Moreover, the course will provide competencies to model simple domains through the UML language and code them into the Java Programming Language, according to the object-oriented paradigm.

Contents

The course teaches object oriented programming and software engineering principles. The student will be able to model problems according to the object oriented paradigm and translate it into Java programs.

Detailed program

- Introduction to basic principles of object oriented programming (information hiding, inheritance, polymorphism) and UML language (Unified Modeling Language).
- Hints on software cycle of life.
- Java as programming language and platform.
- Object Oriented Programming in Java: classes and objects, attributes and methods.
- Advanced Object Oriented Programming in Java: inheritance and polymorphism.
- Exceptions, ArrayList, Geenrics and Collection Framework.

Prerequisites

Structured Programming (Laboratory of Mathematics and Informatics course)

Teaching form

- Lectures: 4 CFU
- Exercise classes: 1 CFU
- Laboratory: 1 CFU

Textbook and teaching resource

All the information about the course as well as the lessons slides and practical exercises will be available through the learning platform of the University, at the elearning.unimib.it link.

The suggested texdtbook will be:

- W. Savitch: "Programmazione di base e avanzata con Java", a cura di Daniela Micucci, 2nd edition, Pearson

Semester

Second semester

Assessment method

Examination type

Written and Oral examination; the oral examination is not mandatory, but necessary to obtain a "cum laude" merit. The mark range is 18-30/30. The oral examination is about both theoretical questions and practical exercises and can increase the result of written examination by at most 4 points.

The written examination is divided into two parts: the first one is devoted to evaluate theoretical skills about object oriented programming, by means of a collection of close-ended questions; the second one concerns the design and implementation of a simple software system, with the aim to demonstrate the student's capability to solve correctly a practical problem, on the basis of object-oriented programming principles considered during the lectures, without generating any kind of error (i.e. compile time, runtime, logical errors).

The arithmetic mean (possibly weighted) of the two marks defines the final mark proposed to the student: in case it is sufficient, the student can accept it as is or modify it by means of an oral examination (possibly decreasing the final mark). Oral examination is possible if and only if written examination is sufficient. The teacher has the faculty to establish mandatory oral examinations for those students whose written examinations, although sufficient, present some criticalities: for example, in case of not sufficient theoretical questions whereas pratical exercises are good, or viceversa.

Five exam sessions are stated: June, July, September, November and January; moreover two partial examinations are proposed to studets during the course.

Office hours

Thursday, between 11 a.m. and 12 a.m., or by appointment.

COMPLEX ANALYSIS (2019/2020)

Teacher: Stefano Meda

Aims

The aim of the course is to make students able to effectively use the powerful methods of complex analysis in theoretic and practical applications.

Specifically, the expected learning outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in Complex Analysis;
- the skill to apply such conceptual background to the construction of concrete examples and to the solution of exercises, ranging from routine to challenging (starting with routine exercise that require straightforward application of the definitions and the results given during the lectures, up to exercise that require deep understanding of the matter and the ability of developing original ideas).

Contents

This is a basic course in one complex variable. It includes holomorphic functions, power series, Cauchy's theorem and applications, isolated singularities, zeroes of entire functions and applications.

Detailed program

Part 1. Preliminaries. Holomorphic functions: definition and examples. Entire functions. Holomorphic functions and diffrentiable maps on R 2. Cauchy–Riemann equations. Power series. Hadamard's formula for the radius of convergence. Series expansions of e z , sin z and cos z. Power series define holomorphic function in the disc of convergence. Integration along curves. Parametrized curves, smooth curves, and piecewise smooth curves. Properties of integration along curves. Primitive of a function and its properties. Functions with anishing first derivative are constant in regions,

Part 2. Cauchy's theorem and applications. Goursat's lemma. Local primitives and the Cauchy theorem for discs. Extensions to toy contours. Computations of integrals. Examples. Cauchy's integral foolomorphic functions are locally sums of power series. Liouville's teorem. Fundamental theorem of algebra. Analytic continuation and identity principle for holomorphic functions. Morera's theorem. Uniform convergence on compacta of sequences of holomorphic functions. The symmetry principle and Schwarz's reflection principle. Runge's theorem.

Parte 3. Meromorphic functions and the logarithm. Zeroes and poles. Residues and the residue formula. Isolated singularities of holomorphic functions, and Riemann's theorem on removable singularities. Poles and essential singularities. The Casorati–Weierstrass theorem. Singularities at infinity. Characterisation of meromorphic functions on Riemann's sphere. The argument principle. Rouche's theorem. The open mapping and the maximum modulus theorems. Homotopic paths, and the general form of Cauchy's theorem. The logarithm. Existence of the logarithm in simply connected domains, and related properties.

Parte 4. Entire functions. Jensen's formula. Functions of finite order. Entire functions and its zeroes. Infinite products. Criterion of convergence. Infinite products of holomorphic functions. Product formula for sin. Weierstrass' canonical products. Entire functions with prescribed zeroes. Hadamard's factorization theorem.

Factorization of entire functions of finite order.

Prerequisites

Calculus, linear algebra

Teaching form

Lectures with blackboard

Textbook and teaching resource

Stein and Shakarchi, “Complex analysis”, Princeton University Press.

Semester

I semester

Assessment method

Written examination, including theoretical questions (proofs of part of the results illustrated during the course) and exercises, often similar to those solved during the class hours. In order to get a positive grade, both the parts including theoretical questions and exercises must get a passing grade.

The grade will take into account the exactness of the answers, the clarity of the exposition and the command of mathematical language used.

Office hours

Upon appointment.

DYNAMICAL SYSTEMS AND CLASSICAL MECHANICS (2019/2020)

Teacher: Paolo Lorenzoni

Aims

This course aims to present the basic ideas of Classical mechanics, from the Galielo-Newton formulation to those of Lagrange, Hamilton and Jacobi and to provide the necessary mathematical tools.

The expected learning outcomes include:

- the knowledge and understanding of the basic definitions and statements of different formulations of Classical Mechanics.
- the knowledge and understanding of some key examples (harmonic oscillator, Kepler's problem, Lagrange top).
- the ability to apply the acquired theoretical knowledge to the solution of exercises. In particular the derivation of the Lagrange/Hamilton equations for constrained mechanical systems, the reduction of the degrees of freedom in the presence of symmetries and, in some simple examples, the discussion of the qualitative behaviour of solutions of the equations of motion and/or their reduction to quadrature.

Contents

Newtonian Mechanics (a reminder). Ordinary differential equations. Qualitative analysis. The D'Alembert principle and Lagrangian Mechanics. The two-body problem. The rigid body. Hamiltonian mechanics. Canonical transformations and Hamilton-Jacobi method.

Detailed program

1. A reminder of the theory of ordinary differential equations. Vector fields and systems of first order ODEs. Rectification of a vector field. Equilibria and their stability. Linearization near equilibrium points. Systems with one degrees of freedom: level curves of the energy

2. Lagrangian Mechanics. Euler-Lagrange equations. Particle constrained on a regular curve. Particle constrained on a regular surface. The D'Alembert principle for general holonomic constraints. Equilibrium points and small oscillations. Variational formulation of Euler-Lagrange equations. One-parameter group of diffeomorphisms, symmetries and Noether's theorem. The two-body problem and the Kepler laws.

3. Transition matrix and angular velocity. Inertial and non inertial frames. Mechanics of rigid bodies. The inertia operator. K¨onig's tehorem. Euler's equations for rigid bodies. The Euler angles and the Lagrange top.

4. Hamiltonian Mechanics. The Legendre transformation. Hamilton's equations. Poisson bracket and Lie bracket. Symmetries and conservation laws in Hamiltonian Mechanics. Liouville's theorem. Variational formulation of Hamilton's equations. Canonical transformations and Jacobi's integration method. Separation of variables in the Hamilton-Jacobi equation. The case of spherical, parabolic and elliptic coordinates.

Prerequisites

Calculus I, Linear Algebra and Geometry, Physics I.

Teaching form

Lectures: 8 CFU

Exercise classes: 4 CFU

Textbook and teaching resource

The course is based on lecture notes provided by the instructor.

The following books are also recommended:

1. V. I. Arnold, Metodi matematici della meccanica classica, Editori Riuniti.

2. S. Benenti, Modelli matematici della Meccanica, Quaderni di matematica per le scienze applicate. Celid

3. A. Fasano e S. Marmi Meccanica Analitica Bollati-Boringhieri 2002.

4. L.D. Landau. E. M. Lifshits, Meccanica, Editori Riuniti.

5 N.M.J. Woodhouse, Introduction to analytical dynamics, Oxford Science Publications. The Clarendon Press, Oxford University Press, New York, 1987.

Semester

Second semester.

Assessment method

The exam consists of two parts: a written test and an oral test.

The written test requires the solution of 3 problems (a dynamical system in the plane, a problem of Lagrangian Mechanics and a problem of Hamiltonian Mechanis). The duration is typically two and half hours. Correct answers without clear explanation will not receive full marks. The minimum grade to pass to the oral part is 15/30.

During the oral exam the students will be asked to state and prove the theorems carried out in class and to illustrate their meaning with significant examples. The oral exam will evaluate the knowledge of the theoretical aspects of the course, as well as the ability to expose it in a well-organized and consistent way.

The written an the oral exams equally contribute to the final grade. The oral examination can be taken in the same session of the written test, as well as in the subsequent one.

During the course, two written partial tests will be offered, each referred to one half of the course. To pass the written examination through the partial tests, the student needs to pass each of them with the minimum grade of 15/30. In this case oral examination must be taken within the exam session of September.

Office hours

By appointment.

GEOMETRY I (2019/2020)

Teacher: Alberto Della Vedova

Aims

Give an elementary introduction to geometry and topology.

Contents

Fundamentals of point-set topology and some aspects of euclidean and projective geometry will be discussed.

Detailed program

Topological spaces and continuous functions. Metric topology. Topological spaces. Basis of a topology. Subsets of a topological space. Continuous functions and homeomorphisms.

Examples of topological spaces. Subspaces. Products. Quotients.

Topological properties. Separation axioms and Hausdorff spaces. Compactness. Completeness and compactness in metric spaces. Connected and path-connected spaces. Locally Euclidean spaces and topological manifolds.

Euclidean and projective spaces. Geometry of euclidean and projective spaces.

Prerequisites

Limits and continuity of real functions. Linear Algebra.

Teaching form

Classroom lectures will be split into: theoretical sessions (discussion of relevant results of the theory, examples, and counterexamples), exercises sessions (training how to solve exercises and problems).

Textbook and teaching resource

Textbook:

- E. Sernesi, Geometria, vol. I-II. Bollati-Boringhieri (1989, 1994).

Further readings:

- C. Kosniowski, A first course in algebraic topology. Cambridge University Press (1980).
- J. R. Munkres, Topology, 2nd edition. Prentice Hall (2000).
- M. Manetti, Topologia, 2a edizione. Springer-Verlag (2014).

Semester

Spring.

Assessment method

- The exam is split into three parts.

Preliminary test - Ten multiple-choice questions. Correct answer +3 pts, wrong answer -1 pts, no answer 0 pts. Candidates will pass the test if they score at least 15 pts.

Written part - Solve some exercises in 2h. The score is assigned on the base of completeness, correctness, rigor, and clarity of the solutions. Max score is 30 pts. Candidates will pass the written part if they score at least 15 pts.

Preliminary test and written part will take place on the same day.

Oral part - Answer questions on the arguments discussed during the classes or on the exercises solved in the written part. In order to take the oral part, candidates need to have earned at least 15 pts in both the preliminary test and the written part. Answers will be evaluated on the base of their completeness, correctness, rigor, and clarity. The final score will take into account the scores earned in the preliminary test and the written part.

- Partial exemptions.

  Exemption from the oral part - Let T and S be the scores in the preliminary test and the written part respectively. If both T and S are not less than 20 then the candidate can avoid the oral part and get as the final score the minimum among (T+S)/2 and 27.

  Exemption from preliminary test and written part - Partial tests during the class period will give exemption from preliminary test and written part. The candidate can take directly the oral part in June or July examinations days.

Office hours

By appointment.

GEOMETRY II (2019/2020)

Teachers: Sonia Brivio, Roberto Paoletti

Aims

The aim of the course is to introduce the foundation of the theory and of the use of differential forms on open sets of Euclidean spaces, as a basis for the general treatment in the context of differentiable manifolds.

Differential forms are a tool of pervasive and fundamental importance in Geometry, Differential Topology, and Analysis. The theory will be developed from first principles, that is, from the basic notion of a tensor in linear algebra.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory of differential forms; the knowledge and understanding of some of the most relevant basic applications, notably to the study of smooth proper maps between opens sets of Euclidean spaces; the knowledge and understanding of some of the key foundational examples of the theory;
- the ability to apply the acquired abstract knowledge to the solution of simple computational exercises and theoretical problems, referring in a precise and well-organized manner to the pertinent results being used; the ability to master the algebraic, differential and integral calculus of differential forms, and to use it in the some simple practical situations, such as the study of proper maps between open sets of Euclidean spaces; the ability to apply the theoretical background to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

Alternating multilinear algebra; differential forms on Euclidean space and their operations; Poincaré Lemma; applications to physics; integration; change of variables; degree of a proper differentiable map between open sets in Euclidean spaces and applications; Theorems of Gauss-Green and Stokes; De Rham Theory (brief outline).

Detailed program

Exterior algebra of a vector space and its operations: exterior product, contractions; oriented Euclidean vector spaces and their volume elements; vector fields and differential forms; exterior differential; closed and exact forms; winding number and applications; gradient, rotor and divergence; differential forms under smooth maps: pull-back; integration; integration and homotopy; change of variable formula; Poincaré Lemma; Poincaré Lemma with compact support; integration on oriented parametrized varieties; Theorems of Gauss-Green and Stokes; degree of a proper smooth map between open sets in an Euclidean space and its computation; invariance under smooth proper homotopy; applications: the Fundamental Theorem of Algebra and the Brower Fixed Point Theorem.

Prerequisites

The content of the courses of Analysis I and (in part) II, Linear Algebra and Geometry, Geometry I.

Teaching form

Lectures: 6 CFU, Exercise classes: 2 CFU

Textbook and teaching resource

Reference text: teacher's notes on e-learning

Recommended reading:

the following book is especially pertinent to the content of this course:

- V. Guillemin and P. Haine, Differential forms, World Scientific 2019

Further recommended textbooks are:

- M. Do Carmo, Differential forms and applications, Springer Verlag 1996;
- V. Guillemin, A. Pollack, Differential Topology 1974;
- W. Fulton, Differential Topology, a first course, Springer Verlag 1995.

Semester

2nd semester

Assessment method

During the course, two written partial tests will be offered, each referred to one half of the course. Each partial test will consist of a balanced flexible combitation of computational exercises and theoretical questions. The exercises and theoretical questions in these tests will be along the lines of those offered in the practical and theoretical tests of the regular exam sessions (see below), except that they will deal with only one half of the course.

The two partial tests will contribute equally to the final grade. To pass the exam through the partial tests, the student needs to pass each of them, thus obtaining a grade of at least 18/30 in both.

Alternatively, students may pass the exam through the regular exam sessions that follow the end of the course. Each session comprises two written tests, a practical and a theoretical one, each referred to the whole course. In the practical test, the student will be asked to solve various computational exercises, while in the theoretical test there will be questions involving definitions, statements of theorems, proofs, construction of examples and counterexamples, and simple theoretical problems.

The practical tests will measure the student's ability to master the acquired formalism and apply it to some simple computations, to build on the acquired theoretical knowledge, and to invoke it in a pertinent and precise manner.

The theoretical tests will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner.

In order to pass the exam in one of the regular sessions, the student needs to first pass a practical test, thus obtaining a grade of at least 18/30, and then to also obtain the passing grade in the theoretical test of the same session or, upon his/her choice, of the session immediately following.

To each exercise/theoretical question (or problem) a maximum partial grade will be assigned by the commission, depending on its difficulty and length; in the evaluation, every student will be given a grade in correspondence to each exercise/theoretical question (or problem) up to the maximum one, measuring the exactness, the completeness, the rigour, the clarity and the overall coherence of the development.

The duration of each test is typically three hours.

Office hours

Upon appointment.

GEOMETRY III (2019/2020)

Teachers: Diego Conti, Roberto Paoletti

Aims

The aim of the course is to introduce the study of topological spaces by means of their most basic algebraic invariant, that is, the first fundamental group, and to develop the foundations of the theory of differential forms on differential manifolds, which is a much more general and flexible framework than the one considered in Geometry II.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory of first fundamental group and of differential forms; the knowledge and understanding of some of the most relevant basic applications, notably to the study of smooth proper maps between differentiable manifolds; the knowledge and understanding of some of the key foundational examples of the theory;
- the ability to apply the acquired abstract knowledge to the solution of simple computational exercises and theoretical problems, referring in a precise and well-organized manner to the pertinent results being used; the ability to master the algebraic, differential and integral calculus of differential forms, and to use it in the some simple practical situations, such as the study of the first fundamental group of some simples spaces and of proper maps between differentiable manifolds; the ability to apply the theoretical background to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

Topological coverings and first fundamental group; differentiable manifolds; tangent bundles; vector fields and their associated flows; differentiable manifolds; Stokes ' Theorem; De Rham theory; proper maps and degree theory.

Detailed program

Topological coverings and the universal cover; first fundamental group; lifting theorems; Seifert-Van Kampen Theorem; differentiable manifolds; vector fields and tangent bundles; differential forms; pull-back and differential; exterior derivative; Lie derivative and Cartan's magic formula; oriented manifolds and integration; smooth domains and Stokes' Theorem; proper maps and degree theory; proper homotopies; applications (e.g., retractions, vector fields on spheres).

Prerequisites

The content of the courses of Analysis I and II, Linear Algebra and Geometry, Geometry I and II.

Teaching form

Lectures: 6 CFU

Textbook and teaching resource

Reference text: teacher's notes on e-learning

Recommended reading:

the following books are especially pertinent to the content of this course:

- V. Guillemin and P. Haine, Differential forms, World Scientific 2019
- W. Fulton, Differential Topology, a first course, Springer Verlag 1995

Further recommended textbooks are:

· M. Do Carmo, Differential forms and applications, Springer Verlag 1996;

· V. Guillemin, A. Pollack, Differential Topology 1974;

· .

Semester

2nd semester

Assessment method

During the course, two written partial tests will be offered, each referred to one half of the course. Each partial test will consist of a balanced flexible combination of computational exercises and theoretical questions. The exercises and theoretical questions in these tests will be along the lines of those offered in the practical and theoretical tests of the regular exam sessions (see below). The two partial tests will contribute equally to the final grade. To pass the exam through the partial tests, the student needs to pass each of them, thus obtaining a grade of at least 18/30 in both.

Alternatively, students may pass the exam through the regular exam sessions that follow the end of the course, and exactly the same pattern will be offered in every exam session. Thus, each session comprises two written tests, each referred to one half of the course, and consisting of a balanced combination of computational exercises and theoretical questions. The theoretical questions will involve definitions, statements of theorems, proofs, construction of examples and counterexamples, and simple theoretical problems.

The exercises will measure the student's ability to master the acquired formalism and apply it to some simple computations, to build on the acquired theoretical knowledge, and to invoke it in a pertinent and precise manner.

The theoretical questions will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner.

In order to pass the exam in one of the regular sessions, the student needs to obtain a grade of at least 18/30 in each of the two texts, which will contribute equally to the final grade. The two tests needn't be undertaken in the same session. It is also allowed to pass one the tests during the course and the other in a regular exam session.

To each exercise/theoretical question (or problem) a maximum partial grade will be assigned by the commission, depending on its difficulty and length. In the evaluation, every student will be given a grade in correspondence to each exercise/theoretical question (or problem) up to the maximum one, measuring the exactness, the completeness, the rigour, the clarity and the overall coherence of the development.

The exact subdivision of the course in two parts will be communicated well in advance during its duration.

Office hours

Upon appointment.

INTRODUCTION TO NUMERICAL ANALYSIS (2019/2020)

Teachers: Milvia Francesca Rossini, Alessandro Russo

Aims

The aim of this course is to present the basic topics of Numerical Analysis, both from the theoretical and algorithmical point of view, that every Mathematician should know.

The learning outcomes are:

Knowledge:

- Knowledge and understanding of the fundamental techniques of Numerical Analysis, including theorems and proofs;
- Knowledge and understanding of the scientific problems related to numerical analysis.

Skills:

- Being able to translate the theoretical results into algorithms;
- Choose the right numerical method to solve a given problem;
- Critical analysis of the results;
- Being able to present in a clear and precise way the theoretical results shown in the course and their practical applications.

Contents

The main topics are:

- Floating Point Arithmetic
- Numerical Linear Algebra: linear systems, eigenvalues computation
- Root-finding algorithms
- Polinomial Interpolation
- Least squares method and QR decomposition
- Quadrature Formulas

Detailed program

- **Floating Point Arithmetic:** Representation of real numbers, Representable numbers, Approximation of real numbers on a computer, Floating point arithmetic, Rounding to even, Computation of elementary functions;

- **Gaussian elimination and the decomposition PA=LU:** Linear systems, Gaussian elimination, Decomposition PA=LU;

- **Topics in linear algebra:** Scalar products and norms, Norms in R^n, Matrix norms;

- **Stability of the Gaussian elimination:** Perturbation of a linear system, Application to the Gaussian elimination;

- **Cholesky decomposition:** Symmetric and positive definite matrices, Cholesky decomposition, Application to the solution of a linear system;

- **Iterative methods for linear systems:** Motivations, Iterative methods for systems of linear equations, Stopping criteria;
- **Eigenvalues:** Gershgorin circles, Perturbation analysis of eigenvalues, Power method;
- **Root-finding:** Bisection method, Newton methods and variations, Experimental measurement of the order of convergence, Brent method, MATLAB implementation;
- **Polynomial interpolation:** Weierstrass theorem, Interpolation, Analysis of interpolation algorithms, Conditioning, Interpolation of functions;
- **Splines;**
- **Least squares and QR factorization:** Overdetermined linear systems, Geometrical interpretation, QR decomposition, Linear regression, Using QR factorization to solve linear systems;
- **Quadrature formulas:** Interpolatory quadrature formulas, Trapezoidal rule, Simpson rule, Newton-Cotes formulas, Adaptive quadrature formulas.

Prerequisites

First year courses *Analisi 1* and *Algebra Lineare e Geometria.*

Teaching form

Lessons (8 CFU), exercise classes with blackboard and computer (4 CFU).

The course is taught in italian.

Textbook and teaching resource

Teacher's notes are available on the web site of the course. Lectures will be videotaped.

Semester

1st semester.

Assessment method

Practice test on the computer followed by oral examination. Final mark out of 30.

In the first part of the examination the student will be assigned a problem in numerical analysis to be solved on the computer using the MATLAB codes developed during the course.

The second part is a standard oral examination which requires the exposition of statements and proofs of the theorems, definitions, examples/counterexamples and computational techniques.

In order to take the oral examination, the students need to pass the practice test with a mark of at least 18 out of 30. The final mark will take into account both tests.

The practice test and the oral examination must be taken in the same session (january-february, june-july, september).

Office hours

On appointment.

LABORATORY OF MATHEMATICS AND INFORMATICS (2019/2020)

Teacher: Fabio Sartori

Aims

This course aims at introducing the basic *knowledge* of computer systems architecture and networks, as well as different programming pardigms. Moreover, the course will provide *competencies* to identify algorithms to solve simple problems and implementing them into the Java Programming Language, according to the imperative programming paradigm.

Contents

Von Neumann's Model of Calculators. Components and functionalities of operating systems. Introduction to Computer Neworks. Programming Languages. Structured Programming in Java.

Detailed program

Architecture of Calculators

- The Von Neumann model and basic notions on information representation
- Introduction to Operating Sytstems
- Basic notions of Computer Networks

Structured Programming in Java

- Programming languages and translators taxonomy
- The Java Virtual Machine
- Algorithms and programs
- Primitive Data types in Java.
- Flow Control in Java
- Arrays of Primitiva Data Types
- Methods in Java: definition and invocation
- Introduction to recursive algorithm design and implementation

Prerequisites

Nothing

Teaching form

- Lessons, 4 credits
- Laboratory, 2 credits

Textbook and teaching resource

All the information about the course as well as the lessons slides and practical exercises will be available through the learning platform of the University, at the elearning.unimib.it link.

The suggested texdtbook will be:

W. Savitch: "Programmazione di base e avanzata con Java", a cura di Daniela Micucci, 2nd edition, Pearson

Semester

Second semester

Assessment method

Examination type

Written and Oral examination; the oral examination is not mandatory, but necessary to obtain a "cum laude" merit. The mark range is 18-30/30. The oral examination is about both theoretical questions and practical exercises and can increase the result of written examination by at most 4 points.

The written examination is divided into two parts: the first one is devoted to evaluate theoretical skills aobut structured programming, by means of a collection of close-ended questions; the second one concerns the design and implementation of a simple software program, with the aim to demonstrate the student's capability to solve correctly a simple practical problem, on the basis of programming principles considered during the course, without generating any kind of error (i.e. compile time, runtime, logical errors).

The arithmetic mean (possibly weighted) of the two marks defines the final mark proposed to the student: in case it is sufficient, the student can accept it as is or modify it by means of an oral examination (possibly decreasing the final mark). Oral examination is possible if and only if written examination is sufficient. The teacher has the faculty to establish mandatory oral examinations for those students whose written examinations, although sufficient, present some criticalities: for example, in case of not sufficient theoretical questions whereas pratical exercises are good, or viceversa.

Five exam sessions are stated: June, July, September, November and January; moreover two partial examinations are proposed to studets during the course.

Office hours

Thursday, between 11 a.m. and 12 a.m., or by appointment.

LINEAR ALGEBRA AND GEOMETRY (2019/2020)

Teachers: Sonia Brivio, Diego Conti, Francesca Dalla Volta

Aims

In line with the educational objectives of the Degree in Mathematics, the course aims to provide an introduction to linear algebra with applications to geometry, essential to prepare the student to understand the mathematics that will be taught in other courses.

Students are expected to gain knowledge of fundamental notions on vector spaces, diagonalization of endomorphisms and scalar products. They are also expected to gain the ability to reproduce the proofs presented in the course, to solve easy problems using the techniques they have learned, and to delve further, with or without guidance, into some of the results presented during the course.

Contents

Vector spaces; systems of linear equations and affine geometry. Linear maps, matrices; diagonalization of an endomorphism. Scalar products.

Detailed program

- Matrix calculus.
- Systems of linear equations.
- Affine subspaces of R^n and their representations. Distance and orthogonality in R^n
- Vector spaces.
- Linear maps and matrices.
- Determinants.
- Eigenvalues, eigenvectors, characteristic polynomial, diagonalization.
- Dual space.
- Scalar and Hermitian products; Sylvester Theorem.
- Self-adjoint, orthogonal, unitary operators.
- Spectral Theorem.

Prerequisites

A good knowledge of mathematics studied in higher school.

Teaching form

Lectures: 48 hours (6 CFU) in which definitions, results and relevant theorems will be presented, providing examples and problems making use of the notions introduced.

Exercise classes: 24 hours (2 CFU) in which exercises related to the subject matters presented in the lectures are presented and solved. In order to encourage student participation, some exercises are left for the students to solve.

A tutor will aid the students in solving the exercises published on the e-learning website every week.

Textbook and teaching resource

Reference book:

- S. Lang, Algebra Lineare, Boringhieri, III edizione.

Other resources:

- M. Abate, Geometria, McGraw Hill, 2002.

Lecture notes on e-learning webpage.

Semester

First semester.

Assessment method

Written and/or oral exam

The written exam consists of two parts:

a. exercises (with open-ended questions) which allow the teachers to evaluate the student's ability to apply the theory in solving problems;

b. a theoretical question where the student is asked to give complete definitions, statements of theorems and/or provide examples and motivations.

   The examination lasts two hours. The total score (33 points) is divided into 27 points for the exercises and 6 points for the theoretical part. The test is evaluated in terms of correctness. completeness, accuracy and clarity of the solutions.

Oral exam.

Students are admitted to the oral test only if the written exam's score is at least 15 points. This test consists in a first part given by the discussion of the written test and in a second part consisting in the verification by the teachers of knowledge and mastery of definitions, theorems and proofs in the program. Both parts are taken into account in forming the final mark, which is the average of the written and oral exam's scores. The test is evaluated in terms of correctness. completeness, accuracy and clarity of the answers. The exam is passed if the final score is at least 18 points.

Exemption from the oral test.

Students passing the written test with a mark in the range 21-25, with at least 3 points in the theoretical part, are exempted from the oral test, the final mark being equal to the mark obtained in the written test, rounded down. If the score of the written test is greater than 25 it is still possible to be exempted from the oral test, however the final mark in this case will be 25/30.

Exemption from the written test.

In the middle and at the end of the course there will be two *midterm written exams*, passing both midterms allows students to be admitted to the oral test. The assessment criterion is the same as the one outlined above. Access to the oral test is granted with a minimal score of 13 points and an average of at least 15 points. The exemption from the written test is available only for winter session (January or febuary). In this case too the student may be exempted from the oral test, with the same rules written above.

Office hours

By appointment.

MATHEMATICAL ANALYSIS I (2019/2020)

Teachers: Veronica Felli, Graziano Guerra, Simone Secchi

Aims

- To understand the basic concepts and the rigorously developed theory of modern mathematical analysis for functions of a single real variable.
- To master the contents and the techniques in order to solve mathematical problems and to apply them to different contexts.
- To acquire the ability of independently make judgments in the application of the learned methodologies to the solution of mathematical problems.
- To be able to express in a precise, rigorous and exhaustive way both the acquired theoretical knowledge and the solutions, independently worked out, of exercises and problems.
- To be able to learn the contents of the following courses delivered within the Mathematics Degree Course.

Contents

Real and complex numbers. One-variable calculus: limits, continuity, differential calculus, integration. Sequences and series.

Detailed program

1. **Natural numbers.** Peano Axioms, Induction Principle, recursive definitions.

2. **Real numbers.** Field axioms, order axioms, rational numbers, the completeness axiom, Dedekind cuts. The Archimedean property of the real-number system. Supremum and infimum of a set, properties of the supremum and the infimum. Existence of roots of nonnegative real numbers. Rational and real powers. Binary and decimal representation of real numbers.

3. **Complex numbers.** Definition, algebraic form, modulus, conjugate of a complex number, real part and imaginary part, triangle inequality. Trigonometric and exponential form of a complex number, products and power of complex numbers in trigonometric/exponential form. Complex exponentials. Roots of complex numbers. Fundamental theorem of algebra.

4. **Topology of the Real Numbers**. Distance, neighborhoods, interior points, boundary points. Open sets, closed sets. Accumulations points, isolated points. Density of rational numbers in the real numbers. Bolzano–Weierstrass Theorem.

5. **Functions.** Definition, domain, codomain, and range. Injective and surjective functions, bijections. Composition of functions, inverse functions, restriction. Countable sets. Countability of rational numbers and uncountability of irrational numbers. Real-valued functions of one real variable, the graph of a function. Monotonic functions, supremum and infimum, maximum and minimum. Elementary functions and their graphs (powers, exponentials, logarithms, trigonometric functions and their inverses, absolute value function, integer part, fractional part, sign function).

6. **Limits.** Definitions, examples, properties: uniqueness of the limit, Sign Permanence Theorem, Squeeze Theorem. Limit of sum, product, quotient and composition of functions. Special limits.

One-side limits. Limits of monotonic functions. Landau symbols. Comparison of infinitesimals.

7. **Numerical sequences.** Limits of sequences. Boundedness of converging sequences. Subsequences. Existence of a convergent subsequence for a bounded sequence. Heine-Borel theorem. Monotonic sequences. The number *e*. Cauchy sequences. Upper and lower limits.
8. **Continuity.** The definition of continuity of a function. Composite functions and continuity. Sign Permanence Theorem. Bolzano's theorem. The intermediate-value theorem. Continuity of the inverse function. Continuity of elementary functions: powers, exponentials, logarithms, trigonometric functions and their inverses. Sequential criterion for the continuity of a function. Weierstrass theorem. Uniform continuity. Heine-Cantor theorem. Discontinuities. Lipschitz continuity.
9. **Series.** Definition. Convergent series, divergent series. Telescoping series, geometric series. Necessary condition for convergence of series. Absolute convergence. Series of nonnegative terms: comparison test, root test and ratio test. Alternating series: Leibniz's test.
10. **Differential calculus.** The derivative of a function. Geometric interpretation of the derivative as a slope. Left-hand and right-hand derivatives. Continuity of differentiable functions. The algebra of derivatives. The chain rule for differentiating composite functions. Derivatives of inverse functions. Derivatives of elementary functions. Extreme values of functions. Fermat's theorem. Rolle's theorem. The mean-value theorem for derivatives and applications. Relation between monotonicity and sign of the derivative. Cauchy's generalized mean value theorem. De l'Hôpital's rule. Convex and concave functions. The sign of the second derivative and the convexity/concavity of a function. Inflection points. Taylor's formula with Peano form of the remainder. Taylor's formula with mean-value form of the remainder.
11. **Integral calculus.** Step functions, definition of the integral for step functions. Properties of the integral of a step function. Upper and lower integrals on bounded intervals. Riemann integral. Properties of the Riemann integral (linearity, monotonicity). Integrability of the positive/negative part and of the modulus of an integrable function. Integrability of the restriction of an integrable function, integral over oriented intervals, additivity with respect to the interval of integration. Integrability of monotonic functions and continuous functions. Mean-value theorems for integrals. Fundamental theorem of calculus. Antiderivatives. Integration by parts, change of variable. Integration of rational functions. Improper integrals.

Prerequisites

Elementary algebra, elementary trigonometry, elementary analytic geometry.

Teaching form

Lessons (8 CFU), exercise classes (4 CFU).

The course is delivered in Italian

Textbook and teaching resource

Textbook: E. Giusti, Analisi Matematica I, Bollati Boringhieri.

Suggested readings:

- G. De Marco: Analisi Uno, Zanichelli Decibel.
- C. D. Pagani, S. Salsa: Analisi matematica 1, Zanichelli.

Exercise books:

- E. Giusti: Esercizi e complementi di analisi matematica, volume 1, Bollati Boringhieri.
- G. De Marco, C. Mariconda: Esercizi di calcolo in una variabile, Zanichelli Decibel.
- S. Salsa, A. Squellati: Esercizi di analisi matematica 1, Zanichelli.
- E. Acerbi, L. Modica, S. Spagnolo: Problemi scelti di analisi matematica. Vol. 1, Liguori.

Semester

First year, First semester.

Assessment method

Written and oral examination (18-30/30).

The written examination evaluates the knowledge of the course contents and the ability to apply them to problem solving. The oral examination requires the exposition of statements and proofs of the theorems, the definitions, the examples / counterexamples and the calculation techniques.

In both examinations, the correctness of the answers, the mathematical language as well as the rigor and clarity of the exposition will be evaluated.

To pass the exam, a score of at least 15 must be obtained in both the practical and theoretical examinations, the arithmetic mean of the two scores must be at least 18. It constitutes the final grade of the exam.

Office hours

By appointment.

MATHEMATICAL ANALYSIS II (2019/2020)

Teachers: Andrea Giovanni Calogero, Maria Gabriella Kuhn

Aims

A basic course in integral and differential calculus in several variables and ordinary differential equation

Expected learning outcomes include

Knowledge: acquiring the notion of Banach space with some classic examples in mind:

continuous / limited functions on a compact inetrval. Understanding of the definitions and main

results of the differential calculus in several variables and of the theory of ordinary differential equations

Capacity: acquire the main integration techniques for functions of several variables in domains delimited by

regular curves as well as the ability to apply the aforementioned abstract knowledge to concrete problems.

Contents

Complete metric spaces and Banach spaces: examples. Sequences and series of functions. Directional derivatives, differentiable functions, higher order derivatives, critical points and local extrema. Integral calculus in several variables: Fubini's theorem; change of variables; polar, spherical and cilindrical coordinates. Ordinary differential equations: existence, uniqueness and continuous dependence on initial data. Implicit function theorem and Lagrange multipliers.

Detailed program

1. Differential calculus in more than one variable.
    1. Directional derivatives. Differentiable functions. Link between the directional derivatives in the case of differentiable functions. Bonds between continuity and derivability and differentiability. Higer order derivatives. Derivatives of compound functions.
    2. Maximum and minimum in open sets: necessary condition for differentiable functions and level curves.
    3. Positive defined/semidefinite matrices and related Matrix criteria Hessian. Taylor's formula arrested on the second order. Convexity and related criteria for the recognition of theirs extrema.
    4. Recognition of the maximums and minima by the Hessian matrix
2. Integral calculus in more than one variable.
    1. Definition of Riemann integral for a real valued function of several variables.
    2. Measurable sets according to Peano-Jordan: necessary and sufficient condition for measurability in the case of bounded sets. Examples of measurable and non-measurable sets.
    3. Reduction method (Fubini's theorem) for multiple integrals. The center of gravity of a two-dimensional measurable set. Solid volume.
    4. Change of variables in double and triple integrals (*): polar, spherical and cylindrical coordinates. Volume of rotation solids: Guldino's theorem.
    5. Improper integrals.

3. Curves and surfaces.
    1. Regular / piecewise regular curves in R^n. Length of a curve: equivalent definitions and independence from the parametrisation (proof of the Theorem 15.4 of the book of Giusti is not required). Curvilinear abscissa
    2. Theorem of implicit functions (Dini) in the two-dimensional case. Cartesian surfaces; sufficient conditions for one surface to be locally a chart. Vector product and theorem of Dini in the case of a function from R^3 to R^2 (without proof)
    3. Maximum and minimum in compact sets: Lagrange multipliers (proof only in the case of a constraint in R^2).
4. Sequences and series of functions
    1. Normed vector spaces: examples (C ([a, b]), B (I) (with I interval) C^n([a, b]). Banach spaces.
    2. Theorem of contractions.
    3. Punctual and uniform convergence for sequences / series of functions. Weierstrass criterion for the series of functions. Uniform convergence and boundedness/ continuity of the limit function. Pointwise / uniform convergence for derivatives of functions. Limits of sequences of integrable functions.
    4. Power series: convergence radius. Taylor series. approximation of integrals through the use of power series.
5. Differential equations
    1. Cauchy problem for first order equations. Theorem of existence and local uniqueness. Extension of solutions. Maximal solution and related properties (*). Sublinearity condition. Theorem of existence and global uniqueness (proof is not required in the sublinear case).
    2. Equations of order n: equivalence with a system of the first order. Linear systems of the first order. Structure of the space of the solutions of a homogeneous system of the first order. Solutions in the non-homogeneous case. The Variation of constants method in the case of an equation of order n.
    3. Exponential matrix: definition and properties. Calculation of the exponential matrix in the case where the matrix is diagonalizable on the real field. The calculation is required in the other cases only for 3 x 3 matrices.
    4. Solution methods for some particular equations and systems.

Prerequisites

Analysis I, Linear Algebra, Geometry I

Teaching form

lessons (8cfu), exercises (4cfu) and tutoring. Lessons in Italian

Textbook and teaching resource

Textbook: C.Pagani; S.Salsa: Analisi Matematica 2 Ed. Zanichelli

Other teaching resources

Enrico Giusti: Analisi Matematica II ed. Bollati Boringhieri.

A. Bacciotti; F. Ricci: Lezioni di Analisi Matematica 2 Ed. Levrotto & Bella /Torino

C.Pagani; S.Salsa: Analisi Matematica 1 Ed. Zanichelli

Enrico Giusti: Analisi Matematica 2, old edition, Bollati Boringhieri

Semester

First semester.

Assessment method

Oral and written exam: weighted 1/2 and 1/2 of the final mark

In the written test it is required to demonstrate to be able to apply the theoretical contents of the course to solve problems. The oral test requires the ability to expose the statements and proofs of the theorems, the definitions, the examples / counterexamples and the calculation techniques introduced.

During the year there are 5 exam sessions in the following periods: two in the month of February, one in June, one in July and one in September. Each exam session includes a written test and then, in case of passing the written test, a theoretical / oral test a few days away. During the period of the lessons there will be two partial written tests which, in case of a positive overall result, will allow to directly support the oral exam in the month of February.

Minimum grade in the written part: 16/33 to be admitted to the oral exam

Office hours

By appointment.

MATHEMATICAL ANALYSIS III (2019/2020)

Teacher: Bianca Di Blasio

Aims

The course aims at providing the knowledge about the fundamental concepts and statements of advanced mathematical analysis. It will also build the skills needed to understand and use the most important proving arguments and techniques in the theory and the ability to solve exercises and deal with problems exploiting them.

Contents

Banach Spaces. Lp spaces. Hilbert spaces. Fourier series. Baire's Theorem. Open mapping Theorem. Banach Steinhaus Theorem. Hahn Banach Theorem. Dual space. weak convergence.

Detailed program

Definition of Banach space. Examples.

Definition of L ^ p (X, μ), μ positive measure.

Holder and Minkowski inequalities.

Completeness of L ^ p (X, μ).

Inclusions of spaces L ^ p (X, μ), finite μ.

Inclusions of spaces L ^ p (Z).

Relations between pointwise convergence, convergence in Lp, and in measure.

Density of Cc (Rn), Coo (Rn) and of the Schwartz space in L p (Rn).

Duality of Lp spaces (only statement).

Hilbert spaces.

Inner product.

Cauchy-Schwarz Inequality.

Hilbert space.

Points of minimum distance from a closed convex.

Projection theorem.

Bessel inequality.

Complete orthonormal systems.

Parseval formula.

Gramschmidt process.

Fourier series for functions on the thorus

Dirichlet kernel.

Convergence in L2.

Pointwise convergence.

Linear operators between normed vector spaces.

Dual space.

Baire's theorem.

The Banach-Steinhaus Theorem.

Divergence of the Fourier series.

Open Mapping Theorem.

Closed Graph Theorem.

Non surjectivity of the Fourier transform from L 1 (T) into c_0 (Z).

The Hahn-Banach Theorem. Weak convergence.

Prerequisites

Elementary topology. Linear Algebra. Differential calculus to one and more variables. Integral calculus. Measure theory. Complex numbers.

Teaching form

Lectures in the classroom, divided into: theoretical lessons in which the knowledge about definitions, results and relevant examples is given and other lessons in which students solve exercises at the blackboard showing their abilities to use the previous notions to deal with analitical problems.

Textbook and teaching resource

W. Rudin "Real and Complex Analysis"

H. Brezis "Analyse fonctionelle. Théorie et applications"

Notes

Semester

Second semester

Assessment method

Written and oral exam.

During the course students are invited to perform exercises (previously assigned) on the blackboard . For each exercise performed on the blackboard, a point is awarded (for a maximum of 3) which is then added to the written score.

Written exam

The written exam consists of exercises aimed at verifying the understanding of the course contents, the ability to apply the learning demonstration technique, the exposition clarity . Each exercise will be given a maximum partial score, due to its difficulty and length. In the evaluation of the student a score will be assigned based on the accuracy, completeness, rigor, clarity and organic nature of the performance. The maximum grade for the written exam is 33.

The proposed exercises are in line with those carried out during the lessons.

The student is admitted to the oral exam with an evaluation of at least 16 (before adding the points for the exposition on the blackboard).

The oral exam consists in a discussion of the written exam and in theoretical questions (definitions and theorems with proofs). In the oral exam the knowledge and understanding of the course content will be evaluated, as well as the ability to organize a coherent and punctual exhibition in a lucid, effective and well-structured manner.

The final grade is given by the average of the grade of the written exam (including points for the resolution of exercises on the balckboard) and the grade of the oral test.

Office hours

By appointment.

MATHEMATICAL PHYSICS (2018/2019)

Teacher: Giovanni Ortenzi

Aims

Learning the methods for the solution of partial differential equations of Mathematical Physics.

Contents

Introduction to classical partial differential equations of mathematical physics and to the related models: Laplace equation, heat equation and wave equation. Solution methods.

Detailed program

- Introduction to partial differential equations:
  - Maxwell equations, transport equation and Euler equations
- Transport equation
  - characteristics and solution of the initial value problem
- Wave equation
  - Physical models (D'Alembert e Lagrange)
  - Characteristics and casual cone
  - Dependence on the space dimensions: Huygens principle and Kirchhoff solution
  - Lorentz invariance
  - Effects of sources and boundaries (Neumann e Dirichlet)
  - Well-posedness
- Heat equation (Diffusion equation)
  - Physical models (Fick law and probabilistic derivation à la Einstein)
  - Self-similar solutions
  - Fundamental solution and solution of the initial value problem
  - Weak maximum principle
  - Effects of sources and boundaries (Neumann e Dirichlet)
  - Well-posedness
- Comparison between wave and heat equation. Dispersion relation.
  - Hints about the Schroedinger equation
- Laplace equation
  - Radial solutions
  - First and second Green identities
  - Mean property for harmonic functions
  - Strong maximum principle for harmonic functions
  - Dirichlet principle
  - Neumann boundary condition (compatibility conditions) and Dirichlet boundary conditions
  - Poisson equation: representation formula and general solution
  - Green functions

    - Method of images
- Distributions
    - Definition and main properties
    - Dirac delta and Green functions
    - Fourier transform method for computation of propagators
    - Weak solutions
- Burgers-Hopf equation
    - Characteristics and initial value problem.
    - Shocks and their regularization.

Prerequisites

Elements of classical Analysis (I & II). Elements of finite dimensional Euclidean geometry. Elements of Physics (I & II)

Teaching form

Lectures and Exercises.

Textbook and teaching resource

Textbook:

W. Strauss Partial differential equations, Wiley&Sons

Suggested readings:

S.Salsa, Partial differential equations in action, Springer

L.C. Evans, Partial differential equations, AMS

Semester

2nd semester

Assessment method

Oral exam: solution of exercises, statements and proofs of theorems, relevant examples and physics derivation of equations, solutions of exercises proposed in class.

Five exam sessions (January-February, June, July, September).

Office hours

By appointment.

MATHEMATICAL STATISTICS (2019/2020)

Teacher: Daniela Bertacchi

Aims

Knowledge of the basic tools of inferential statistics.

Acquire the ability of applying this knowledge to situation where we have a sample of observations, in order to provide, after a proper choice of the random model, estimates of the unknown parameters and reasonable oinions on the properties of these parameters.

Contents

Statistical models. Inferential statistics: estimators, confidence intervals, hypothesis testing.

Detailed program

The program is the same for attending and non-attending students

Introduction to Statistics

1. Target population, random sample. Sampling problem. Statistics. Density dependent on unknown parameters.

Point estimation of parameters

1. Estimator, unbiased estimator, mean square error, consistency in quadratic mean, necessary and sufficient condition for the consistency in quadratic mean of a sequence of estimators.
2. Estimators for the moments of a VA: sample moments. Sample mean.
3. Unbiased estimator of the variance: sample variance.
4. Method of moments for the construction of estimators.
5. Likelihood function. Maximum likelihood estimator.
6. UMVUE, the lower limit of variance (Cramér-Rao).
7. Invariance property of maximum likelihood estimators. Asymptotic properties of maximum likelihood estimators.
8. Sampling from Normal random variables: the law of the sample mean. Square law of a Normal (0,1): chi-square law with 1 degree of freedom. Law of the sum of squares of independent standard normals: Chi-square law with k degrees of freedom. Law of the sample variance of a normal sample. Student's t law.

Confidence intervals

1. Definition, confidence level.
2. Intervals for the mean of the normal population (known or unknown variance).
3. Intervals for the variance of the normal population (mean known or unknown).
4. Pivotal quantity and its use for the calculation of confidence intervals.
5. Confidence intervals for large samples (especially for frequencies or parameters of Bernoulli).
6. Pivotal quantity for samples from the absolutely continuous laws.

Hypothesis testing

1. Tests for statistical hypothesis, non-randomized tests and the critical region. Significance level, the p-value. Uniformly more powerful tests.
2. Test for a mean of the normal population (variance known or unknown).
3. Test for the variance of a normal population (mean known or unknown).
4. Test for difference of means for normal populations.
5. Test on a frequency and on two frequencies (large sample).
6. Test of simple and generalized likelihood ratio. Neyman-Pearson Theorem.
7. Pearson chi-square test for adaptation (with or without parameters estimated).
8. Pearson chi-square test for independence.

Linear regression

1. Simple and multiple linear regression: definition, interpretation, testing.

Prerequisites

Mathematical analysis I and II, in particular integral calculus.

Basic probability: laws of discrete and continuous random variables. Expected value and variance. Law of functions of random variables. Independence. Convergence of sequences of random variables.

Teaching form

Lessons and exercises in the classroom. Students are advised to exercise at home with the exercises which are provided through the elearning platform.

Textbook and teaching resource

Textbook:

Introduzione alla statistica di A.M.Mood, F.A.Graybill, D.C.Boes, 1991, McGraw-Hill Italia,ISBN: 9788838606618

Other material: slides of the lessons and the exercises on elearning.unimib.it

Semester

Second semester.

Assessment method

Written exam with

1. multiple choiche questions (on the theoretical aspects described in the course)
2. open questions (on the theoretical aspects described in the course, includeing the requirement of writing definitions and statement and proof of theorems)
3. written exercises: application of the theoretical concepts and of techniques like the ones used in the exercises which are assigned in the classroom and at home.

The composition of the written exam may vary in the proportion of the three parts in different sessions, but all the three parts will always be represented. The written exam usually last from a minimum of 1h30' to a maximum of 2h20' (depending on the length of the questions/exercises). In the multiple choice questions we evaluate the capacity of recognizing the correct answers among wrong answers, and the capacity of understanding under which circumstances some properties of the objects, studied in the course, are valid or not. In the rest of the written exam we evaluate the correctness of the answers, the clarity and their completeness. We also evaluate the capacity of discussiong when certain statistical inference methods are more appropriate and when they are only an approximation.

An oral examination may be required by the student and/or the teacher and is a discussion based on the written examination, on the subjects treated during the lessons and eventually on the subject of the linear regression. In the oral examination we evaluate the same qualities of the answers of the written part. Students with an insufficient evaluation but larger or equal to 16/30 may ask to be evaluated orally, and so may all the other students with positive evaluation in the written examination. In case of an oral examination, its mark will have a weight of 1/4 (and 3/4 for the written part). Usually sufficient marks (including the maximum of the gradings) may be confirmed without an oral examination, but the teacher can request an oral examination in all the situations where she judges necessary to ask for explanations on the written exam.

There are two intermediate examinations, which subdivide the programme of the course into two parts. Students attending the lectures may choose to attend these two examinations instead of the final exam. The average mark of the two intermediate examinations substitute for the mark of the final exam. The two intermediate exams usually take place in April and June.

During the year there are 5 examination, usually in the following months: June, July, September, November and January (or February).

Office hours

By appointment.

MEASURE THEORY (2019/2020)

Teacher: Luigi Fontana

Aims

The students will learn the theoretical aspects and the basic analytic applications of Measure and Intrgration Theory.

They will have to understand and be able to present with clarity and rigour the main definitions, theorems and proofs. They also should be able to use what they have learned to solve exercises and problems given during class and at the exam.

Finally they should be ready to apply the ideas and techniques of Measure Theory to the various mathematical and scientific fields where they play a role.

Contents

- The Riemann integral and the problems with passing to the limit.
- Algebras, sigma-algebras and measures. Measurable functions.
- Outer measures, premeasures, extension theorem. Borel and Lebesgue measures.
- Abstract integration. Convergence theorems
- Integration in several variables. Fubini-Tonelli theorem. Change of variables.
- Completeness of L^1.

Detailed program

1. The Riemann integral (an overview) and the problems with passing to the limit. Need of an integral better suited to deal with pointwise convergence of sequences of functions. A possible approach and an obstacle: Vitali's set.
2. Abstract measure theory. Algebras, sigma-algebras and measures. Basic properties and examples. Complete measures. Borel sigma-algebra. Product of sigma-algebras. Measurable functions. Simple functions. Measurability of the pointwise limit of a sequence of measurable functions. Measurable functions as pointwise limit of simple functions.
3. How to construct relevant measures. Outer measures and a way to generate some of them. Caratheodory condition and theorem. Premeasures and the extension theorem. Borel and Lebesgue measures.
4. Abstract integration. Integration of non negative functions. Monotone convergence theorem, Fatou's Lemma. Integration of complex valued functions. The dominated convergence theorem.
5. Integration in several variables. Fubini-Tonelli theorem. Change of variables.
6. Completeness of L^1.

Prerequisites

The basic courses in Analysis of one and several real variables. A good knowledge of general topology and to be familiar with abstract algebra is also recommended.

Teaching form

Frontal teaching.

Textbook and teaching resource

Notes from the instructor, collections of previous written tests, teaching resource from past years.

Main reference text: Folland, Real Analysis, Wiley

Other texts:

- Ambrosio - Da Prato - Mennucci, Introduction to Measure Theory and Integration, Edizioni della Normale.
- Rudin, Real and Complex Analysis,
- Stein - Shakarchi, Real Analysis, Measure Theory, Integration and Hilbert spaces, Princeton

Semester

Spring semester. March-June 2019.

Assessment method

The exam consists of a written part and an oral part. The written part tests how well the student can use in problem solving what he/she has learned in the course. During the oral test, there will be a discussion of the written test and then the student will be asked to present with clarity and rigour some definitions and theorems with their proofs. Passing the written test is mandatory to be admitted to the oral test. The written test is past if the grade obtained is at least 18. In that case the student can choose in which session, within the same academic year, he/she prefers to take the oral test.

The student is admitted to the oral test also with grades 15, 16, 17, but in such a case, he/she MUST take it in the current session.

To pass the exam is required: knowledge and correct use of the convergence theorems; a clear and precise picture of the abstract theory of measure and integration; a good understanding of Borel and Lebesgue measures in one and higher dimensions. The grade will depend on how the student can state, use and prove the main theorems.

The written and oral tests have the same weight in determining the final grade.

There will be six exam sessions in one academic year: june, july, september, november, february/march, april.

Office hours

By appointment, mostly on thursdays 14.00 - 16.00.

NUMERICAL ANALYSIS (2019/2020)

Teacher: Lourenco Beirao Da Veiga

Aims

In line with the educational objectives of the Bachelor Degree in Mathematics, the course aims at providing the basic knowledge, with a deep theoretical support, about the topics of the course (mainly optimization problems, and also discretization of ordinary differential equations). It will also build the skills needed to understand, analyse and compare the different methods, in addition to implementing them in the computer.

Contents

The main part of the course is about optimization problems in R^n, whose resolution is a fundamental step in many applied math problems. We will consider the following topics: search for zeros of functions, then minima of functions, finally constrained minima. The last part of the course will instead consider the discretization of ordinary differential equations.

The course will provide a rigorous theoretical support of the methods considered, together with a computational lab part in MATLAB.

Detailed program

All the topics developed in class will have also a coding part in the computer Lab (MATLAB language). Some labs will consider PDE problems that, after discretization/approximation by some numerical scheme, become optimization problems in R^N. We will consider the following topics. Iterative methods for fixed points, local and global convergence properties. Search of zeros of vector valued functions, quasi-Newton methods, examples, local convergence, modifications for global convergence. Search of minima of functions (in open sets), line search methods, examples, convergence properties. Search of constrained minima, Kuhn-Tucker and lagrangian theory, projected gradient, Uzawa method, convergence properties. Ordinary differential equations, one step methods, convergence theory, absolute stability, RK methods.

Prerequisites

The standard knowledge of a third year math student is sufficient

Teaching form

Standard blackboard classes, plus practice classes in the computer Lab.

Textbook and teaching resource

• C.T. Kelley, “Iterative methods for linear and nonlinear equations”, SIAM

• J. Nocedal, S.J. Wright, “Numerical Optimization”, Springer

• P.G. Ciarlet, “Introduction to numerical linear algebra and optimizations”, Cambridge Texts in Applied Math

• Uploaded pdf text on the Ordinary Diff. Eq. part

Semester

Second semester.

Assessment method

The exam is an oral examination, and is divided into two parts. In the first part, the student presents a matlab laboratory project (to be developed individually), chosen by the teacher among a set of three previously selected by the student (these are 3 among the projects developed in the Lab during the course). The second part is an evaluation of the critical and operational knowledge of the definitions, results and proofs presented during the course. The relative weight of the project and the theoretical examination are roughly 30% and 70%, respectively. In the project discussion the teacher will evaluate the exactness of the results and the comprehension of the practical/computational aspects of the adopted numerical method. During the theoretical part of the exam, the teacher will mainly evaluate the comprehension of the topic and the mathematical rigour in presenting the numerical methods and the associated proofs.

Office hours

Flexible, arranged directly via email.

PHYSICS I (2019/2020)

Teacher: ---------------

Aims

Provide the students with good knowledge of the basics of classical mechanics, thermodynamics and, for the students of the Degree in Physics, of special relativity. Acquire the ability to schematize a phenomenon to identify the laws that govern it. Acquire the ability to identify the relevant physical laws for the resolution of different types of exercises.

Contents

Kinematics and dynamics of the massive particle, work of a force, energy.

Kinematics and dynamics of systems of massive particles and of a rigid body.

Kepler's laws for the motion of the planets and Newton's law of gravitation.

Ideal gas, first and second principle of thermodynamics, entropy.

Lorentz transformations for time and space. Mass and energy in relativity.

Detailed program

The experimental method and the operational definition of measurable quantities. Systems of units, fundamental units, lengths, times, masses.

Vectors:

Properties of a vector space, the vector displacement, sum, difference and multiplication of vectors, scalar product and vector product among vectors. Components of a vector.

Equations of components for a basis change.

Kinematics of a particle:

Vector position, and displacement. Definition of instantaneous velocity and acceleration. One-dimensional case: uniform motion, motion with constant acceleration.

Uniform motion and motion with constant acceleration in three dimensions with the example of the parabolic motion of a projectile.

Derivative of a vector, intrinsic representation of velocity and acceleration, tangential and normal components of the acceleration.

Polar coordinates.

Circular motion, velocity and angular acceleration, centripetal and tangential acceleration in circular motion. Uniform and uniformly accelerated circular motion.

Harmonic motion, characteristics of motion for x, v, a. Differential equation of harmonic motion.

Relative motions (only translation of origin), transformations of Galileo for r, v, a between O and O '. Galileo's principle of relativity.

The vector angular velocity and description of the motion of a particle through the angular velocity.

Velocity and acceleration in a moving reference frames (rf) (with rotations and translations), Coriolis acceleration.

Dynamics of the material point:

Newton's law, inertial reference frames (irf).

Concept of interaction, static operational definition of force, experimental observations on forces and accelerations that lead to Newton's II law.

Fundamental forces and empirical forces. Weight, normal force (as a constraint on the motion).

III Newton's law.

Static and dynamic friction forces, motion on an inclined plane.

Frictional force in fluids: F = -kv, equation of motion.

Tension of an ideal string, example with the Atwood machine.

Description of the motion of a pendulum.

Elastic force and ideal spring.

Work, definition with integral curvilinear.

Relationship between work and kinetic energy.

Conservative forces, potential energy, mechanical energy.

Examples for a constant force, weight strength, elastic force.

F = - grad U, equilibrium (stable or unstable).

Description of the dynamics in non-inertial rf: fictitious (inertial) forces.

Definition of momentum, average force.

Systems of particles:

Definition of center of mass (CM) for a system of particles and for a body, momentum of a system and relationship with Forces.

Angular momentum of a system, torque of a forces, relationship between angular momentum and torque.

Angular momentum in the CM reference frame. Koenig's theorem for angular momentum.

Kinetic energy of a system. Koenig's theorem for kinetic energy.

Work of in a system of particles (external and internal). Potential energy in a system.

Description of the motion of a two-body system in the absence of external forces and reduced mass.

Impulsive forces in collisions. Elastic and anelastic collisions. Elastic collisions in the CM rf, discussion for 1D collisions, in the CM rf and in the laboratory rf.

Dynamics for rigid bodies:

Definition of a rigid body and degrees of freedom of a rigid body.

Motion of translation, motion of rotation around a fixed axis (RFA) or combined.

Moment of inertia. Kinetic energy and angular momentum (along the axis of rotation) for RFA. Examples with L not parallel to the rotation axis. Dynamic equations for a rigid body , work in a RFA.

Huigens-Steiner's theorem.

Physical pendulum.

Static of a rigid body, levers.

Effect of an impulse on a rigid body free or bound to RFA.

Examples of collisions between rigid bodies.

Rolling without slipping motion, example on an inclined plane.

Variable mass systems: example of the rocket.

Harmonic oscillator:

Equation for a free harmonic oscillator. Dumped harmonic oscillator: equation, complex solutions, large dumping and small dumping.

Forced and dumped oscillator equation and solution, transferred power and resonance.

Gravitation:

Kepler's laws, derivation of Newton's law of gravitation.

Newton's law of gravitation, two-body problem.

Gravitational potential energy, potential energy and trajectories,

Potential, kinetic and mechanical energy for circular orbits.

Gravitational force potential energy of an extended body.

Gravitational force for a homogeneous sphere.

Example with determination of the tidal force for the earth (moon and sun) system.

Derivation of the equations of planets starting from Newton's law.

Waves:

Concept of wave, progressive and regressive wave, D'Albert's equation.

Sinusoidal waves. Wave on a string. Waves on a solid bar. Energy carried by the waves. Average power for sinusoidal waves. Reflection of waves in a string . Impedance of a medium.

Reflection and transmission to the interface of two media with different Zs.

Pressure waves in gases. Acoustic waves (decibels).

Coupled pendulums, normal modes, energy in normal modes.

Overlap of waves: stationary waves, example with a string, interference, beats.

3D waves (hints), planar and spherical waves.

Doppler effect and Mach cone.

Thermodynamics:

System and environment, thermodynamic variables, equilibrium states, thermodynamic transformations, principle 0, thermometric quantities and temperature, thermometers. Pressure.

Work of a gas. Internal energy and the first principle of thermodynamics. Historical definition of heat. Calorimetry, thermal capacity and specific heat. Phase transitions.

Boyle's law, Gay-Lussac's law, equation of state of an ideal gas. Work (W) of a gas for isocore, isobar and isothermal transformations (reversible). Internal energy of an ideal gas. Relationship Cp = Cv + R. Equation of a reversible adiabatic transformation.

Gas transformations: Q, ΔU, W for isochor, isobar, isotherm and adiabatic transformations.

Cyclic transformations, efficiency for thermal cycles, coefficient of performance and heat pumps.

Second principle of thermodynamics (Kelvin-Planck and Clausius), Carnot's theorem, thermodynamic temperature. Clausius theorem. Definition of entropy.

Examples of entropy variation: gas transformations, temperature variations of solids or liquids, phase transformations.

Clayperion equation.

Transformations in the T-S plane.

Heat propagation mechanisms: convection, conduction, irradiation.

Kinetic theory of gases, relationship between T and average quadratic velocity and average kinetic energy for monoatomic gases,

Interpretation of internal energy and CV. Average kinetic energy and T for diatomic and polyatomic / solid gases, mentioning quantum effects.

Maxwell-Boltzmann distribution for gas velocity.

Hints on statistical interpretation of entropy.

The following topics concern only the students of the Physic degree.

Fluids:

Definition of fluid, normal stresses and shear stresses, volume forces.

Stevino's law, Pascal's law. Link between pressure and volume forces.

Case of conservative forces. Changes in a non-inertial frame (example rotating fluid).

Archimede's law. Thrust center. Example in non-inertial frame.

Ideal fluid in motion: Bernoulli equation.

Real fluids, viscosity, hydraulic impedance, Poiseuille law, Reynolds criterion.

Special relativity:

Newton's laws and invariance for Galileo's transformations, Galileo's principle of relativity.

Maxwell equations and incompatibility with of Galileo's transformations, ether theory, Michelson-Morley measure. Principles of relativity (constancy of c), deduction of Lorentz transformations, proper time and time dilatation, length contraction, Doppler effect for electromagnetic waves.

Principle of conservation of relativistic momentum, mass and energy. Conservation of E and p in relativistic collisions. Space-time diagrams, space type and time type separations. Relativistic invariants. Hints of quadrivector and Minkowski metric formalism.

Prerequisites

Basic knowledge of mathematic (capability to solve equations and sistems of equations).

A basic knowledge of calculus (differential and integral) is recommended.

Teaching form

Lectures and exercise sessions.

Textbook and teaching resource

- Mazzoldi, Nigro, Voci, Fisica 1, EdiSES (Meccanica e termodinamica). (avalaible only in Italian)
- Halliday, Resnick, Krane, Fisica 1, Ambrosiana. (also in the original version in English)
- For the relativity topic: R.Resnik, Introduzione alla relatività ristretta. (also in the original version in English)

Semester

October - November: mechanics and dynamics of a massive particle (4 CFU).

December - January: mechanics and dynamics of systems of massive particle and rigid bodies (4 CFU).

March - April: Mechanical waves and thermodynamics (4 CFU).

May - June: Fluid Mechanics and Special Relativity (4 CFU).

Assessment method

A written test and an oral test are required after passing the written test.

The written test can be replaced by four tests on specific topics during the course, two per semester. The students of the Degree Course in Mathematics following only the first 12 credits, must take only the first three tests.

The written test is considered replaced if the result is not insufficient in 3/4 of the tests [or 2/3 of the tests for those who follow only 12 credits]. Absence counts as insufficient.

After passing the written test it is possible to take the oral exam in any exam session, within the academic year. The passing of the written test remains valid even after a failure to pass the oral exam.

The oral ones are made starting from the calendar for the session and in the following days. Normally, after the closing date to subscribe a given session, a detailed calendar of the oral exams dates will be announced via e-learning.

Office hours

Usually the teacher is always available for reception, however the presence is guaranteed only if previously arranged, either in classroom or by e-mail.

PHYSICS II (2019/2020)

Teacher: Alessandro Tomasiello

Aims

Maxwell Equations, Special Relativity.

Expected learning targets:

- knowing how to solve simple problems in electrostatics, magnetostatics, magnetic induction, RLC circuits;

- knowledge of Maxwell equations, their conceptual base and derivation, relationships among them; knowledge of the basics of special relativity, and of its relationship with electromagnetism.

Contents

Electrostatics: Coulomb's law, Gauss' Law. Electric currents: Ohm's law.

Special relativity. Magnetostatics: Biot-Savart equation, Ampere's Law.

Magnetic induction; Faraday's law. LRC circuits.

Maxwell's equations. Electromagnetic waves. Poynting vector. Relativistically covariant notation for electromagnetism.

Detailed program

- Electrostatics. Coulomb's law; electric field, electric potential. Gauss' Law. Poisson's equation; Laplacian. Energy of the electric field. Curl of the electric field. Harmonic functions. Conductors. Capacitors. Exterior calculus.
- Moving charges. Electric current; Ohm's law. RC circuits.
- Special relativity. Lorentz transformations; four-vector notation.
- Magnetostatics. Deduction of the existence of magnetic field; its divergence and curl. Vector potential.
- Magnetic induction. Circuits moving in a magnetic field; Faraday's law. Inductance. Energy of the magnetic field. LRC circuits. Applications: power lines, radio.
- Maxwell's equations. Time-dependent currents. Electromagnetic waves. Poynting vector. Relativistically covariant notation for the electromagnetic field and for Maxwell's equations. Exterior calculus in spacetime.

Prerequisites

Physics I, Analysis I & II

Teaching form

Lectures (6 CFU); Exercise sessions (2 CFU)

Textbook and teaching resource

Lecture notes available at https://www.dropbox.com/s/s2kvegmy9t0xc5t/EM.pdf?dl=0

D. J. Griffiths, Introduction to electrodynamics. Prentice Hall, 1999.

E. M. Purcell and D. J. Morin, Electricity and magnetism. Cambridge University Press, 2013.

Semester

first semester.

Assessment method

Written and oral exam, of equal weight in the final evaluation, not necessarily in the same call.

Written exam: four exercises, three hours. Admission to the oral exam with 14/30. Object of evaluation will be the logic used in the resolution of the problems.

Oral exam: open questions on the course's topics, unrelated to the written exam. Object of evaluation will be the candidate's knowledge of the theoretical part of the program.

Office hours

by appointment.

PROBABILITY THEORY (2019/2020)

Teachers: Elena Bandini, Francesco Caravenna

Aims

The course aims at providing students with the basic concepts and tools of probability theory, together with an illustration of some applications. At the end of the course students will have acquired the following:

- *knowledge*: language, definitions and statements of the fundamental results in probability theory;
- *competence*: operational understanding of the main proof techniques;
- *skills*: ability to apply theoretical notions to the solution of exercises and the analysis of problems.

Contents

1. Probability spaces
2. Random variables
3. Convergence of random variables
4. Introduction to Markov chains
5. Examples of probability models

Detailed program

1. *Probability spaces*

- Introduction to probability: mathematical models for a random experiment
- Axioms of probability
- Basic properties of probability, continuity from above and below
- Combinatorics, uniform probability spaces
- Conditional probability, Bayes theorem
- Independence of events, Bernoulli trials

2. *Random variables*

- Reminders of measure theory
- Important distributions, discrete and continuous, on the real line
- Random variables
- Marginal laws and joint law
- Independence of random variables
- Transformations of random variables
- Expected value, variance and covariance
- L^p spaces, inequalities (Jensen, Cauchy-Schwarz, Hölder)
- Correlation coefficient and linear regression (hints)

3. *Convergence of random variables*

- Reminder on convergence theorems in the theory of integration
- Borel-Cantelli lemma

- Weak and strong law of large numbers
- Notions of convergence for sequences of random variables (a.s., in probability, in L^p)
- Weak convergence of probabilities, convergence in law of random variables
- Law of small numers (weak convergence of the binomial to the Poisson distribution)
- Central limit theorem via Lindeberg's principle
- Central limit theorem via characteristic functions (hints)
- The method of normal approximation
- Independence of sigma-algebras, Kolmogorov's 0-1 law

4. *Introduction to Markov chains*

- Introduction to stochastic processes, finite-dimensional distributions
- Markov chains, transition matrix, Markov property
- Recurrent and transient states, invariant and reversible measures
- Convergence theorem (hints): convergence to equilibrium, law of large numbers
- Absorption probabilities (hints)
- Random walks on graphs (hints)

5. *Examples of probability models (presented alongside the theory)*

- Classical paradoxes (birthdays, Monty-Hall, Borel, Bertrand)
- Random permutation and fixed points
- Concentration properties of volume in high-dimensions
- Weierstrass' approximation theorem and the law of large numbers
- Simulation of random variables, the Monte Carlo method
- Simple random walk in one and more dimensions
- Gambler's ruin
- The PageRank algorithm

Prerequisites

The knowledge, competences and skills taught in the courses of the first two years, in particular *Linear Algebra, Analysis 1 and 2 (= calculus in one and more variables), Measure Theory*.

Teaching form

Lectures and recitations in the classroom, divided into:

- theoretical lectures (10 ects) focused on the knoledge of definitions, results and relevant examples, as well as the competences linked to their comprehension;
- recitations (2 ects) focused on the skills necessary to apply the theoretical knoledge and competencies to the solution of exercises.

The course is given in Italian.

Textbook and teaching resource

Reference textbooks

- F. Caravenna, P. Dai Pra. *Probabilità. Un'introduzione attraverso modelli e applicazioni.* Springer-Verlag Italia, Milano (2013).
- D. Williams. *Probability with Martingales.* Cambridge University Press (1991).

Other dydactical material (available on the e-learning page of the course)

- Notes by the teacher on specific arguments
- Weekly exercise sheets (with detailed solutions)
- Written exams from previous years (with detailed solutions)
- List of proofs for the oral examination

Semester

Third year, First (Fall) Semester.

Assessment method

Written examination (or midterms) and oral examination, with the rules described in the sequel. The aspects that will be evaluated are the correctness of the answers, the creativity, the precision, the clarity of exposition. There will be 5 exam sessions (two in February, one in July, one in September, one in January).

- The *written examination* lasts 3 hours and gets a mark out of 30. This examination tests practical skills (solving exercises) and also theotetical knoledge and competencies (definitions, examples and counter-examples). The written examination is passed with a minimal mark of 15/30 and allows to be admitted to the oral examination.
- In the middle and at the end of the course there will be two *midterm written exams,* which last 1.5 hours each and get a mark out of 15. Passing both midterms with a minimal mark of 7,5/15 is equivalent to passing the written examination (with the "sum" of the marks) and allows to be admitted to the oral examination.

- The *oral examination* lasts 30-45 minutes and gets a mark out of 30. It can be given (after passing the written examination) in any exam session of the same academic year. The oral examination tests the knowledge of a selection of proofs as well as a working knowledge of the notions of the course. The oral examinations is passed with a minimal mark of 15/30.
- The final mark results from the average between the marks of the written and oral examinations. The exam is passed with a minimal mark of 18/30.

Exemption from the oral examination. Passing the written examination with a mark in the range 20-27/30 allows to be exempted from the oral examination, the final mark being equal to the mark obtained in the written examination; with a mark greater than 27/30 it is still possible to be exempted from the oral examination, however the final mark in this case will be 27/30; finally, with a mark smaller than 20/30 it is necessary to take the oral examination.

Office hours

To be fixed at the beginning of the course and communicated in the e-learning page.

GRADUATE PROGRAM (F-40)

Students are given the opportunity to draw up a personalized curriculum with courses of their own choice, offered in the Mathematics graduate degree course and in other degree courses in Bicocca University (see tables A-B-C below) and Politecnico of Milano (table D). Personalized curricula shall include a combination of theoretical and practical courses in different academic disciplines, totalling a minimum amount of ECTS (120 ECTS over 2 years including ICT – 1 credit and preparation of dissertation/thesis – 39 ECTS).
It is recommended that students contact lecturers for advice on the choice of subjects.
The curriculum shall be submitted to the relevant Committee for approval.
On request, the courses will be given in English.

First year

Course	ECTS
6 courses from table A: - at least two courses from the Academic Disciplines MAT/02-MAT/03-MAT/05 - at least one course from the Academic Disciplines MAT/06-MAT/07-MAT/08	48
2 courses from table B	16

Second year

Course	ECTS
2 course from tables A or B or other courses of the University, or given in other Universities and for which a specific convention exists, provided they conform to the structure and aims of the program	16
Elaboration of Mathematical Texts (ICT)	1
Thesis	39

Table A

Course	Academic Discipline	ECTS
ALGEBRAIC COMBINATORICS	MAT/02	8
APPROXIMATION OF ORDINARY DIFFERENTIAL EQUATIONS	MAT/08	8
COMPLEX GEOMETRY*	MAT/03	8
DIFFERENTIAL GEOMETRY	MAT/03	8
FUNCTIONAL ANALYSIS	MAT/05	8
GEOMETRY AND PHYSICS	MAT/07	8
HARMONIC ANALYSIS	MAT/05	8
HIGHER ANALYSIS	MAT/05	8
HIGHER MECHANICS	MAT/07	8
METHODS OF MATHEMATICAL PHYSICS	MAT/07	8
MODELS AND METHODS OF APPROXIMATION	MAT/08	8
NUMBER THEORY AND CRIPTOGRAPHY	MAT/02	8
NUMERICAL LINEAR ALGEBRA	MAT/08	8
NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS	MAT/08	8
REAL ANALYSIS AND DIFFERENTIAL EQUATIONS	MAT/05	8
REPRESENTATION THEORY	MAT/02	8
STOCHASTIC METHODS IN FINANCE	MAT/06	8
STOCHASTIC PROCESSES	MAT/06	8
SYMPLECTIC GEOMETRY**	MAT/03	8

Table B

Insegnamento	ECTS

ADVANCED NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS	8
CALCULUS OF VARIATIONS	8
ELEMENTARY MATHEMATICS	8
GEOMETRIC GROUP THEORY (delivered in English)	8
HISTORY OF MATHEMATICS	8
HISTORY OF MATHEMATICS - ELEMENTS	4
MATHEMATICAL METHODS FOR ECONOMIC ANALYSIS – OPTIMAL CONTROL	8
MATHEMATICAL METHODS FOR ECONOMIC ANALYSIS – OPTIMIZATION AND CONVEX ANALYSIS	8
MATHEMATICAL METHODS FOR MODERN PHYSICS (delivered in English)	8
PREPARATION OF DIDACTIC EXPERIMENTS	8
TEACHING MATHEMATICS	8
TOPICS IN GEOMETRY AND TOPOLOGY	8

*** Given in the first year**

**** Given in the second year**

Table C

Courses	**ECTS**
METHODS OF SCIENTIFIC CALCULUS (GRADUATE PROGRAM - INFORMATION TECHNOLOGY)	6

Table D

Courses offered in the Graduate program in Mathematical Engineering at Politecnico-Milano

Courses	**ECTS**
ADVANCED PROGRAMMING FOR SCIENTIFIC COMPUTING	10
ALGORITHMS AND PARALLEL COMPUTING	10
APPLIED STATISTICS	10
BAYNESIAN STATISTICS	10
BIOMATHEMATICAL MODELING	8
COMPUTATIONAL FINANCE	10
COMPUTATIONAL FLUID DYNAMICS	10
COMPUTATIONAL MODELING IN ELECTRONICS AND BIOMATHEMATICS	8
DISCRETE DYNAMICAL MODELS	8
FINANCIAL ENGINEERING	10
FLUIDS LABS	10
GAME THEORY	8
MATHEMATICAL AND PHYSICAL MODELING IN ENGINEERING	10
MATHEMATICAL FINANCE II	10
OPTIMIZATION	8
REAL AND FUNCTIONAL ANALYSIS	8
STOCHASTIC DIFFERENTIAL EQUATION	8

GRADUATE COURSES DESCRIPTION

ADVANCED NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS
ALGEBRAIC COMBINATORICS
APPROXIMATION OF ORDINARY DIFFERENTIAL EQUATIONS
CALCULUS OF VARIATIONS
COMPLEX GEOMETRY
DIFFERENTIAL GEOMETRY
ELEMENTARY MATHEMATICS
FUNCTIONAL ANALYSIS
GEOMETRIC GROUP THEORY
GEOMETRY AND PHYSICS
HARMONIC ANALYSIS
HIGHER ANALYSIS
HIGHER MECHANICS
HISTORY OF MATHEMATICS
HISTORY OF MATHEMATICS - ELEMENTS
MATHEMATICAL METHODS FOR ECONOMIC ANALYSIS – OPTIMAL CONTROL
MATHEMATICAL METHODS FOR ECONOMIC ANALYSIS – OPTIMIZATION AND CONVEX ANALYSIS
MATHEMATICAL METHODS FOR MODERN PHYSICS
METHODS OF MATHEMATICAL PHYSICS
NUMBER THEORY AND CRIPTOGRAPHY
NUMERICAL LINEAR ALGEBRA
NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS
PREPARATION OF DIDACTIC EXPERIMENTS
REAL ANALYSIS AND DIFFERENTIAL EQUATIONS
REPRESENTATION THEORY
STOCHASTIC METHODS IN FINANCE
STOCHASTIC PROCESSES
SYMPLECTIC GEOMETRY
TEACHING MATHEMATICS
TOPICS IN GEOMETRY AND TOPOLOGY

ADVANCED NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS (2019/2020)

Teacher: Lourenco Beirao Da Veiga

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing knowledge on some important advanced aspects of the finite element method, building a strong theoretical basis but also a good critical sense for applications. It will also build the skills needed to understand, analyse and compare the different methods, in addition to implementing and using them in the computer.

Contents

This course is about the approximation of problems in partial differential equations through the finite element method, and can be considered a second and more advanced stage of the course "Approximation of Partial Differential Equations". In particular, the course will treat important topics such as time dependent problems and problems in mixed form, that play a key role in many applications (such as fluidodynamics). Part of the course will be in the computer lab (MATLAB).

Detailed program

Brief review of the fundamental notions of the finite element method and main results for standard elliptic problems. The (non-stationary) heat diffusion problem, discretization in time and space, theoretical analysis of the method, implementation in MATLAB. A posteriori error analysis in the stationary case, theoretical analysis, implementation, adaptive algorithm. Problems in mixed form, Stokes as a model problem, discretization and difficulties, general theory of mixed methods, implementation. Diffusion in mixed form, theoretical analysis, implementation. Possible additional topics will be treated at the end of the course.

Prerequisites

Basic notions of functional analysis are needed. It is moreover required to have followed the course "Approximation of Partial Differential Equations". The course will have a strong theoretical component.

Teaching form

Standard blackboard lessons and computer practice labs.

Textbook and teaching resource

- D. Braess, “Finite Elements: theory, fast solvers, and applications in solid mechanics”, Cambridge University Press (alternative: P.Ciarlet “The finite element method for elliptic problems” oppure S.Brenner e R.Scott, “The mathematical theory of finite element methods”)
- D. Boffi, F. Brezzi, M. Fortin, “Mixed finite element methods and applications”, Springer
- V. Thomee, “Galerkin Finite Element Methods for Parabolic Problems”, Springer

Semester

First semester.

Assessment method

The exam is an oral examination, and is divided into two parts. In the first part, the student presents a matlab laboratory project, that the student choses among some projects proposed by the teacher at the end of the course. The students can work in groups of 1-3 members for the development of the project (is thus allowed to work individually or as a team, but the discussion will be anyway personal). The second part of the examination is an evaluation of the critical and operational knowledge of the definitions, results and proofs presented during the course. There relative weight of the two parts, project and theory, is roughly 40% and 60%, respectively.

Office hours

Email appointment.

ALGEBRAIC COMBINATORICS (2019/2020)

Teacher: Andrea Previtali

Aims

In line with the aims of the CdS, the course will provide students the knowhow necessary to deal with transmission of information via noicy channels, in order to analyze optimal error-correcting and -detecting procedures. Time permitting some rudiments of programming languages as Magma and Gap will be imparted. These tools serve to emphasize sperimental aspects of mathematical discovery. We will also impart the necessary skills to comprehend and analyze the main technical and proof methods.

Those will be tested via problem solving and resolutionef exercises related to the contents of the course.

Contents

Information Theory, messages transmission, error probability, entropy, Shannon's Theorem, symmetric channel, Error-correcting codes, alphabet, finite fields, linear codes, Hamming codes, cyclic codes, Reed-Solomon and Reed-Muller codes, Weight preserving maps, MacWilliams' Theorems, Invariant Theory of Finite Groups.

Detailed program

1. Information Theory, messages transmission, noisy channels, error probability, entropy, Shannon's Theorem, symmetric channel.
2. Error-correcting codes, alphabet, finite fields, linear codes, Hamming codes, cyclic codes, Reed-Solomon and Reed-Muller codes.
3. Upper and lower Bounds, Sphere Packing, Gilbert-Varshamov, Perfect codes and their classification.
4. Weight preserving maps, MacWilliams' Theorem, Monomial maps, Wilson Theorem.
5. Weight enumerator polynomials, MacWilliams' Theorem, self-dual codes, isotropic vectors, Witt Theorem,.
6. Invariant Theory of Finite Groups, primary and secondary invariants, Cohen-Macaulay rings, groups generated by pseudo-reflection, Shephard-Todd Theorem.

Prerequisites

Algebra I and II, Linear Algebra, Group Theory, Finite Field Theory, Elementary notions of Thermodynamics and Probability.

Teaching form

Lessons: 8 credits

Textbook and teaching resource

Textbooks:

- Hall, Notes on Coding Theory, 2005
- Tablet taken notes available on this platform.

Further Readings:

- Huffman and Pless, Fundamentals of error-correcting codes, 2010
- MacWilliams and Sloane, The Theory of Error-Correcting Codes, 1977
- Smith, Polynomial invariants of finite groups, 1995

Semester

Second semester.

Assessment method

The exam starts with a discussion concerning the solution with the support of the programming language Magma of a previously assigned problem.

This will immediately be followed by an oral enquiry assessing both the student's acquisition of the course contents and her/his capabilities of analyzing and solving problems. Both aspects equally contribute to the final score.

In case of failure the student will be told if a new problem (or a better solution for the old one) is requested in order to face a new oral exam.

Office hours

By appointment.

APPROXIMATION OF ORDINARY DIFFERENTIAL EQUATIONS (2019/2020)

Teacher: Blanca Pilar Ayuso De Dios

Aims

The main goals of the course are:

- Knowledge (and understanding) of the different numerical methods for ODEs
- Ability to construct and analyse numerical methods for approximating systems of ordinary differential equations
- Ability to choose the appropriate numerical method for concrete problems
- Ability to interpret and analyse numerical results
- Ability to implement the resulting numerical algorithms efficiently

Contents

This course is concerned with the development and analysis of numerical methods for differential equations. Topics covered include: well-posedness of initial value problems, analysis of Euler's method, Runge-Kutta methods, methods for stiff problems and Geometric numerical integration approaches.We shall cover the subject from mathematical point of view, studying how to construct modern computational algorithms, exploring their properties and validating the algorithms in concrete problems.

Detailed program

0- Introduction.

Recap of the theory of ordinary differential equations and systems of ODEs. Well posedness results. Recap on numerical integration (numerical quadrature)

1. -One-step schemes:

Euler method. Convergence theory. Explicit Runge-Kutta (RK) methods. Convergence Theory. Hint on Order conditions . Richardson extrapolation. Embedded Runge-Kutta methods.

2.- Collocation methods(I).

Recap on Gaussian Quadrature. Construction of collocation methods. Convergence analysis of implicit RK.

3.- Linear Stability and Stiffness.

Linear Stability lineare. Stability of RK methods. Stiff problems.

BDF method (Backward Differential Formula).

4.- Collocation methods (II). Implementation of implicit RK. Partitioned methods and Splitting methods: Trotter-Lie and Strang splittings.

5.- Geometric integrators.

Hamiltonian Systems. Numerical conservation of invariants. Symmetric integrators. Symplectic integrators. Stormer-Verlet method.

Depending on how the course proceed and the interest of the students we might cover either:

- Introduction to basic numerical integrators for Stochastic differential equations

or

- Application of the contents of the course for the solution of evolutionary PDEs: Hyperbolic-type, Parabolic-type, Conservation Laws,…

Prerequisites

Solid knowledge of Analysis, Linear Algebra and basic Numerical Analysis.

Solid knowledge of Ordinary differential equations and basic knowledge of MATLAB

Teaching form

Lectures in class and in the Lab.

We will use MATLAB for all computer examples, exercises and projects.

Textbook and teaching resource

Different material will be provided during the course. The course has a big practical component for which we will use MATLAB

We will use several books(several chapters in each of them to cover the different topics)

Bibliography:

-E. Hairer and S. P. Norsett and G. Wanner, "Solving Ordinary Differential Equations I ", Springer, Berlin, 1993.

-E. Hairer and G. Wanner, "Solving Ordinary Differential Equations II ", Springer, Berlin, 1996.

-E. Hairer, C. Lubich and G. Wanner, "Geometric Numerical Integration", second edition, Springer, Berlin, 2006.

-B. Leimkuhler and S. Reich, "Simulating Hamiltonian Dynamics", Cambridge University

Press, 2005.

Semester

First semester

Assessment method

The evaluation of the course has two parts:

1- the development of a small project

2- a small (oral or written) exam. Specifics on the oral or written exam will be given later on during the course.

The small project could be chosen from a list of projects that will be made available to the students towards the end of the course. Students are encouraged to work on the project **in groups** of at most two or three people. The project should be handed four days before the before the date of the small exam. Part of the small exam will be devoted to the discussion of the project, allowing to validate the knowledge and capabilities of the students related to the course.

Office hours

By appointment (that should be fixed by writing an email to me)

CALCULUS OF VARIATIONS (2019/2020)

Teachers: Mauro Garavello, Simone Secchi

Aims

The objectives of the course are the following.

Knowledge and understanding. The student will learn the modern theory of Calculus of Variations and will become acquainted with advanced tools and techniques in the study of variational problems.

Applying knowledge and understanding. By means of several examples and exercises, the student will develop the ability of applying the theorical results presented in the lectures to specific problems of minimization and optimal control.

Making judgements. The student will be able to face critically variational, minimization, and control problems, identifying by himself/herself the most appropriate tools among those introduced in the course.

Communication skills. The student will become familiar with the language and formalism of Calculus of Variations, which will make him/her able to communicate with rigor and clarity the acquired knowledge.

Learning skills. The student will be able to apply the acquired knowledge to different contexts and to examine in depth some related topics by autonomous reading of scientific literature.

Contents

- The problem of the existence of a solution. The Direct Method. The minimal surface problem.
- Necessary conditions.
- Regularity of solutions.
- Connections with optimal control.
- Semilinear elliptic equations.

Detailed program

- The Direct Method of the Calculus of Variation.
- Existence of solutions for minimum problems.
- The Euler-Lagrange equation: classic and weak formulations.
- Validity of the Euler-Lagrange equation.
- The minimal time problem.
- Minimization problems without solutions.
- Lavrentiev's phenomenon.
- The minimal surface problem.
- Regularity of solution of minimum problems.
- Control problems and optimal control problems.
- Pontryagin Maximum Principle.
- Semilinear elliptic equations: existence of solutions via minimization, min-max methods, constrained minimization .

Prerequisites

Basics of Analysis and Functional Analysis.

Teaching form

Lectures: 8 ECTS credits.

Textbook and teaching resource

- A. Ambrosetti, A. Malchiodi. *Nonlinear analysis and semilinear elliptic problems*. Cambridge University Press, 2007.
- B. Dacorogna. *Introduction to the calculus of variations*. Third edition. Imperial College Press, London, 2015.
- B. Dacorogna. *Direct methods in the calculus of variations*. Second edition. Applied Mathematical Sciences, 78. Springer, New York,2008.
- L. C. Evans. *Partial differential equations*. Second edition. Graduate Studies in Mathematics, 19. American Mathematical Society, Providence, RI, 2010.
- O. Kavian. *Introduction à la théorie des points critiques*. Springer, 1993.
- M. Struwe. *Variational methods. Applications to nonlinear partial differential equations and Hamiltonian systems*. Fourth edition. Springer-Verlag, Berlin, 2008.
- M. Badiale, E. Serra. *Semilinear Elliptic Equations for Beginners*. Springer-Verlag, London, 2011.

Semester

Second semester.

Assessment method

The assessment of the achievement of learning objectives is carried out by a written test, which is designed to evaluate the level of theoretical knowledge and skills gained by the student. The student is asked to develop two topics out of three proposed at the exam. The written discussion must be precise, detailed, comprehensive and consistent with the proposed topic. Moreover it must contain some of the most significant proofs.

Office hours

By appointment.

COMPLEX GEOMETRY (2019/2020)

Teacher: Diego Conti

Aims

This course is an introduction to the geometry of complex manifolds, focused on both their general properties (algebraic structure of the ring of germs of holomorphic functions, correspondence between divisors and line bundles, hermitian metrics and curvature) and the construction of examples (submanifolds of projective space, hermitian symmetric spaces).

Students are expected to gain knowledge of fundamental notions relative to complex geometry. They are also expected to gain the ability to analyse and reproduce the proofs presented in the course, and to delve further, with or without guidance, into some of the results presented during the course.

Contents

Functions of several complex variables, complex manifolds, vector bundles, hermitian symmetric spaces.

Detailed program

- Holomorphic functions and Hermitian linear algebra.
- Weierstrass polynomials.
- Complex manifolds, holomorphic vector bundles.
- Connections; Dolbeault cohomology.
- Divisors and line bundles; global holomorphic sections and projective morphisms.
- Semisimple Lie groups, hermitian symmetric spaces.

Prerequisites

Vector spaces, rings, topological spaces, differential and integral calculus, differentiable manifolds, functions of one complex variable.

Teaching form

Lectures: 8 CFU

Textbook and teaching resource

D. Huybrechts, Complex Geometry. An Introduction, Springer 2005

A. Borel. Semisimple groups and Riemannian symmetric spaces, Hindustan Book Agency 1998

Semester

II semester

Assessment method

Oral examination with questions on definitions, theorem statements and proofs. Evaluation will be based on correctness, completeness and exactitude of the answers.

Office hours

By appointment

DIFFERENTIAL GEOMETRY (2019/2020)

Teacher: Roberto Paoletti

Aims

The aim of the course is to introduce the foundations of the theory of Riemannian manifolds, that is, manifolds endowed with a Riemannian metric, which consists in the assignment to each tangent space of a smoothly varying Euclidean product. The students will familiarize with the most basic concepts and techniques of differential geometry, moving from the foundational concept of Levi-Civita connection. Starting from the latter, the basic local curvature invariants and the notion of geodesic will be introduced. A key aspect which we propose to illustrate is the interplay between local aspects of the Riemannian metric, and the global topological structure of the underlying manifold.

The expected learning outcomes include the following:

- the knowledge and understanding of the basic definitions and statements, as well as of the basic strategies of proof in the theory in differential geometry; the knowledge and understanding of some of the key foundational examples of the theory;
- the ability to apply the acquired abstract knowledge to the solution of simple computational exercises and theoretical problems, referring in a precise and well-organized manner to the pertinent results being used; the ability to apply the theoretical background to the construction and discussion of simple examples and solution of exercises; the ability to expose and communicate effectively and clearly the theoretical content of the course.

Contents

Differentiable manifolds and Riemannian metrics, connections, curvature invariants, hypersurfaces and Lie groups, Riemannian submersions, Riemannian submanifolds, parallel transport and geodesics. Some notable global results, such as the Theorems of Hopf-Rinow, Hadamard, Bonnet-Myers.

Detailed program

Brief recalls on differentiable manifolds; Riemannian metrics; the fundamental theorem of Riemannian Geometry and the Levi-Civita connection; the curvature tensor and local curvature invariants; examples: Lie groups, hypersurfaces, metrics with rotational symmetry; shape operator; equations of Gauss and Codazzi-Mainardi; Theorema Egregium; Riemannian submersions and the formula of Gray and 'O Neill; the Hopf map; parallel transport and geodesics, existence and uniqueness; examples; the exponential map; normal coordinates,; Jacobi vector fields; Theorems of Hopf-Rinow, Hadamard, Bonnet-Myers.

Prerequisites

The content of the courses of the first two years of the Laurea Triennale in Mathematics should in principle be an adequate background. However, those students who do not have the basic notions on differentiable manifolds offered in, say, the course of Geometry III of the same program should expect to do some parallel extra work, since the discussion on prerequisites will be limited to some brief recalls.

Teaching form

Lectures: 8 CFU

Textbook and teaching resource

M. Do Carmo, Riemannian Geometry, Birkhauser

Recoomended reading:

M. Do Carmo, Differential forms and applications, Springer Verlag 1996;

P. Petersen, Riemannian Geometry, Springer Verlag 2006

Semester

2nd semester

Assessment method

During the course, two written partial tests will be offered, each referred to one half of the course. Each partial test will consist of a balanced flexible combination of computational exercises and theoretical questions. The exercises and theoretical questions in these tests will be along the lines of those offered in the practical and theoretical tests of the regular exam sessions (see below). The two partial tests will contribute equally to the final grade. To pass the exam through the partial tests, the student needs to pass each of them, thus obtaining a grade of at least 18/30 in both.

Alternatively, students may pass the exam through the regular exam sessions that follow the end of the course, and exactly the same pattern will be offered in every exam session. Thus, each session comprises two written tests, each referred to one half of the course, and consisting of a balanced combination of computational exercises and theoretical questions. The theoretical questions will involve definitions, statements of theorems, proofs, construction of examples and counterexamples, and simple theoretical problems.

The exercises will measure the student's ability to master the acquired formalism and apply it to some simple computations, to build on the acquired theoretical knowledge, and to invoke it in a pertinent and precise manner.

The theoretical questions will evaluate the knowledge and understanding of the conceptual framework of the course, as well as the ability to expose it in a well-organized, consistent and effective manner.

In order to pass the exam in one of the regular sessions, the student needs to obtain a grade of at least 18/30 in each of the two tests, which will contribute equally to the final grade. The two tests needn't be undertaken in the same session. It is also allowed to pass one the tests during the course and the other in a regular exam session.

To each exercise/theoretical question (or problem) a maximum partial grade will be assigned by the commission, depending on its difficulty and length. In the evaluation, every student will be given a grade in correspondence to each exercise/theoretical question (or problem) up to the maximum one, measuring the exactness, the completeness, the rigour, the clarity and the overall coherence of the development.

The exact subdivision of the course in two parts will be communicated well in advance during its duration.

Office hours

Upon appointment

ELEMENTARY MATHEMATICS (2019/2020)

Teachers: Leonardo Colzani, Giancarlo Travaglini

Aims

To introduce several classical and elementary results in Number Theory and Geometry. Historical and didactical aspects will be discussed as well as connections with other mathematical topics. The students will acquire skills related to several topics in number theory, geometry, calculus, numerical analysis, and most of all skills on a few connections between the above topics. Since the course has very mild prerequisites, the students will be able to use part of it for high school teaching, and for their own aptitude and preparation for this job.

Contents

- First part: Integer points. Polyhedra. Riemann sums.
- Second part: Elementary geometry.

Detailed program

- First part:
    - Simpson's paradox.
    - Prime numbers, arithmetic functions and integer points.
    - Integer points in polyhedra, the Frobenius Coin Problem.
    - Farey sequences.
    - Convexity and diophantine approximation.
    - Uniformly distributed sequences and normal numbers.
    - Riemann sums and integrals.
- Second part:
    - Euclid's Elements.
    - Hilbert's Foundations of Geometry.
    - Compass and straightedge constructions.
    - Polyhedra.
    - The fifth postulate and non-Euclidean geometries.

Prerequisites

Undergraduate mathematics. A large part of the course has essentially no prerequisites.

Teaching form

Lesson.

Textbook and teaching resource

Detailed notes will be provided during the classes.

M. Beck, S. Robins, *Computing the continuous discretely. Integer-point*

enumeration in polyhedra. Springer (2015).

M. Bramanti, G. Travaglini, *Studying Mathematics: The Beauty, the Toil and the Method*, Springer (2018).

J. Sally, P. Sally, *Roots to research. A vertical development of mathematical problems*. Amer. Math. Soc. (2007).

G. Travaglini, *Number Theory, Fourier Analysis and Geometric Discrepancy*, Cambridge Univ. Press (2014).

Semester

Second semester.

Assessment method

Oral exam. The student will be requested to understand all the contents of the course; he/she will be also requested to hold a seminar. He/she will be evaluated considering her/his mastering of the contents of the course as well as the teaching skills he/she will exhibit through the seminar. Mark out of thirty, the exam is passed if the evaluation is at least 18/30.

Office hours

By appointment.

E-Mail: giancarlo.travaglini@unimib.it

E-Mail: leonardo.colzani@unimib.it

FUNCTIONAL ANALYSIS (2019/2020)
Teacher: Paolo Maurizio Soardi

Aims

Consistent with the educational objectives of the Master's Degree in Mathematics, the course aims to provide students with the knowledge concerning the definitions and the basic statements of the Functional Analysis. The skills needed to understand and analyze the main techniques and demonstration methods related to the theory, and the skills to apply them to face problems will also be provided. Particular emphasis will be placed on topological aspects.

Contents

Locally compact Hausdorff spaces. Spaces of continuous functions. Spaces L^p. Compactness in L^p and in C^0. Weak and weak star topology. Riesz representation theorems.

Detailed program

An overview on abstract integration theory and L^p spaces.

Spaces of continuous functions in locally compact Hausdorff spaces. Urysohn lemma and Lusin Theorem. Density of continuous compactly supported functions in L^p. Separability of spaces of continuous functions. Separability of L^p. Sobolev spaces: definition and first properties. Compactness and non compactness in Banach spaces. The Ascoli-Arzelà theorem in C(X) and its extensions. The Riesz-Kolmogorov theorem in $L^{p.}$ Linear functionals and weak topology on a normed. Sub additive positively homeoenous functionals. The Hahn-Banach theorem: generall form. Convexity and separation by hyperplanes. The Mazur theorem. The weak-star topology. Dual and bidual. Product topology and Tichonov theorem. Alaoglu theorem. Re lexive spaces. Uniform convexity. Kakutani theorem.

Prerequisites

Elements of the theory of abstract integration, elements of L ^ p space theory , elements of general topology. Basic knowledge of Banach spaces and Hilbert spaces.

Teaching form

Lectures held by the teacher with discussions in the lessons on the topics covered.

Textbook and teaching resource

Bibliographic references:

- H. Brezis, Functional Analysis.
- H. Royden, Real Analysis.
- W. Rudin, Real and Complex Analysis
- P. Lax, Functional Analysis

Semester

First semester.

Assessment method

The exam is solely oral and consists of a colloquium with assessment, and is divided into a series of oral questions designed to verify the student's knowledge and mastery of the theorems with related demonstrations carried out during the course.

In the oral exam it is assessed whether the student has acquired the necessary skills to present a selection of the demonstrations carried out in the classroom, and, above all, the critical and operational knowledge of the definitions and results of the course, also by illustrating examples and counter-examples. The exam is passed if the vote is at least 18/30.

Office hours

The teacher is available after each lesson for clarifications and explanations. Students can also make an appointment with the teacher for further explanations.

GEOMETRIC GROUP THEORY (2019/2020)

Teacher: Thomas Stefan Weigel

Aims

In coherence with the principal educational objectives of the master program, the main scope of the course is to provide the students with the necessary mathematical knowledge, i.e., definitions, notions, and the statement of the theorems of Bass-Serre theory on groups acting on trees. Apart from the necessary theoretical competences allowing the student to follow the proof of the main results of the theory, we also aim to provide the student with the ability to apply the theory in exercises and open problems (problem solving). The course will finish with a general discussion on important applications of the theory in group theory, like the discussion of the trefoil knot group, Ihara's theorem, the boundary of a tree, etc.

Contents

Graphs, paths, connectivity, trees; actions of groups on graphs, Cayley graphs; free groups, free products and amalgams, HNN-extensions; quotient graphs, graphs of groups, the fundamental group of a graph of groups, the fundamental theorem for groups acting on trees.

Detailed program

- graphs, paths, connection, trees.
- group actions on graphs, Cayley graphs;
- free groups, free products (with amalgamation), HNN-extensions.
- quotient graphs, graph of groups; the fundamental group of a graph of groups; the main theorem of Bass-Serre theory.

Prerequisites

Algebra I, Geometria I.

Teaching form

Lessons 8 CFU (ECTS).

Textbook and teaching resource

- J-P. Serre: Trees, Springer-Verlag, Berlin, 1980.
- J. Meier: Groups, Graphs and Trees, London Mathematical Society, Student Texts, 73, CUP, 2008.
- O. Bogopolski: Introduction to Group Theory, EMS Textbooks in Mathematics, 2008.

Semester

1st semester

Assessment method

A presentation of 20 minutes on an application of Bass-Serre theory previously discussed with the lecturer, and an oral examination on the content of the course in which the students capacity to explain and to apply

Bass-Serre theory is validated. The seminar will contribute 20% to the final mark, 80% will contribute the oral exam.

Office hours

On appointment.

GEOMETRY AND PHYSICS (2019/2020)

Teacher: Franco Magri

Aims

The course has the aim of presenting the mathematical tools and the conceptual ideas that are required to understand the modern formulation of Maxwell's electromagnetic theory and the theory of Einstein's gravitational fields.

Objective: at the end of the course the student must be able to explain the intrinsic spacetime formulations of the main theories of Classical Physics, without reference to any spacetime observer.

Abilities: at the end of the course the student should be able to : 1) to explain the evolution of the concept of time from Newton to Einstein and to describe the experiment of Hafele-Keating; 2) to explain the concepts of relativistic mass and energy of a particle; 3) to write the Maxwell equations with the use of differential forms; 4) to explain why the distribution of mass and energy in the Universe determine the curvature of spacetime; 5) to write the Einstein equations in the spacetime regions void of matters and to discuss the Schwarzschild solution; 6) to know the definition of black hole.

Contents

The arguments dealt with in the course are:

1. The electromagnetic theory of Maxwell
2. The gravitational theory of Einsteiniana
3. Black holes.

The Maxwell's equations have been written by Maxwell, in 1867, in the form of a system of vector differential equations on the fields E and B. The advent of Special Relativity and of the spacetime point of view has allowed to better understand the structure of these equations and has changed drastically their form. Nowadays the Maxwell's equations are written in the compact form

$dF = 0$, $dM = 0$,

by using a new and more powerful mathematical formalism. The aim of the first part of the course is to explain the meaning of modern interpretation of Maxwell's theory.

The equations of the gravitational field generated by a planet and the equations of motion of a satellite orbiting around the planet have been written by Einstein in 1915. The meaning of the Einstein's equations has been summarized by J.A. Wheeler in the following aphorism: “ The planet says to spacetime how to curve ; the spacetime says to the satellite how to move” . This aphorism enphasizes the central point of the Einstein's theory, according to which the gravitational field reveals itself as a curvature of spacetime. The aim of the second part of the course is to give a precise sense to the Wheeler's aphorism.The lectures will be available on the web page of the lecturer

One of the most astonishing phenomena entailed by the Einstein's theory of gravitation is the appearance of “ black holes” , that is of regions of spacetime delimited by an “ event horizon” that forbids all signals to leave the black hole and to reach farther observer. The aim of the third part of the course is to present two specific examples of Einstein's gravitational fields, the Schwarzschild metric and the Kerr metric, and to discuss the geometry of the black hole associated with at least one of them.

Finally, time permitting, the course could end with a very terse introduction to the more recent evolution of Einstein's central idea, accordind to which forces manifest themselves as curvatures. This idea is the starting point of gauge theories, that can be seen as the modern attempt to unify the fundamental interactions.

Detailed program

Newton:

The gravitational law in the Principia

Newtonian spacetime.

Minkowski:

Radar observers and the Doppler effect

K-calculus and the Lorentz transformations

The geometrical structure of the Minkowski's spacetime

Physics in the Minkowski's spacetime:

Four-vectors and the dynamics of a relativistic particle

Differential forms and the Maxwell's electromagnetic theory

Einstein:

The peculiarities of the gravitational force

Freely falling observers

Geodesic motions

Tidal forces and the curvature of spacetime

Gauss and Riemann:

The concept of curvature for embedded surfaces

Gauss and the Theorema Egregium

Riemannian manifolds

Connection, curvature, parallelism and geodesic curves

The Physics in Einstein's spacetime:

The Einstein's field equations in vacuum

Schwarzschild's solution

The motion of satellites in the Schwarzschild's gravitational fields

Black holes

Prerequisites

The prerequisites are the knowledge of the basic facts of the geometry of surfaces embedded into the Euclidean space, of classical electromagnetism, and of Special Relativity. Some of these notions will be tersely reviewed during the course.

Teaching form

Lectures: 7 cfu, Exercises: 1 cfu

Textbook and teaching resource

References :

1. R.Geroch, Relativity from A to B

It is a nice introduction to the geometry of spacetime and its physical interpretation.

2. G.Ellis,R.Williams, Flat and curved spacetimes

Mainly chapter 3, on the geometry of Minkowski's spacetime (studied by the K-calcolo of Bondi) and chapter 5, on the relation between the gravitational force and the curvature of spacetime.

3. W. Kuhnel, Differential Geometry (Curves-Surfaces-Manifolds)

Mainly: sections 3A e 3B on the theory of principal curvatures ; sections 4A,

4B, 4C on the theory of embedded surfaces by Gauss; sections 5A, 5B, 5C, 5D on Riemannian manifolds

The lectures will be available on the web page of the lecturer.

Semester

First semester.

Assessment method

Oral examination. The student has the faculty of choosing two themes amon the six arguments of the program (a theme in the first three points and the other theme among the remaining three points). The student will present these themes in the form and in the order he likes.

During the discussion the student must be able to stress the links with the other points of the program.

The time at disposal of the student for the presentation may vary between 45 minutes and i hour.

The evaluation criteria are : 1) the capability of stessing the conceptual structure of the theory he is discussing; 2) clarity and conciseness; 3) methodological precision; 4) appropriate use of the (specialistic) language.

Office hours

The students are received on their request, by an email message.

HARMONIC ANALYSIS (2019/2020)

Teacher: Stefano Meda

Aims

The aim of the course is to illustrate some basic material in Fourier analysis, of wide use in analysis and applications. Applications to signal processing will be given. At the end of the course the student will be able to understand the basic issues concerning the theory of signal processing, with emphasis on musical applications. No specific knowledge of musical theory is assumed.

Specifically, the expected learning outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in Fourier Analysis and Signal Processing, focusing on the convergence of Fourier series and integrals (pointwise, in mean, uniform), the treatment of the Discrete Fourier Transform and the Fast Fourier Transform, and the diffusion of sound waves;
- the skill to apply such conceptual background to the construction of concrete examples and to the solution of exercises, ranging from routine to challenging (starting with routine exercise that require straightforward application of the definitions and the results given during the lectures, up to exercise that require deep understanding of the matter and the ability of developing original ideas).

Contents

Basics on Fourier series and integrals. Applications to signal analysis, and to music.

Detailed program

- Orthonormal systems.
- Basic properties of Fourier series in one variable. Mean and pointwise convergence.
- The Fourier transform in one variable. The Schwarz space. Plancherel and inversion formulae.
- The Fourier transform in several variables. The wave equation and propagation of sound.
- Sampling and quantizing.
- The discrete and the fast Fourier transform.
- The Shannon sampling theorem.
- Applications to music and the digitalization of sound.

Prerequisites

In order to be able to successfully attend the course, the student should know basics of Analysis and Linear Algebra: calculus for functions of several viariables, pointwise and uniform convergence of series of functions, the Lebesgue integral and basics of measure theory, matrix calculus. A knowledge of the main properties of the space L2 and of the elementary theory of Hilbert spaces will be valuable.

Teaching form

Lectures at the blackboard. The teaching hours will be dedicated either to the illustration of main results in the theory, or to the solution of exercises (previously assigned) containing (possibly fine) applications of the theory.

Textbook and teaching resource

- E. Del Prete, Analisi di Fourier, Una introduzione alla teoria e applicazioni, Notes available on this site
- J. Duoandikoetxea, Fourier Analysis, American Mathematical Society, Graduate Studies in Mathematics, Vol. 29, 2001
- Stein-Shakarchi, Fourier Analysis, Princeton University Press
- Steiglitz, A Digital Signal Processing Primer, Princeton University Press
- A. Visintin, Fourier Series and Musical Theory, available at https://www.science.unitn.it/~visintin/Fourier-Mus.pdf

Further references concerning specific parts of the program will ge given during the class hours.

Semester

II semester.

Assessment method

Written examination, including theoretical questions (proofs of part of the results illustrated during the course) and exercises, often similar to those solved during the class hours. In order to get a positive grade, both the parts including theoretical questions and exercises must get a passing grade. The two parts of the written examination will contribute in the same amount to the determination of the final grade.

The grade will take into account the exactness of the answers, the clarity of the exposition and the command of mathematical language used.

Office hours

Upon appointment.

HIGHER ANALYSIS (2019/2020)

Teachers: Veronica Felli, Stefano Meda

Aims

The aim of the course is to introduce some basic material of wide use in analysis. This is done by illustrating how the theories explored interact with the Dirichlet problem for the Laplacian.

The expected learning outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof of modern analysis; the knowledge and understanding of some crucial examples in which the theory manifests itself;
- the ability to recognize the role that concepts and techniques from modern analysis introduced in the lectures (such as convolution, Fourier transform, distributions, Sobolev spaces) play in various areas of pure and applied mathematics (numerical analysis, mathematical physics, probability); the skill to apply such conceptual background to the construction of concrete examples and to the solution of exercises; the ability to communicate and explain in a clear and precise manner both the theoretical aspects of the course and their applications to specific situations, possibly to different contexts.

Contents

Basics on convolution and Fourier transform, the Dirichlet problem in the ball and in the half space, distributions, regularity of distributions, Sobolev spaces, second order elliptic problems.

Detailed program

Chapter 0. **Preliminaries**

Convolution. Hypersurfaces of class C^k in R^n. The divergence theorem and Green formulas. Complex measures.

Chapter 1. **The classical Dirichlet problem**

Harmonic functions. Mean value theorems for harmonic functions. Characterization of harmonic functions via mean value theorems.The maximum principle for harmonic functions and the uniqueness of the Dirichlet problem.The Newtonian potential. Green's representation formula. The Green's function. The Poisson kernel. Green's function and the Poisson kernel for the half-space. Further properties of harmonic funcitons: estimates for derivative, Schwarz's reflection principle and Liouville's theorem. Classical solution for the Dirichlet problem on the sphere and on the half-spce.

Chapter 2. **L^p data and convergence to boundary**

Poisson integral of measures and L^p functions. Weak $*$ convergence. Solution of the Dirichlet problem with L^p boundary data. Operators of weak type (1, 1). Marcinkiewicz interpolation theorem. The Hardy–Littlewood maximal operator. A covering lemma. Boundedness properties of the Hardy–Littlewood maximal function. Lebesgue's differentiation theorem. Nontangential convergence of Poisson integrals.

Chapter 3. **Distributions and their derivatives**

Distributions. Examples. Derivatives of distributions. Examples.

Chapter 4. **Sobolev spaces**

Motivations, definitions and properties. Properties of Sobolev spaces: $W^{k,p}(\Omega)$ is a Banach space, approximation by smooth functions, product and composition of Sobolev spaces. Sobolev spaces in dimension 1: existence of a continuous representative and fundamental theorem of calculus for $W^{1,p}(a, b)$ functions. Morrey Theorem. Sobolev inequality (Sobolev-Gagliardo-Nirenberg Theorem). Sobolev embeddings. Extension operator and Extension Theorem for half-spaces and bounded regular domains. Global approximation by smooth functions. Sobolev embeddings for extension domains. Embeddings for higher order Sobolev spaces. Rellich-Kondrachov Theorem. Existence of the Trace Operator $\gamma_0^p: W^{1,p}(\Omega) \to L^p(\partial\Omega)$ with $1\leq p < +\infty$ and Ω being a half-space or bounded regular domain. Fractional Sobolev spaces and Gagliardo Theorem (hints). Characterization of $W^{1,p}_0(\Omega)$ by traces.

Chapter 5. **Second order elliptic problems**

Lax-Milgram Lemma. Second order elliptic problems: variational formulation, existence of solutions. Poincaré inequality. Dirichlet Principle. Elliptic problems with Neumann boundary conditions: variational formulation and some hints on H(div,Ω) spaces. Poincaré-Wirtinger inequality. Existence of solutions for the Neumann problem under compatibility conditions

Prerequisites

Calculus in several variables, linear algebra, basics of Hilbert and L^p spaces.

Teaching form

Lectures, with blackboard. The teaching hours will be dedicated either to the illustration of main results in the theory, or to the solution of exercises (previously assigned) containing (possibly fine) applications of the theory.

Textbook and teaching resource

- Notes available on the teacher website.
- A. Bressan. *Lecture Notes on Functional Analysis*. American Mathematical Society, 1900.
- H. Brezis. *Functional analysis, Sobolev spaces and partial differential equations*. Springer Science & Business Media, 2010.
- L.C. Evans. *Partial differential equations*, American Mathematical Society.

Semester

I semester.

Assessment method

The exam consists of a written test, aimed at verifying the level of knowledge, the ability to apply it to the resolution of exercises, the student's independence in making judgements, as well as his/her communication skills. The test is divided into two parts: the first part contains theoretical questions (proofs of part of the results illustrated during the course), while the second part contains exercises, often similar to those solved during the class hours. The two parts will contribute equally to the determination of the final grade.

Office hours

By appointment.

HIGHER MECHANICS (2019/2020)

Teacher: Diego Davide Noja

Aims

First orientative tour in the ideas and methods of Quantum Mechanics, with an attention to the rigorous mathematical formulation. The comparison with Classical Mechanics, when suitable, will be emphasized. Knowledges given to the students will be mainly in operator theory needed to formulate in a rigorous way Quantum Theory. The students will develop skills useful to understand the main concepts and techniques and the abilities useful to apply them in treating simple examples and solving well chosen problems.

Contents

The present course is an introduction to Quantum Mechanics suitable for Mathematics students at the master level. The purpose is to give a synthetic and mathematically rigorous exposition of the elements of Quantum theory and of some of its more interesting consequences. So we will renounce, after a brief preliminary introduction, to follow the historical development of the subject and we will prefer a more systematical treatment. We will give instead some conceptual and mathematical needed premises from Classical Mechanics.

Detailed program

- The mathematical structure of Classical Mechanics
- Integrable systems and the Arnold-Liouville theorem
- Many body systems and Classical Statistical Mechanics
- The quantum phenomenology and the birth of Quantum Mechanics
- Introduction to states, evolution and observables; Heisenberg principle
- The Schroedinger equation. First examples in one dimension: free particle, harmonic oscillator, scattering and tunneling
- The Schroedinger equation; Hydrogen atom and radial problems
- Summary of operator theory and the problem of dynamics in Quantum Mechanics
- Mathematical structure of Quantum Mechanics. Axioms and interpretation;
- Comparison with the mathematical structure of Classical Mechanics
- Symmetry and Groups in Quantum Mechanics: Angular Momentum and Spin
- Stability of Matter in Quantum Mechanics*
- Problems of interpretation: Einstein-Podolski-Rosen paradox and Bell inequalities*

Prerequisites

The only prerequisites are the mathematical concepts and tools learned during the three-year grade, especially in Analysis. Giving this, the course it is self contained. It will be useful, but not necessary, some familiarity with the main definitions and properties of Hilbert space and the definition and properties of Fourier transfom.

Teaching form

Lectures and exercise session lead by the teacher responsible of the course. Exercise sheets will be periodically proposed to the students during the course. They have to provide written solution to the exercise sheets autonomously or as a result of a group work. In this second case however every student has to write his own final version of exercises. Solutions to selected exercises will be then discussed in class.

Textbook and teaching resource

Lectures are based on several sources. Notes will be distributed on some parts of the course.

Bibliography:

- Basdevant J-L.: Lectures on Quantum Mechanics, Springer (2007)
- Caldirola P.: Prosperi A.M., Cirelli, R.: Introduzione alla Fisica Teorica, Utet, (1982)
- Dirac P.A.M.: Principles of Quantum Mechanics 4th revised ed. OUP (1982)
- Faddeev, L.D.: Yakubovsky O.A.: Lectures on Quantum Mechanics for Mathematics Students, Student Mathematical Library, AMS (2009)
- Galindo A., Pascual P.: Quantum Mechanics I & II, TMP, Springer (1990)
- Gallone F.: Hilbert Spaces and Quantum Mechanics, World Scientific (2015)
- Hall, B.C.: Quantum Theory for Mathematicians, GTM, Springer (2013)
- Hannabuss K.: An Introduction to Quantum Theory, OUP (1997)
- Onofri E. Destri C.: Istituzioni di Fisica Teorica, Carocci, (1996)
- Thaller B.: Visual Quantum Mechanics Springer (2000)
- Thaller B.: Advanced Visual Quantum Mechanics, Springer (2005)
- Teschl, G.: Mathematical methods in Quantum Mechanics with application to Schroedinger operators, II ed. AMS (2014)
- Teta, A.: A Mathematical primer on Quantum Mechanics, Springer (2018)
- Thirring W.: Quantum Mathematical Physics, Springer 2nd Ed. (2001)

Semester

II semester.

Assessment method

Oral examination. Mark out of thirty.

The main goal is to ascertain if the student has learnt concepts and mathematical techniques introduced in the lectures. Moreover during the colloquium the student it is required to be able to correctly set simple problems giving essential solutions to them with a correct interpretation, in particular to explain the solution to the homework exercises proposed during the course. The choice and discussion of problems is based on the ones given in the exercises sheet proposed by the lecturer during the course.

The final evaluation will result from the average between the evaluation of the written exercises sheets proposed during the course and that of the oral examination. The exam is passed if the evaluation is at least 18/30.

There will be 5 exam sessions (in June, July, September, January, February).

Office hours

By appointment.

HISTORY OF MATHEMATICS (2019/2020)

Teacher: Leonardo Colzani

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing the knowledge of some important chapters in the history of mathematics. It will also build the skill and ability to connect some classical results to more modern theories. In other words, the aim is to present a number of classical and elementary results that you always wanted to know (but were afraid to ask).

Contents

Squaring the circle

Algebraic equations.

Prime numbers.

Proofs will be required.

Detailed program

Squaring the circle and the hyperbola. Computing pi (Archimedes, Huygens, Newton).

Rational and irrational numbers, algebraic and transcendental (Pytagoras, Liouville, Cantor).

Irrationality and transcendence of e (Eulero, Hermite), and pi (Lambert, Lindemann).

Algebraic equations and the fundamental theorem of algebra (d'Alembert, Gauss).

Equations of first,second third and fourth degree (Tartaglia, Cardano, Ferrari).

Equations of fifth degree (Ruffini, Abel, Galois).

Roots of a polynomial in an interval (Cartesio, Sturm).

Prime numbers. The fundamental theorem of arithmetic (Euclid, Gauss).

Primes are infinite (Euclid, Eulero). Primes in arithmetic progressions (Dirichlet).

Distribution of prime numbers (Riemann, Hadamard, de la Vallée Poussin).

The seminars of the students are part of the program.

Prerequisites

The algebra, analysis, and geometry in standard undergraduate mathematical courses. Some complex analysis may be

helpful. In case of problems, the lecturer may provide help.

Teaching form

(1) The student is expected to read and study some books on the history of mathematics.

(2) The student is expected to write a report and give a seminar on an original memoir.

(3) The teacher will present in the classroom, with proofs, a certain number of classical theorems and the genesis of some theories encountered in the mathematical curriculum.

Textbook and teaching resource

Some books on the history of mathematics:

C.Boyer "A history of Mathematics".

M.Kline "Mathematical thought from ancient to modern times".

V.J.Katz "A history of mathematics".

Notes provided by the teacher.

Semester

1^{st} semester.

Assessment method

The exam consists of two parts, a written report with a seminar on a subject in agreement with the lecturer, and an oral examination. These two parts, the witten report with seminar and the oral exam, can be taken at different times. The final evaluation results from the average between the parts of the examination. Mark out of thirty. The exam is passed if the evaluation is at least 18/30.

Office hours

On appointment. E-Mail: leonardo.colzani@unimib.it

HISTORY OF MATHEMATICS - ELEMENTS (2019/2020)

Teacher: Leonardo Colzani

Aims

The course of History of Mathematics - Elementi (4 CFU) is associated to the couse of History of Mathematics (8 CFU), with a reduced program. In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing the knowledge of some important chapters in the history of mathematics. It will also build the skill and ability to connect some classical results to more modern theories. In other words, the aim is to present a number of classical and elementary results that you always wanted to know (but were afraid to ask).

Contents

Squaring the circle

Algebraic equations.

Prime numbers.

Proofs will be required.

Detailed program

Squaring the circle and the hyperbola. Computing pi (Archimedes, Huygens, Newton).

Rational and irrational numbers, algebraic and transcendental (Pytagoras, Liouville, Cantor).

Irrationality and transcendence of e (Eulero, Hermite), and pi (Lambert, Lindemann).

Algebraic equations and the fundamental theorem of algebra (d'Alembert, Gauss).

Equations of first,second third and fourth degree (Tartaglia, Cardano, Ferrari).

Equations of fifth degree (Ruffini, Abel, Galois).

Roots of a polynomial in an interval (Cartesio, Sturm).

Prime numbers. The fundamental theorem of arithmetic (Euclid, Gauss).

Primes are infinite (Euclid, Eulero). Primes in arithmetic progressions (Dirichlet).

Distribution of prime numbers (Riemann, Hadamard, de la Vallée Poussin).

Prerequisites

The algebra, analysis, and geometry in standard undergraduate mathematical courses. Some complex analysis may be

helpful. In case of problems, the lecturer may provide help.

Teaching form

(1) The student is expected to read and study some books on the history of mathematics.

(2) The teacher will present in the classroom, with proofs, a certain number of classical theorems and the genesis of some theories encountered in the mathematical curriculum.

Textbook and teaching resource

Some books on the history of mathematics:

C.Boyer "A history of Mathematics".

M.Kline "Mathematical thought from ancient to modern times".

V.J.Katz "A history of mathematics".

Notes provided by the teacher.

Semester

1^{st} semester.

Assessment method

The exam consists is a colloquium. Mark out of thirty. The exam is passed if the evaluation is at least 18/30.

Office hours

On appointment. E-Mail: leonardo.colzani@unimib.it

MATHEMATICAL METHODS FOR ECONOMIC ANALYSIS – OPTIMAL CONTROL (2019/2020)

Teacher: Andrea Giovanni Calogero

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing the *knowledge* about the fundamental concepts and statements of the theory of optimal control using the varionational approach and using the dynamic programming. Moreover, we will introduce the students to the theory of the differential games. It will also build the *skills* needed to understand and use the most important proving arguments and techniques in the theory and the *ability* to solve exercises and deal with several models and applications.

Contents

Optimal control problems with the variational approach: theory and economic models. Optimal control problems with dynamic programming: theory and economic models. Introduction to differential games.

Detailed program

1. INTRODUCTION TO THE OPTIMAL CONTROL

a. Some problems.

The moon landing problem, in boat with Pontryagin, a model of optimal consumption, the “lady in the lake”.

b. Statement of an optimal control problem

Definition of control, dynamics, trajectory, control set. Admissible control. Importance of the case of linear dynamics.

2. THE OPTIMAL CONTROL WITH THE VARIATIONAL APPROACH

a. The simplest problem of optimal control

The theorem of Pontryagin (PROOF in the case of control set U = R): comments and consequences of the Maximum principle. Extremal control, associated multiplier. Normal and abnormal controls. Sufficient conditions of optimality: the condition of Mangasarian (PROOF). Concave functions, upgradient, upgradiente for differentiable function, theorem of Rockafellar. The sufficient condition of Arrow (PROOF). Transversality conditions for problems with fixed initial/final points. On the minimum problems. An example of abnormal control. *A two sector model with investment and consumption goods. A model of inventory and production I.*

b. The simplest problem of the calculus of variations

Euler's theorem (PROOF as a particular case of the theorem of Pontryagin). Transversality conditions for problems with fixed initial/final points. Sufficient conditions for the simplest problem using concavity/convexity. *Curve of minimal length*.

c. Singular and bang-bang controls.

Definition of bang-bang control, switching time and singular controls. *The construction of a mountain road with minimal cost.*

d. More general problem of optimal control

Problems of Mayer, Bolza and Lagrange: their equivalence (PROOF). A necessary condition for the problem of Bolza with final time fixed/free. *The adjustment model of labor demand (Hamermesh).*

Time optimal problems: *In boat with Pontryagin.* Singular time optimal problems: *The Dubin car.*

Problems with infinite horizon: counterexample of Halkin; sufficient condition (PROOF). Current Hamiltonian and current multiplier. Models of economic growth: the utility functions. *A model of optimal consumption with log-utility.*

e. Problems of existence and controllability

Examples. Gronwall inequality. Theorem of existence of optimal control for the problems of Bolza: the case of closed control set and the case of compact control set.

3. OPTIMAL CONTROL WITH THE METHOD OF DYNAMIC PROGRAMMING

a. The value function of its properties in the simplest problem of optimal control.

Definition of the function value. The first necessary condition for the value function (PROOF). Bellman's principle of optimality (PROOF). The properties of the value function: the equation of Hamlton-Jacobi-Bellmann (PROOF). The Hamiltonian of Dynamic Programming. Sufficient conditions of optimality (PROOF). On the problems of minimum.

The value function for problem with fixed final time and free final value, is Lipschitz (PROOF). Definition of viscosity solution; the vaue function is the unique viscosity solution of BHJ equation. *Problem of business strategy of production / sale II.*

b. More general problem of optimal control

Necessary and sufficient conditions for more general optimal control problems. *A model of inventory and production II.*

Infinite horizon problems: the current value function and its BHJ equation. *A problem of optimal consumption with HARA utility.* An idea of the stochastic situation: *the model of Merton.*

c. Relations between the variational approach and Dynamic Programming

Interpretation of the multiplier as a shadow price (PROOF).

4. DIFFERENTIAL GAMES

a. Introduction

Statement of a differential game with two players. Symmetric games, fully cooperative games, zero-sum games. Concepts of solutions: Nash equilibrium, Stackelberg equilibrium. Types of strategies: open-loop and Markov.

b. Nash equilibrium

* Open loop strategies. Definition, use of the variational approach and sufficient condition for an open-loop

strategy. *The model "workers versus capitalists" of Lancaster. Two fishermen at the lake I. A model on international pollution.*

** Markovian strategies. Because the variation technique is not particularly useful. Definition of the value function on a Nash feedback equilibrium. Necessary and sufficient conditions with dynamic programming. Value Functions for Affine-Quadratic Two-Player Differential Game Problems. The current value functions for infinite and discounted horizon games. *A problem of production for two competing companies*. *Two fishermen at the lake II.*

c. Stackelberg equilibrium.

Leader and follower players, the set of best replies. Open-loop solution with the variational approach. *A model on International pollution with hierarchical relations.*

d. Zero sum games

Nash equilibrium optimal control as a saddle point. Non anticipative strategies. Upper value function V^+, lower value V^- and their relation.

Upper Hamiltonian of Dynamic Programming H^+_{PD} and Lower H^-_{PD}: their relation (PROOF). Properties of V^+ and V^+: Lipschitz properties ans viscosity solution for the Isaacs's equations.

Isaacs condition (minimax condition) and the Hamiltonian of Dynamic Programming H_{PD}. Definition of value function V. A geometric proof that V satisfies the Isaacs equation (PROOF). *War of attrition and attack.*

e. Pursuit and evasion games.

Statement, target set, exit time. The value function and the Isaacs' equation for the autonomous case (PROOF). *Lady in the lake.*

Prerequisites

The tools and the knowledge of the courses of the first degree are a sufficient basis.

Teaching form

Lectures with exercises.

Textbook and teaching resource

[C1] A. Calogero "*Notes on optimal control theory*", avaible in web site.

[C2] A. Calogero "*A very short tour on differential games*", avaible in web site.

[C3] A. Calogero "*Exercises of dynamic optimization*", avaible in web site.

Other references:

[BO] T. Başar, G.O. Olsder "*Dynamic noncooperative game theory*", SIAM Classic in Applied Mathematics, 1998

[B] A. Bressan "*Noncooperative differential games. A Tutorial*", Milan Journal of Nathematics, vol 79, pag

357-427, 2011.

[E] L.C. Evans “*An introduction to mathematical optimal control theory”*, disponibile gratuitamente in rete.

[FR] W.H. Fleming, R.W. Rishel *"Deterministic and stochastic optimal control*", Springer-Verlag, 1975

[KS] M.I. Kamien, N.L. Schwartz “*Dynamic optimization*” Elsevier, second edition, 2006

[SS] A. Seierstad, K Sydsæter “*Optimal control theory with economics applications*” Elsevier Science, 1987

Semester

First semester.

Assessment method

The examination starts with a written test. The evaluation will be in thirty: it is possible to take the oral examination only with evalutation major then 26 in the written part.

WRITTEN TEST: (3 hours) contains the following arguments:

- definitions, theorems and proofs (the proofs required are denoted with PROOF in the program of the course);
- economic and more general models, as in the program of the course;
- exercises of optimal control, using the variational approach and the dynamic programming: Such exercises will be chosen in the list of exercise [C3] located in the page of the course.

ORAL TEST: (the date will be fixed together with the student). It consists in a discussion starting to the written part.

The student can reject a sufficient evaluation at most 2 times.

Office hours

On appointment.

MATHEMATICAL METHODS FOR ECONOMIC ANALYSIS – OPTIMIZATION AND CONVEX ANALYSIS (2019/2020)

Teacher: Rita Pini

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing the *knowledge* about the fundamental concepts and statements of the theory of optimization and convex analysis in the Euclidean setting. It will also build the *skills* needed to understand and use the most important proving arguments and techniques in the theory and the *ability* to solve exercises and deal with problems exploiting them. Particular emphasis will be put on the theory of nonlinear programming and its relationship with convexity, as well as some results of duality.

Contents

Finite-dimensional optimization, elements of convex analysis, duality theory, introduction to game theory

Detailed program

Introduction to optimization problems. Basic calculus tools in R^n.

Unconstrained optimization.

Ekeland variational principle.

Transposition theorems.

Convex analysis for sets and functions.

Linear and nonlinear programming.

Duality theory and convex programming.

Strategic games.

Nash equilibrium.

Two-players zero-sum games.

Mixed strategies in finite games.

Prerequisites

Basic concepts and results of linear algebra and analysis in finite-dimensional spaces.

Teaching form

Lectures

Textbook and teaching resource

O. Guler, Foundations of Optimization, Springer, 2010

D.P. Bertsekas, Convex Analysis and Optimization, Athena Scientific, Belmont, Mass., 2003

J. Gonzalez-Diaz, I. Garcia-Jurado, M.G. Fiestras-Janeiro, An Introductory Course on Mathematical Game Theory, American Mathematical Society

Semester

II

Assessment method

Examination type:

Written and oral examination.

a) The written part consists of exercises where the students show their ability in using methods and tools introduced in the course (80%), as well as questions (20%). If the mark of the written exam is between 18/30 and 26/30, then the final grade is the grade of the written exam. If the grade of the written part is greater than or equal to 27/30, the student obtains at most 27/30 as final grade unless he/she decides to undergo the oral part.

b) The oral part consists of statements and proofs of theorems from a detailed list, as well as theorical exercises. It is only for students with mark not less than 27/30 in written examination. Its relative weight is 25%. It consists in:

- discussion about the written part;
- the student must show his competence about subjects considered in the lectures (i.e., statements and proofs of theorems from a detailed list, theorical exercises)

If the grade of the written part is more, or equal to 18, the student can decline it only once.

Office hours

by appointment

MATHEMATICAL METHODS FOR MODERN PHYSICS (2019/2020)

Teacher: Renzo Ricca

Aims

According to the educational objectives of the Course, the taught material aims to provide students with the *basic notions* regarding the definitions and the fundamental results for a geometric and topological approach to the study of classical field theory, with particular emphasis on classical vortex dynamics, ideal magnetohydrodynamics and quantum hydrodynamics. The course aims to provide also the necessary *competences* to understand and use standard techniques and the demonstration methods involved in the theory, as well as the *capabilities* to use them to solve exercises and tackle problems.

The expected outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in classical geometric and topological field theory; the knowledge and understanding of some key examples where theory is fully applied;
- the ability to recognize the role that concepts and techniques from a geometric and topological approach play in various areas of mathematics (such as vortex theory, ideal magnetohydrodynamics, quantum fluids), and in the mathematical modelling of physical situations (vortex dynamics, relations between energy and complexity, topological defect production, knotting and linking); skills to apply the basic concepts to the elaboration of practical examples and to the solution of posed questions; the ability to communicate and explain in a clear and precise manner both the theoretical aspects of the course as well as their applications to specific situations, possibly in analogous but different contexts.

Contents

Part I. Fluid flows and diffeomorphisms, Green's identities, conservation theorems, Euler's equations, Helmholtz's conservation laws, Navier-Stokes equations, ideal magnetohydrodynamics, magnetic helicity.

Part II. Elements of knot theory, torus knot solutions to LIA, Gross-Pitaevskii equation, topological defects, helicity and linking numbers, measures of topological complexity.

Detailed program

The course is divided into two parts, the first being of introductory and general character, the second focused on more specific topics of current research.

Part I. Fluid flows and diffeoporphisms, Green's identities, Kelvin's correction for multiply connected domains, kinematic transport theorem, conservation theorems, decomposition of fluid motion, Euler's equations, vorticity transport equation, Helmholtz's conservation laws, Biot-Savart law, Navier-Stokes equations, energy dissipation, Burgers' steady vortex solution, Maxwell's equations, ideal magnetohydrodynamics, magnetic helicity, perfect and non-perfect analogous Euler's flows.

Part II. Localized induction approximation (LIA), elements of knot theory, fluid dynamic interpretation of Reidemeister moves, inflexional configuration and twist energy, torus knot solutions to LIA, fluid dynamics interpretation of Gross-Pitaevskii equation, topological defects, helicity and linking numbers, writhe, twist, knot polynomial invariants, measures of topological complexity.

Prerequisites

Elements of differential geometry of curves and surfaces in three-dimensional space, elements of mechanics of continuum systems, balance laws in physics.

Teaching form

Front lectures with written material presented on the blackboard, with lectures on theory aimed at providing basic knowledge of fundamental definitions, results and examples through which students acquire and develop the necessary capabilities to solve exercises and tackle proposed problems and applications.

Textbook and teaching resource

Lecture notes by the lecturer handed out during the course.

Semester

II semester.

Assessment method

Written examination paper to be done within 2 hours based on 4 questions of equal weight, each one concerning a specific topic covered during the course. No auxiliary material is permitted during the exam. Specific solutions must reproduce the material presented during the course, including detailed proofs of theorems and statements, complete with explicit computations. There is no oral examination and the final mark is expressed in 30ths.

The written examination paper must show operational *capability* to tackle and solve the proposed questions by using the acquired *knowledge* and the necessary *competence* to reproduce the topics presented during the course.

Office hours

Upon appointment, to be arranged with the lecturer.

METHODS OF MATHEMATICAL PHYSICS (2019/2020)

Teacher: Gregorio Falqui

Aims

The course presents the physical amd mathematical foundations of classical field theory, dealing with the introductory case of continuum Mechanics, with a special focus on Fluid Dynamics. The course will discuss the ideas, the general principles, the constitutive equations in the theory of Elasticity and in Fluid Dynamics, with a special emphasis on the latter, some applications and examples of which will be dealt with in details. A relevant part of the course deals with wave motion: linear and nonlinear waves, dispersionless and dispersive waves, and soliton equations.

The main expected learning outcomes are:

1) The knowledge and understanding of the definitions of "continuum mechanics" and, especially, fluid mechanics; the knowledge of the physical motivations thereof, of the main theoretical results and of the basic strategies for their proofs.

2) The mastering of the different approximations needed in the modelling processes (such as constitutive equations, linearization processes, asymptotic expansions) discussed during the course.

3) The ability to apply such a conceptual background in the analysis of the various applications; the acquisition/improvement of the skill in presenting and clearly discussing both the theoretical contents of the matter and their implementation in specific situations, possibly related with a broader scientific area.

4) The skill to build on the acquired knowledge by further refinements to be used in the analysis of subjects not fully developed during the lectures.

Contents

- The configuration space for continous bodies.
- Continuous bodies and deformation theory.
- Stress and Deformation tensors. Velocity gradient.
- Transport theorems and their geometrical formulation in terms of differential forms in the Euclidean three-space.
- The mass conservation equations, the Cauchy equations, the energy equation and the entropy inequality.
- The concept of pressure as isotropic stress and the Euler equations.
- Static and stationary solutions.
- Bernoulli's theorem and applications.
- Hemholtz equations.
- Theory of aerofoils.
- Sound waves.

- Gravity waves: air-water and stratified fluids.
- Waves in shallow water: the Korteweg-de Vries (KdV), Burgers and Airy equations. The Kadomtsev-Petviashvilii equation (mention).
- Hamiltonian formulation of the KdV equation.
- Viscous stresses and the Navier-Stokes equations. First applications.
- Scalings and the Reynolds number. Boundary layers and Prandtl equations.

Detailed program

The starting point of the course is the analysis of the deformation and of the motion of a continuous body, through the introduction of the notions of deformation gradient and velocity gradient. This part of the course introduces and makes use of methods of differential geometry in the Euclidean three space. Transport theorems of scalar and vector quantities are then discussed and proved as relevanmt part of the kinematics of continuum bodies.

Then dynamics is considered via the study of the external actions on a continuum deformable body. The core is Cauchy's stress theory. The mass conservation law and the balance equations for linear and angular momentum and energy are discussed. The notions of internal energy and entropy are introduced and the first and second principles of thermodynamics are reviewed.

The mechanical (and thermal) properties of solid bodies, liquids and gases are via the constitutive and state equations. The models of elastic bodies and Newtonian fluids (both in the compressible and incompressible regimes) and possibly viscous fluids are considered.

Then the course focusses on the equations describing the dynamics of fluids, starting from a deper analysis of the Euler equations, and deriving models that describe relevant physical phenomena.

Starting from static solutions, we move to an ample section devoted to the study of the Euler equations for the so-called ideal fluids and its consequences and applications. The most relevant are the Bernoulli equation, the Helmholtz laws on the vorticity evolution and Kelvin's circulation theorem.

Then the following points are discussed:

- Compressible elastic fluids and sound waves.
- The incompressibility regime (that is, the so-called incompressible fluids)
- Irrotational planar flows: the stream function ad the complex potential.
- Flow around an obstacle (theory of aerofoils).
- Gravity waves in an incompressible fluids ("water waves").
- Gravity waves in stratified fluids (internal waves).
- Gravity waves in the presence of surface tension.
- Gas dynamics and quasi-linear equations: theory of characteristics and shock waves.
- The Korteweg-de Vries equation for waves in shallow water: non-linearity, dispersion and the one-soliton solution. The Kadomtsev Petviashvilii equation.
- Hamiltonian formulation of KdV and the constants of the motion.

The last part of the course is devoted to the study of some properties of viscous fluids, described by the Navier-Stokes equations, where the following points are introduced and discussed :

- the transport of linear momentum through shear stresses.
- the non conservation of mechanical energy in the Navier-Stokes theory
- vorticity diffusion for a Navier-Stokes fluid.
- Scale transformations, self-similarity and the Reynolds number.
- The boundary layer and the Prandtl equations.

Prerequisites

No course of the Master Degree in Mathematics is strictly required for attending the present course. The basic notions of the courses Mathematical Analysis I and II, Linear algebra and Geometry, Physics I and II and Dynamical Systems and Classical Mechanics of the Batchelor Degree are needed. The prior knowledge of the contents of the courses Complex Analysis and Mathematical Physics of the third year of the Batchelor Degree are advisable.

Teaching form

Lectures (8CFU).

Textbook and teaching resource

Reference texts:

1. P. Chadwick, Continuum Mechanics: Concise Theory and Applications. Dover Publications, 1999.
2. S. Salsa: Partial Differential Equations in Action: from Modeling to theory. Springer, 2008.
3. G. Falkovich, Fluid Mechanics (a short course for physicists). Cambridge University Press, 2011.

The notes of the lectures are published in the e-learning page.

Semester

First semester.

Assessment method

The first part of the examination consists in the presentation of a written homework on a subject chosen within a list provided by the end of the lectures by the instructor. The list will comprise (also) items complementary to those discussed in the lectures. The student should inform the instructor about her/his choice of the subject of the homework at least 10 days befor the discussion date. Also, she/he must send a copy of the homework to the instructor at least 2 days before that date for a preliminary evaluation.

The main aim of this first part mainly regards points 3 and 4 of the above-mentioned "expected learning outcomes". The evaluation will regard, also taking into account the complexity of the chosen homework subject, the clarity of the exposition, the ability to sinthetyze the subject as well as the degree of mastering of the subject acquired by the student.

In the second part, the student will be asked to discuss a few of the main points of the program (at the instructor's choice). This part mainly addresses points 1 and 2 of the "expected learning outcomes".

For what the exam's outcome is concerned, the relative weight of the two parts is equal.

Office hours

Meetings whose schedule is to be agreed either via e-mail (preferred) or this e-learning page.

NUMBER THEORY AND CRIPTOGRAPHY (2019/2020)

Teacher: Francesca Dalla Volta

Aims

In line with the educational objectives of the Degree in Mathematics, the course aims to provide the student with some of the fundamental concepts, methods and some techniques of Number theory, essential for understanding the main asymmetric Cryptographic systems based on modular arithmetic or on elliptic curves over finite fields.

The student is expected to have knowledge of main probabilistic primality tests, of the structure of the group of elliptic curves over finite fields, with applications to the problem of discrete logarithm and of factorisation. He is also expected to have the ability to give proofs presented in the course, using given techniques to solve easy problems and the ability to study some more details of results presented during the course.

Contents

Some classical results in Number Theory are presented, with particular regard to factorizzation methods and primality tests, using modular arithmetic and Elliptic Curves.

Detailed program

- Integers and finite fields; Euler function; modular arithmetic
- Definition of a Cypher: public and private key
- Some topics about Prime numbers: Dirichlet's Theorem; Number Prime Theorem
- Prime numbers and factorization: pseudoprimes; primality tests (Fermat, Jacobj, Miller-Rabin AKS); (p-1)-pollard method for factorization; complexity of the alghoritms.
- Some remark about Riemann's zeta function; Euler'sFactorization; Riemann's hypothesis; extended Riemann's hypothesis and some consequence on primality tests.
- Diffie-Hellman cypher; discrete logarithm
- Elliptic curves; group of the points of an elliptic curve on a finite field.
- Endomorphisms.
- Torsion points and Weil pairing.
- Hasse Theorem
- Cryptosystems on elliptic curves.
- Discrete Logarithm on Elliptic Curves
- Digital Signature: DSA, ECDSA

Prerequisites

Basic Algebra: algebraic structure; abelian groups; finite fields.

Teaching form

Lectures (8 credits). They will be of two different kind: they will give knowledge of basic definitions, relevant results and theorems. On the other side, we intend to give skills to use results and knowledge in solving exercises and analysing problems

Textbook and teaching resource

- N. Koblitz, A course in Number Theory and Cryptography, volume 114 of Graduate texts in Mathematics, Springer-Verlag, second edition, 1994.
- A. Languasco, A. Zaccagnini, Introduzione alla Crittografia, Hoepli Editore, 2004.
- H.E. Rose, A course in Number Theory, II edizione, Oxford: Clarendon press, 1994
- Lawrence C. Washington, Elliptic Curves, Number Theory and Criptogtaphy CRCPress
- Maria Welleda Baldoni, Ciro Ciliberto, Giulia Maria Piacentini Cattaneo, Elementary Number Theory, Cryptography and Codes, 2009 Springer-Verlag Berlin Heidelberg

Semester

II term.

Assessment method

Written and Oral examination.

- The written part consists of exercises where the students show their ability in using methods and tools introduced in the course.
- The oral part consists of two parts:

- discussion about written part;
- the student may decide to have a classical oral exam, where he must show his competence about subjects considered in the lectures, also giving motivations for applications of theoretical topics; alternatively, one student may give a talk about a particular subject, which was considered not very deeply in the course. The final result is achieved considering the average between the mark obtained in written+discussion (together), and the mark obtained in the subsequent oral part.

Mark range: 18-30/30.

Office hours

By direct agreement.

NUMERICAL LINEAR ALGEBRA (2019/2020)

Teacher: Cristina Tablino Possio

Aims

Consistently with the educational objectives of the Master Degree in Mathematics, the course aims at providing the *knowledge* about the advanced iterative methods for solving linear systems. Skills to understand the computational difficulties typical in the resolution of large linear systems and skills to handle the techniques of analysis of the most innovative iterative methods will be provided , so that the student will acquire th*ose abilities* useful in facing the choice of a suitable solver in practical problems.

Contents

Advanced iterative methods proposed in literature are studied and their application to the solution of linear systems arising in the discretization of PDEs and IEs is considered.

Detailed program

- Krylov methods for symmetric and non-symmetric linear systems.
- Spectral analysis and conditioning of linear system arising from PDEs.
- Preconditioning techniques.
- Geometrical and algebraic Multigrid methods.
- Fast Transforms.
- Singular value decomposition and its applications.
- Application of previous techniques to linear systems arising in the approximation of PDEs and IEs.

Prerequisites

Basic courses of the degree in Mathematics (Mathematical Analysis I and II, Linear Algebra, Introduction to Numerical Analysis) and Approximation of Differential Equation, even if not mandatory.

Teaching form

Standard blackboard lessons and computer practice labs in Matlab (8 CFU).

Textbook and teaching resource

- S. C. Brenner, L. R. Scott. *The mathematical theory of finite element methods. Third edition.* Texts in Applied Mathematics, 15. Springer, New York, 2008.
- G. H. Golub, C. F. Van Loan. *Matrix computations. Third edition.* Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, Baltimore, MD, 1996.
- A. Greenbaum. *Iterative methods for solving linear systems.* Frontiers in Applied Mathematics, 17. *Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA*, 1997.
- Y. Saad. *Iterative methods for sparse linear systems. Second edition.* Society for Industrial and Applied Mathematics, Philadelphia, PA, 2003.
- U. Trottenberg, C. W. Oosterlee, A. Schüller. *Multigrid. With contributions by A. Brandt, P. Oswald and K. Stüben.* Academic Press, Inc., San Diego, CA, 2001.

Semester

Second semester.

Assessment method

Written individual project, chosen among two possible projects proposed at the end of the course and to be discussed during the oral examination, and oral examination.

The written project evaluates student's skills in solving problems by using theoretical tools and Matlab codes developed during the course. The original development of the project is encouraged according to personal curiosity and interests.

The oral examination consists in discussing the written project and in a second part where the knowledge and the ability to critically expose the studied arguments and computational techniques is evaluated in order to verify if the student has acquired the critical and operational knowledge of the definitions, methods and results presented during the course.

Mark is out of thirty. The student needs to reach at least 18/30 in both parts to pass the exam. the final mark is the average of the two partial marks. The project with at least 18/30 mark is still valid if the oral test is repeated.

There will be 5 exam sessions (in June, July, September, January, February).

Office hours

By appointment.

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS
(2019/2020)

Teachers: Alessandro Russo, Cristina Tablino Possio

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims to provide the knowledge of the rigorous mathematical theory of the **Finite Element Method** for the approximation of linear elliptic second-order partial differential equations.

At the end of the course the students will have the skills needed to understand more advanced aspects of the method, both with individual work and with other courses.

The method will be implemented in MATLAB, and with the developed codes the students will have the ability to solve simple real-life problems connected with the approximation of partial differential equations.

Contents

- Sobolev Spaces
- Lax-Milgram Lemma
- Galerkin methods
- Cea's Lemma
- Linear Finite Elements
- Lagrange Finite Elements of order k
- error estimates in the energy norm
- Bramble-Hilbert Lemma
- Aubin-Nitsche duality argument

Detailed program

- **Basic concepts.** Presentation in the one-dimensional case of the techniques and the ideas which will be studied in the rest of the course.
- **Sobolev Spaces.** The natural functional environment for the mathematical analysis of the finite element method.
- **Variational Formulation of Elliptic Boundary Value Problems.** Abstract setting for the partial differential equations which will be studied in the course.
- **The Construction of a Finite Element Space.** How to build a finite element.
- **Polynomial Approximation Theory in Sobolev Spaces.** The core of the course. We will study how finite elements (in essence, continuous, piecewise smooth functions) approximate functions in Sobolev Spaces.
- **n-Dimensional Variational Problems.** Examples of partial differential equations which can be approximated with the finite element method.

Prerequisites

Courses of the Laurea Triennale. It is recommended the course Analisi Funzionale of the 1st semester.

Teaching form

Lessons (6 CFU), exercise classes with blackboard and computer (2 CFU).

Textbook and teaching resource

S. C. Brenner e L. R. Scott: The Mathematical Theory of Finite Element Methods, Springer 2008

Semester

2nd semester

Assessment method

The final examination is split into two parts:

- writing and presenting a project;
- oral examitation.

Mark is out of thirty. The student need to reach at least 18/30 in both parts to pass the exam. the final mark is the average of the two partial marks.

The project consists in implementing the approximation of a problem related to partial differential equations, using the codes developed during the course. Tha aim is to test the ability to use the developed instruments. Group working is encouraged (max 3 students) and the quality of the exposition will be part of the mark.

The oral examination will evaluate the knowledge of the definitions, results and rigourous proofs developed in the course; the capacity to understand what are the key points of the theory will also be checke.

There will be 5 exam sessions (in June, July, September, January, February).

Office hours

On appointment.

PREPARATION OF DIDACTIC EXPERIMENTS (2018/2019)

Teacher: Jacopo Parravicini

Missing information

REAL ANALYSIS AND DIFFERENTIAL EQUATIONS (2019/2020)

Teacher: Graziano Guerra

Aims

According to the Mathematics Degree educational objectives, the course aim is the introduction to linear partial differential equations with hints to non--linear ones. The skills needed to understand and analyse the most important techniques in the theory and the ability to solve exercises and problems will be provided.

Contents

Spectral theory for compact and selfadjoint operators. Elliptic equations, maximum principles, Laplacian eigenvalues and eigenfunctions. Linear parabolic and hyperbolic partial differential equations. Hyperbolic systems of first order equations.

Detailed program

Spectral theory: Definitions of adjoint, selfadjoint, compact operators, spectrum. Properties. Spectrum of compact operators. Spectral decomposition of selfadjoint compact operators. Theorem of Fredholm alternative.

Second order elliptic equations: elliptic operators, classical and weak solutions, representation of solutions, maximum principles, Laplacian eigenvalues and eigenfunctions.

Linear parabolic equations: Definitions, weak solutions. Energy estimates, existence and uniqueness of weak solutions. Regularity. Maximum principles.

Linear hyperbolic equations: Definitions, weak solutions. Energy estimates, existence and uniqueness. Finite propagation speed.

Hyperbolic systems of first order equations: Definitions, symmetric hyperbolic systems, systems with constant coefficients.

Prerequisites

Main results in functional analysis, bounded linear operators in Banach spaces, weak topologies, spaces of continuous and H\"older continuous functions, $\mathbf{L}^p$ spaces, their duals and properties, Sobolev spaces and immersion theorems.

Teaching form

Lectures in classroom where definitions, results and relevant examples are illustrated (sometimes with relation to extra-mathematical applications as well).

Course delivered in Italian with the possibility of being delivered in English if foreign students request it.

Textbook and teaching resource

- A. Bressan. Hyperbolic systems of conservation laws: the one-dimensional Cauchy problem. Vol. 20. Oxford University Press on Demand, 2000.
- A. Bressan. Lecture Notes on Functional Analysis. With applications to linear partial differential

equations. American Mathematical Society, 2013.
- H. Brezis. Functional analysis, Sobolev spaces and partial differential equations. Springer Science and Business Media, 2010.
- L. C. Evans, Partial Differential Equations, AMS Graduate Studies in Mathematics, Vol.19. Second Edition, Providence 2010.
- D. Gilbarg, N. S. Trudinger, Elliptic partial differential equations of second order, Reprint of the 1998 edition. Classics in Mathematics. Springer-Verlag, Berlin, 2001.

Course webpage: https://elearning.unimib.it/course/view.php?id=25417

Semester

Second semester.

Assessment method

Written examination. Mark out of thirty. The student is asked to develop two topics out of three proposed at the examination in two hours. The written discussion must be precise, detailed, comprehensive and consistent with the proposed topic. Moreover it must contain some of the most significant proofs. The ability to present a selection of proofs and, above all, the critical and operational knowledge of the definitions and results presented during the course is evaluated, also by the illustration of examples and counterexamples.

Office hours

By appointment.

REPRESENTATION THEORY (2019/2020)

Teacher: Lino Giuseppe Di Martino

Aims

The course is aimed to present the contents and the fundamental methods, as well as some noteworthy applications of the 'classical' theory of representations of finite groups.

Contents

Semisimple rings and modules. Modules and representations. Characters of finite groups. Tensor products of representations. Permutation representations and applications. Direct products. Induction and restriction of representations. Clifford Theory.

Detailed program

Semisimple rings and modules.

Generalities on rings and modules. Artinian and noetherian rings and modules. Semisimple rings and modules. Simple modules. Decomposition of a semisimple modules in isotypic components. Structure of semisimple rings. Wedderburn's theorem. Double centralizer property (DCP). Structure of simple artinian rings.

Modules and represesentations.

The group algebra KG. KG-modules and G-representations. Completely reducible representations. Maschke's theorem. Representations over splitting fields: structure of KG. Frobenius-Schur theorem. Examples of complex representations of finite groups.

Characters of finite groups.

Definition and properties of characters of a group G. The space CF(G) of class functions. CharK = 0 and K splitting for G: characters and modules; the character table. Regular representation, orthogonal idempotents, first orthogonality relations. Irr(G) is an orthonormal basis of CF(G); second orthogonality relations. Algebraic integers and characters; structure constants of the centre of KG. The degree of an irreducible character divides the order of G. Applications:The p^a.q^b Theorem of Burnside. Structural properties of a group detectable from the character table [Remarks on representations of compact groups]

Tensor products of representations.

Tensor products of modules. Tensor products of representations, products of characters. The ring of virtual characters. The Burnside-Brauer theorem. Counting involutions, the Brauer-Fowler theorem and its implications.

Permutation representations and applications.

Permutation groups. Actions on conjugacy classes and characters. Brauer's permutational Lemma. Real characters.

Direct products.

Irreducible characters of a direct product. Application: Burnside's theorem on character degress.

Induction and restriction of representations, Clifford theory.

Representations induced from subgroups. Induced characters. Frobenius reciprocity law and applications. Restriction to a normal subgroup: Clifford's theory. Inertia group, Clifford correspondence. Ito's theorem.

Prerequisites

It is recommended an a priori knowledge of the standard contents of first and second year Algebra courses, plus some extra knowledge of field theory.

Teaching form

Lessons.

Textbook and teaching resource

Reference Textbooks:

C. W. Curtis and I. Reiner, Representation Theory of Finite Groups and Associative Algebras, Wiley Interscience 1962.

C. W. Curtis and I. Reiner, Methods of Representation Theory I, Wiley 1981.

L. Dornhoff, Group Representation Theory, Marcel Dekker 1971.

B. Huppert, Character Theory of Finite Groups, de Gruyter 2011.

I.M. Isaacs, Character theory of finite groups, Academic Press 1976.

Semester

First semester.

Assessment method

The exam is only oral. It consists of a number of questions and an evaluation (marks: 18/30 to 30/30). The questions are aimed to verify that the student has understood the theoretical development of the course and has a good knowledge of the theorems (and their proofs), as given in the lectures.

Office hours

By appointment.

STOCHASTIC METHODS IN FINANCE (2019/2020)

Teachers: Francesco Caravenna, Gianmario Tessitore

Aims

The course aims at providing the student with the definitions and basic properties of the Brownian motion and the fundamental results of the theory of stochastic differential equations. Particular emphasis will be given on the interactions between stochastic differential equations and partial differential equations, and on the applications to the modeling of financial derivatives.

At the end of the course students will have acquired the following:

- *knowledge*: language, definitions and statements of the fundamental results about Brownian motion and stochastic differential equations;
- *competence*: operational understanding of the main proof techniques and of the main financial models to which the theory can be applied;
- *skills*: ability to apply theoretical notions to the analysis of problems and models.

Contents

- Introduction to stochastic processes in continuous time
- Levy processes and Brownian motion
- Ito stochastic integral
- Ito's formula
- Stochastic differential equations (SDEs)
- The associated Kolmogorov differential operator
- The Kolmogorov PDE and the Feynman-Kac formula
- Introduction to continuous time financial markets
- The Black and Scholes formula and the pricing of European options

Detailed program

Il moto browniano. Definizione di processo stocastico. Spazio delle traiettorie, insiemi cilindrici, sigma-algebra prodotto. Legge di un processo stocastico e leggi finito-dimensionali. Vettori aleatori normali. Processi stocastici gaussiani. Definizione di moto browniano. Costruzione del moto Browniano a partire dal Teorema di esistenza di Daniell-Kolmogorov e utilizzando il Teorema di continuità di Kolmogorov. Caratterizzazione del MB come processo gaussiano. Proprietà di invarianza (riflessione spaziale, traslazione e riflessione temporale, riscalamento diffusivo, inversione temporale). Moti Browniani rispetto alla filtrazione naturale e rispetto a una filtrazione qualsiasi. Proprietà delle traiettorie di un moto browniano: non differenziabliltà. Calcolo delle variazione quadratica. Legge del Logaritmo iterato. Moti Browniani in dimensione d.

Processi di Lévy. Generalità sulle filtrazioni (*F*t) indicizzate da un insieme continuo. Filtrazione naturale di un processo stocastico, processi adattati a una filtrazione. Continuità a destra e completezza per una filtrazione (definizione di *F*t+), ampliamento standard. Processi di Lévy rispetto a una filtrazion. Esempi: processo di Poisson, processo di Poisson composto. Indipendenza da *F*0 di un processo di Lévy rispetto a una filtrazione (*F*t). Legge 0-1 di Blumenthal. Tempi d'arresto proprietà di Markov forte.

L'integrale di Ito: Modificazione e indistinguibilità per processi stocastici. Continuità e misurabilità per processi stocastici. Tempi d'arresto. La sigma algebra degli eventi antecedenti a un tempo d'arresto. Martingale a tempo continuo, esempi, modificazioni continue da destra (solo enunciato), teorema d'arresto e disuguaglianza massimale. Processi progressivamente misurabili. I processi semplici, L'integrale di Ito per i processi semplici. L'estensione a M^2 e l'estensione a M^2. Proprietà: località, esistenza della versione a traiettorie continue, proprietà di Martingala. Variazione quadratica. Integrale di Ito di processi a traiettorie continue e somme di Riemann. L'integrale di Wiener. Martingale locali.

La formula di Ito. La formula di Ito per il moto browniano, con dimostrazione. Processi di Ito. Formula di Ito per processi di Ito generali. Applicazione della formula di Ito, Moto browniano geometrico e supermartingala esponenziale. La formula di Ito nel caso multidimensionale. Funzioni armoniche e problema di Dirichlet. Il Teorema di Girsanov. Esempio: il processo di Ornstein-Ühlenbeck. Il Teorema di rappresentazione delle martingale browniane.

Equazioni differenziali stocastiche. Esistenza forte e debole, unicità pathwise e in legge. : Il Teorema di Yamada-Watanabe. Esistenza forte (sketch di dimostrazione) e unicità pathwise (dimostrazione) sotto ipotesi Lipschitz. Proprietà di flusso. ll semigruppo di Kolmogorov. L'equazione alle derivate parziali di Kolmogorov. La formula di Feynamn-Kac.

Applicazione ai mercati finanziari: Sottostanti, opzioni call e put, loro valore (payoff) e significato. Prezzaggio di un'opzione mediante copertura (hedging). Modello di mercato finanziario a tempo continuo basato su un titolo non rischioso (bond) e d titoli rischiosi (stocks) guidati da d moti browniani indipendenti. Misura martingala locale equivalente. Strategie di investimento autofinanzianti e strategie ammissibili. Teorema di assenza di arbitraggio. Prezzaggio e copertura di opzioni europee. Il modello di Black&Scholes (unidimensionale, con tasso d'interesse, drift e volatilità costanti). Formula esplicita per il prezzo delle opzioni call. Un modello di mercato finanziario Markoviano con drift e volatilità dipendenti dal tempo e del sottostante. Formula di rappresentazione per il prezzo delle opzioni Europee e per la strategia di copertura.

Prerequisites

Knowledge of probability calculus and of the theory of discrete-time stochastic processes are needed. It is useful to know definitions and first properties of L^p spaces and Hilbert spaces.

Teaching form

Lectures in the classroom, which illustrate definitions, results proofs, techniques and relevant examples. Some of the lessons will be devoted to the analysis of financial models related to mathematical theory.

Textbook and teaching resource

Lecture notes (available on the e-learning platform)

Brownian Motion, Martingales, and Stochastic Calculus, Jean-François Le Gall, Springer series Graduate Texts in Mathematics (Volume 274, 2016).

Semester

First semester.

Assessment method

Oral exam. Mark out of thirty.

The oral test evaluates the student's ability to critically review the definitions, statements and proofs presented during the course. The ability to expose a topic, among those illustrated during the course, and communicated to the student 15 minutes before the oral examination will also be evaluated.

Office hours

By appointment.

STOCHASTIC PROCESSES (2019/2020)

Teachers: Federica Masiero, Gianmario Tessitore

Aims

In line with the educational objectives of the Master Degree in Mathematics, the course aims at providing the *knowledge* about the fundamental concepts and statements of the theory of stochastic processes in discrete time. It will also build the *skills* needed to understand and use the most important proving arguments and techniques in the theory and the *ability* to solve exercises and deal with problems exploiting them. Particular emphasis will be put on the theory of martingales.

Contents

Conditional law and conditional expectation. Martingales in discrete time. Introduction to the asymptotic behavior of Markov chains. The Poisson process. Examples and applications.

Detailed program

- Conditional law and expectation. Definitions and properties. Existence of conditional expectation of a random variable with respect to a sigma algebra. Fundamental properties: tower property, Jensen inequality, freezing. Limit theorems.
- Discrete-time Martingales. Definition and examples (sums of independent centered r.v.s, products of independent r.v.s with expectation 1, closed martingales). Integral of a predictable process. Stopped Martingales. Optional stopping theorem. Applications: first hitting time of a random walk on Z; the gambler's ruin problem. Upcrossing Lemma. Almost certain convergence of martingales bounded in L^1 norm. Martingales bounded in L^2 norm. Uniform integrability and convergence in L^p. Maximal inequality. Doob's inequality. Examples: Galton-Watson branching processes. Absence of arbitrage in discrete time binomial markets.
- Asymptotic behavior of Markov chains. Definition and properties. Links with martingale and harmonic functions. Invariant measures: existence and uniqueness in the irreducible and positively recurrent case. Ergodic theorem and law of large numbers.
- The Poisson process. Definition and properties. Independence of increments.

Prerequisites

Knowledge of differential and integral calculus for functions of one and more real variables, as well as measure-theoretical probability theory is needed. It is also useful to know definitions and basic properties of L^p spaces and Hilbert spaces.

Teaching form

Lectures in the classroom, divided into: theoretical lessons in which the knowledge about definitions, results and relevant examples is given and other lessons in which we try to give the skills and abilities needed to use the previous notions to solve exercises and to deal with problems (also related to extra-mathematical applications).

Textbook and teaching resource

- D. Williams, Probability with Martingales, Cambridge University Press (1991).
- E. Pardoux, Markov Processes and Applications, Wiley Series in Probability and Statistics (2008).
- Lecture notes (available on the e-learning platform)
- Written tests from previous years, with detailed solutions (available on the e-learning platform).
- List of proofs that may be requested during the oral examination (available on the e-learning platform).

Semester

Second (Spring) semester.

Assessment method

Written and oral exam. Mark out of thirty.

The written test evaluates the operational *ability* to solve exercises, it receives a mark out of thirty. It is necessary to obtain an evaluation of at least 16/30 in the written test to access the oral exam, which evaluates the *capacity* to present a selection of proofs and, above all, the critical and operational *knowledge* of the definitions and results presented during the course, also by means of examples and counterexamples. The final evaluation will result from the average between the evaluation of the written test and that of the oral examination. The exam is passed if the evaluation is at least 18/30.

There will be 5 exam sessions (in June, July, September, January, February).

Office hours

By appointment.

SYMPLECTIC GEOMETRY (2018/2019)

Teacher: Roberto Paoletti

Aims

We aim to discuss and elaborate on the basic concepts of Symplectic Geometry, starting with the local aspects and then moving on to the more global properties. Time permitting, we shall approach the theme of symplectic reduction. One basic goal is to clarify the geometric meaning of some fundamental concepts that are introduced in various contexts, such as moment map, generating functions, canonical transformations, Hamilton-Jacobi equation and theory, *etc*.

The expected learning outcomes include:

- the knowledge and understanding of the fundamental definitions and statements, as well as of the basic strategies of proof in symplectic geometry; the knowledge and understanding of some crucial examples in which the theory manifests itself;
- the ability to recognize the role that concepts and techniques from symplectic geometry play in various areas of mathematics (such as differential equations, Riemannian geometry, complex geometry, representation theory), and in the mathematical modelling of physical situations (mathematical physics); the skill to apply such conceptual background to the construction of concrete examples and to the solution of exercises; the ability to communicate and explain in a clear and precise manner both the theoretical aspects of the course and their applications to specific situations, possibly to different contexts.

Contents

Symplectic vector spaces, symplectic manifolds, Hamiltonian flows and symplectomorphisms, canonical forms of symplectic structures, moment maps and symplectic reductions

Detailed program

· Symplectic linear algebra.

· Cotangent bundles, Hamiltonian equations, Poisson brackets.

· Symplectic manifolds and special submanifolds, neighborhood theorems.

· Isotopies and theorems of Darboux and Moser.

· Generating functions, Hamilton-Jacobi equations and geometric solution;

· Moment maps and their properties, symplectic reduction;

· Compatible complex and almost complex structures; Kähler and quasi-Kähler manifolds.

· Coadjoint orbits and their natural symplectic structure.

Prerequisites

Prerequisites are: a good familiarity with the concepts of linear algebra offered during the Laurea Triennale in Mathematics, since the study of symplectic linear algebra will play a foundational role in the development of the course; the most basic notions on differentiable manifolds and differential forms, as they are commonly treated say in the courses of Geometry II and III. Brief recalls will be given as needed.

Teaching form

Lectures at the blackboard

Textbook and teaching resource

Bibliographic references:

V. Guillemin, S. Sternberg, Symplectic Techniques in Physics, Cambridge University Press

D. McDuff, D. Salamon, Introduction to Symplectic Topology, Clarendon Press, Oxford

Recommended readings:

V. Guillemin, S. Sternberg, Semiclassical Analysis, International Press

J. J. Duistermaat, Fourier Integral Operators, Birkhäuser

Semester

1st semester

Assessment method

The exam will comprise two written tests, each dealing with a part of the course (I and II), aimed at the evaluation of the knowledge, understanding and skills forming the expected learning outcomes of the course. The precise subdivision of the topics will be communicated during the course well in advance with respect to the tests themselves. Each test consists of a flexible combination of theoretical questions (including definitions, statements and proofs) and more practical ones (including exercises, examples and counterexamples). Each test will be evalueated independently, and will concur in the same amount to the determination of the final grade.

Office hours

upon appointment

TEACHING MATHEMATICS (2019/2020)

Teachers: Davide Luigi Ferrario, Franco Magri

Aims

Contents

Introduction to the methods, the ideas, the historical developtment and the theoretical frameworks useful in teaching secondary school mathematics. This course will be delivered only in Italian language. The course is held in Italian language. **Please refer to the page of the Italian part of the document for further details**.

Detailed program

Prerequisites

Teaching form

Textbook and teaching resource

DIDATTICA DELLA MATEMATICA, di Roberto Natalini, Anna Baccaglini-Frank, Pietro Di Martino, Giuseppe Rosolini (Mondadori 2018).

Semester

2S

Assessment method

Office hours

TOPICS IN GEOMETRY AND TOPOLOGY (2019/2020)

Teacher: Davide Luigi Ferrario

Aims

The aim of the course is to take some classic topics in algebraic topology of simplicial complexes, introducing homology theory, cohomology theory and some aspects of homotopy theory, with some recent applications.

(Further verbose details in the Italian version.)

Contents

Simplicial complexes, homology and cohomology of polyhedra, triangulable manifolds, homotopy groups, applications to data analysis and dynamical systems.

Detailed program

Fundamental concepts: topological spaces, connectedness, compactness, function spaces, general ideas on Categories, push-out diagrams. Euclidean and abstract simplicial complexes. Introduction to homological algebra. Homology with coefficients. Category of polyhedra. Cohomology of polyhedra. Cohomology ring, cap product. Triangulable manifolds. Surfaces and classification. Poincaré Duality. Fundamental group of polyhedra. Fundamental group and homology. Homotopy groups. Obstruction theory. Applications to: computational homology, persistent homology, data analysis and dynamical systems.

Prerequisites

Basic topics covered in bachelor courses of geometry and algebra

Teaching form

Lectures: 8 ECTS credits.

Textbook and teaching resource

Ferrario, Piccinini, "Simplicial structures in topology". CMS Books in Mathematics, Springer, New York, 2011. xvi+243 pp. ISBN: 978-1-4419-7235-4

Semester

1S

Assessment method

Oral examination on the topics covered in the course, with in-depth analysi and re-elaboration of them with a personal perspective. The date and the content of the seminar, which is part of the exam, have to be first discussed with the teacher.

Office hours

By appointment, or Mondays 15:30.

Ph.D PROGRAM

The Department of Mathematics and its Applications contributes, together with the Department of Mathematics of the University of Pavia and INdAM, to the organization of a Ph.D. program in Mathematics. The program has a three-year duration and it is the traditional gateway to research in Mathematics. It also provides various scholarships.

The PhD Program in Mathematics aims to transmit to the PhD students the research techniques and methodologies in the current fields of Mathematics and its applications. For more specific information, refer to the website: https://sites.google.com/view/jointphd/home