Titolo nota

MICROECONOMETRIA - LEZIONE 2 (LAB 1)

10/05/2019

STATA

SUM

LIST

REG

PREDICT

$$y = X\delta + u$$

$$\hat{y} = X\hat{\delta} \quad \text{(FITTED VALUES)}$$

PREDICT UHAT, RESIDUALS

SAVE

STIMATORE SUR

$$y_{it} = \alpha_i + \beta_{2i}\, x_{it} + \beta_{3i}\, x_{it}^2 + u_{it}$$

$i = 1 \ldots N = 3$

$t = 1 \ldots T = 146$

$$\underset{(T\cdot 1)}{y_i} = \underset{1\cdot 1}{\alpha_i}\, \underset{(T\cdot 1)}{J} + \underset{T\cdot 2}{x_i^*}\, \underset{2\cdot 1}{\beta_i} + \underset{(T\cdot 1)}{u_i} = \boxed{X_i \delta_i + u_i}$$

$$\mathrm{DOVE}\quad \underset{T\cdot 2}{x_i^*} \equiv \left(\underset{T\cdot 1}{\overset{\mathrm{LOG}\left(\frac{GDP}{POP}\right)}{x_i}} \;\vdots\; \underset{T\cdot 1}{\overset{\left(\mathrm{LOG}\left(\frac{GDP}{POP}\right)\right)^2}{x_i^2}} \right) \quad \mathrm{E}\quad X_i \equiv \left(J \;\vdots\; x_i^* \right)$$

$$\underset{NT\cdot 1}{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}} = \underset{NT\cdot NK}{\begin{pmatrix} \overset{T\cdot 3}{X_1} & & 0 \\ & X_2 & \\ 0 & & X_3 \end{pmatrix}} \underset{NK\cdot 1}{\begin{pmatrix} \overset{3\cdot 1}{\delta_1} \\ \delta_2 \\ \delta_3 \end{pmatrix}} + \underset{NT\cdot 1}{\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}}$$

DOVE $T=146;\ N=3;\ K=3$

$$y = X\delta + U$$

$NT \cdot 1$ $NT \cdot NK$ $NK \cdot 1$ $NT \cdot 1$

$$E(UU') = \Phi = \Sigma_N \otimes I_T$$

$NT \cdot NT$

DOVE $$\Sigma_N = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix}$$

$N=3$

$$\hat{\delta}_{SUR} = \left(X' \Phi^{-1} X\right)^{-1} X' \Phi^{-1} y$$

STIMA DI Φ

$$\hat{\sigma}_i^2 = \frac{RSS_i}{T-K}, \quad T = 146, \; k = 3$$

$$\hat{U}_i = y_i - X_i \hat{\delta}_i \text{ OLS}$$

$$\widehat{\sigma_{ij}} = \frac{\hat{u}_i' \hat{u}_j}{T-K}, \quad i \neq j$$

$$\Downarrow$$

$$\hat{\Sigma}_N = \begin{pmatrix} \ddots & & \hat{\sigma}_{ij} \\ & \hat{\sigma}_i^2 & \\ & & \ddots \end{pmatrix} \Rightarrow \hat{\Phi} = \hat{\Sigma}_N \otimes I_T$$

$$\Downarrow$$

$$\hat{\delta}_{SUR} = (X' \hat{\Phi}^{-1} X)^{-1} X' \hat{\Phi}^{-1} y$$

COSA SUCCEDE ALLO STIMATORE SUR SE
i REGRESSORI SONO GLI STESSI NELLE DIVERSE
EQUAZIONI ?

ES1: DOMANDE DI BENI DI CONSUMO, IN CUI
LE DOMANDE/QUANTITÀ SONO FUNZIONE DEI
PREZZI BENI E REDDITO CONSUMATORE

ES2. $z_t = A z_{t-1} + U_t$ VAR(1)

$N=2$

DOVE $z_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}$; $z_{t-1} = \begin{pmatrix} y_{t-1} \\ x_{t-1} \end{pmatrix}$

$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$; $U_t = \begin{pmatrix} U_{yt} \\ U_{xt} \end{pmatrix}$

$$\begin{cases} y_t = a_{11}\, y_{t-1} + a_{12}\, x_{t-1} + u_{yt} \\ x_t = a_{21}\, y_{t-1} + a_{22}\, x_{t-1} + u_{xt} \end{cases}$$

VAR(1)

RISPOSTA : SUR = OLS (EQUAZIONE PER EQUAZIONE)

INDIPENDENTEMENTE DALLA FORMA DI Φ, CIOÈ

INDIPENDENTEMENTE DALLA FORMA DI Σ_u

$$y = X\delta + U$$

$$\hat{\delta}_{sur} = \left(X'\Phi^{-1}X\right)^{-1} X'\Phi^{-1}y$$

$$X = \begin{pmatrix} X_1 & & 0 \\ & \ddots \; X_i \; \ddots & \\ 0 & & X_N \end{pmatrix} = \begin{pmatrix} \tilde{x} & & 0 \\ & \tilde{x} & \\ 0 & \ddots & \tilde{x} \end{pmatrix} = I_N \otimes \tilde{x}$$

Done $X_1 = X_2 = \cdots = X_i = \cdots = X_N = \tilde{x}$

$$\Phi = \Sigma_N \otimes I_T \Rightarrow \Phi^{-1} = \Sigma_N^{-1} \otimes I_T$$

$$X = I_N \otimes \tilde{x}$$

$$\hat{\delta}_{sur} = \left[\left(I_N \otimes \tilde{x}'\right)\left(\Sigma_N^{-1} \otimes I_T\right)\left(I_N \otimes \tilde{x}\right)\right]^{-1} \cdot$$

$$\left(I_N \otimes \tilde{x}'\right)\left(\Sigma_N^{-1} \otimes I_T\right) y \quad =$$

$$= \left[\left(\cancel{I_N} \Sigma_N^{-1} \cancel{I_N} \right) \otimes \left(\tilde{x}' I_T \tilde{x} \right) \right]^{-1}$$

$$\left(\cancel{I_N} \Sigma_N^{-1} \right) \otimes \left(\tilde{x}' I_T \right) y$$

$$= \left[\Sigma_N^{-1} \otimes \tilde{x}' \tilde{x} \right]^{-1} \cdot \Sigma_N^{-1} \otimes \left(\tilde{x}' I_T \right) y$$

$$= \cancel{\Sigma_N} \otimes \left(\tilde{x}' \tilde{x} \right)^{-1} \cancel{\Sigma_N^{-1}} \otimes \left(\tilde{x}' I_T \right) y =$$

$$\Downarrow$$

$$\boxed{\hat{\delta}_{SUR} = I_N \otimes (\tilde{x}'\tilde{x})^{-1}\tilde{x}'y}$$

$$\hat{\delta}_{SUR} = \begin{pmatrix} (\tilde{x}'\tilde{x})^{-1}\tilde{x}'y_1 \\ (\tilde{x}'\tilde{x})^{-1}\tilde{x}'y_2 \\ \vdots \\ (\tilde{x}'\tilde{x})^{-1}\tilde{x}'y_N \end{pmatrix} \Longrightarrow SUR = OLS$$

(EQUAZIONE PER EQUAZIONE)