MICROECONOMETRIA – LEZIONE 7

Titolo nota 21/05/2019

MODELLO PANEL EFFETTI RANDOM / ERROR COMPONENT MODEL

$$\underset{T\times 1}{y_i} = \underset{T\times K}{x_i}\,\underset{K\times 1}{\delta} + \underset{T\times 1}{v_i}, \quad \text{dove } v_i = \mu_i \iota + u_i$$

(ERRORE COMPOSITO)

ETEROGENEITÀ INDIVIDUALE

NON CORRELATA CON x_i

$$\hat{\delta}_{GLS} \Longleftarrow$$

$$\Downarrow$$

$$\underset{T\cdot T}{E(v_i v_i')} \equiv V = \sigma_1^2 C_T + \sigma_u^2 D_T$$

Dove $\boxed{\sigma_1^2 \equiv T\sigma_\mu^2 + \sigma_u^2}$

$$\text{GLS} \xrightarrow[\sigma_\mu^2 \text{ E } \sigma_u^2]{\text{STIMA DI}} \text{FGLS}$$

STIMA DI σ_1^2

SI SFRUTTA LA TRASFORMAZIONE BETWEEN
(TRASFORMAZIONE IN MEDIE RISPETTO A t)

MODELLO RANDOM EFFECTS (FORMA SCALARE)

(1) $$y_{it} = \alpha + \sum_{z=2}^{K} \beta_z x_{zit} + \underbrace{(\mu_i + v_{it})}_{\nu_{it}}$$

TRASFORMIAMO y_{it}, x_{kit} $(k=2 \sim K)$, μ_i, u_{it} CON <u>BETWEEN</u>:

(2) $$y_{i\bullet} = \alpha + \sum_{k=2}^{K} \beta_k x_{ki\bullet} + \underbrace{(\mu_i + u_{i\bullet})}_{\nu_{i\bullet}}$$

DOVE $z_{i\bullet} = \frac{1}{T} \sum_t z_{it}$

$\rightarrow$ CROSS-SECTION $(i = 1 \sim N)$

(3) $\mathrm{VAR}(\nu_{i\cdot}) = \mathrm{VAR}(\mu_i + v_{i\cdot}) = \mathrm{VAR}(\mu_i) + \mathrm{VAR}(v_{i\cdot})$

(μ_i E v_{it} NON CORRELATI PER IPOTESI)

$$= \sigma_\mu^2 + \mathrm{VAR}\left(\frac{1}{T}\sum_{t=1}^{T} v_{it}\right)$$

$$= \sigma_\mu^2 + \frac{1}{T^2}\,\mathrm{VAR}\left(\sum_t v_{it}\right)$$

$$= \sigma_\mu^2 + \frac{1}{T\cancel{^2}} \cdot \cancel{T}\,\sigma_u^2$$

σ_1^2

$$T\,VAR(\bar{r}_{i\cdot}) = T\sigma_\mu^2 + \sigma_u^2$$

CONCLUSIONE : STIMANDO $VAR(\bar{r}_{i\cdot})$ E MOLTIPLICANDOLA PER T SI OTTIENE LA STIMA DI σ_1^2

PER STIMARE $VAR(\bar{r}_{i\cdot})$ USIAMO LA SEGUENTE PROCEDURA:

1) STIMARE CON OLS IL MODELLO (2), OTTENENDO I

RESIDUI $\hat{v}_{i\cdot} = y_{i\cdot} - \sum_{r=2}^{k} \hat{\beta}_r x_{ri\cdot} - \hat{a}$

2) SULLA BASE DEI $\hat{v}_{i\cdot}$, OTTENIAMO LA VARIANZA STIMATA $\widehat{var}(\hat{v}_{i\cdot})$

3) $\hat{\sigma}_1^2 = T \, \widehat{var}(\hat{v}_{i\cdot})$

$$\Downarrow$$

$$\hat{V} = \hat{\sigma}_1^2 C_T + \hat{\sigma}_u^2 D_T \Longrightarrow \underline{\underline{FGLS}}$$

INDIRETTAMENTE OTTENIAMO:

$$\hat{\sigma}_\mu^2 = \frac{\hat{\sigma}_1^2 - \hat{\sigma}_u^2}{T}$$

STIMA DEGLI EFFETTI INDIVIDUALI RANDOM ($\hat{\mu}_i$)

PREVISIONI CON MODELLI DI REGRESSIONE LINEARI

$N = 1$ (NO DATI PANEL)

$t = 1 \ldots T$ (ORIZZONTE DI STIMA)

$t = T+1 \ldots T+G$ (ORIZZONTE DI PREVISIONE)

MODELLO DI REGRESSIONE "CLASSICO"

$$\underset{T\cdot 1}{y} = \underset{T\cdot K}{X}\,\underset{K\cdot 1}{\beta} + \underset{T\cdot 1}{U} \qquad t = 1 \ldots T$$

$$E(UU') = I_T \quad (\sigma_u^2 = 1)$$

$$\underset{G\cdot 1}{y_F} = \underset{G\cdot K}{X_F}\,\underset{K\cdot 1}{\beta} + \underset{G\cdot 1}{U_F} \qquad t = T+1 \ldots T+G$$

$$E(U_F U_F') = I_G$$

STABILITÀ DEI PARAMETRI (PER IPOTESI)

$E(UU_F') = 0$

<u>PREVISIONI</u> : $\underset{G\cdot 1}{\hat{y}_F} = \underset{G\cdot K}{X_F} \underset{K\cdot 1}{\hat{\beta}}$, DOVE $\underset{K\cdot 1}{\hat{\beta}} = (X'X)^{-1}X'y$

VALORI "FUTURI" DELLE X SONO NOTI.

STABILITÀ DEI β

MODELLO DI REGRESSIONE GLS

$$y = X\beta + U \quad , \quad E(UU') = \Omega \neq I_T \quad (t = 1 \ldots T)$$

"GLS"

$$\hookrightarrow \hat{\beta}_{GLS} = (X'\Omega^{-1}X)^{-1} X'\Omega^{-1} y$$

$$\underset{G\cdot 1}{y_F} = \underset{G\cdot K}{X_F}\,\underset{K\cdot 1}{\beta} + \underset{G\cdot 1}{U_F} \quad , \quad E(U_F U_F') = \Omega_G \quad (t = T+1 \ldots T+G)$$

DOVE $E(UU_F') \neq 0 \quad Q \neq 0$

RESIDUI GLS $t = 1 \ldots T$

PREVISIONI:

$$\underset{G\cdot 1}{\hat{y}_F} = \underset{G\cdot K}{X_F}\ \underset{K\cdot 1}{\hat{\beta}_{GLS}} + \underset{G\cdot T}{Q'}\ \underset{T\cdot T}{\Omega^{-1}}\ \underset{T\cdot 1}{\hat{u}_{GLS}}$$

MODELLO PANEL EFFETTI RANDOM $(i = 1 \ldots N)$

$$\underset{T\cdot 1}{y_i} = \underset{T\cdot K}{x_i}\ \underset{K\cdot 1}{\delta} + \underset{T\cdot 1}{v_i} \quad , \quad \underset{T\cdot 1}{v_i} = \underset{1\cdot 1}{\mu_i}\ \underset{T\cdot 1}{J} + \underset{T\cdot 1}{u_i} \quad (t = 1 \ldots T)$$

IPOTESI: $G = 1$ (PREVISIONI UN PERIODO IN AVANTI)

$$\underset{(1\cdot 1)}{y_{iT+1}} = \underset{1\cdot K}{x_{iT+1}} \, \underset{K\cdot 1}{\delta} + \underset{(1\cdot 1)}{v_{iT+1}}$$

y_{iT+1} ?

(SCALARE)

PREVISIONE:

$$\underset{(1\cdot 1)}{\hat{y}_{iT+1}} = \underset{(1\cdot K)}{x_{iT+1}} \, \underset{(K\cdot 1)}{\hat{\delta}_{GLS}} + \underset{(1\cdot T)}{Q'} \, \underset{(T\cdot T)}{V^{-1}} \, \underset{T\cdot 1}{\hat{v}_i}$$

RESIDUI DEL MODELLO EFFETTI RANDOM

DOVE $\hat{v}_i = y_i - x_i \hat{\delta}_{GLS}$

$$E\underbrace{Q}_{(T\cdot 1)} = E\Big(\underbrace{v_i}_{(T\cdot 1)} \cdot \underbrace{v_{iT+1}}_{(1\cdot 1)}\Big)$$

$$= E\Big[\Big(\underbrace{\mu_i}_{(1\cdot 1)}\underbrace{J}_{T\cdot 1} + \underbrace{U_i}_{T\cdot 1}\Big)\Big(\underbrace{\mu_i}_{(1\cdot 1)} + \underbrace{U_{iT+1}}_{(1\cdot 1)}\Big)\Big]$$

$$= E(\mu_i^2)J = \underbrace{\sigma_\mu^2 J}$$

$$V = \sigma_1^2 G_T + \sigma_w^2 D_T \Longrightarrow V^{-1} = \frac{1}{\sigma_1^2} G + \frac{1}{\sigma_w^2} D_T$$

$$\Downarrow$$

$$\hat{y}_{iT+1} = x_{iT+1}\hat{\delta}_{GLS} + \left(\sigma_\mu^2 J\right)'\left(\frac{1}{\sigma_1^2}G_T + \frac{1}{\sigma_w^2}D_T\right)\hat{\nu}_i$$

$$= x_{iT+1}\hat{\delta}_{GLS} + \left[\frac{\sigma_\mu^2}{\sigma_1^2}\underbrace{J'C_T}_{J'} + \frac{\sigma_\mu^2}{\sigma_w^2}\underbrace{J'D_T}_{0}\right]\hat{\nu}_i$$

<u>N.B.</u> $J'C_T = J' \cdot \frac{JJ'}{T} = \frac{\cancel{J'J}^{\,T}}{\cancel{T}} \cdot J' = J'$

$$J' D_T = J'(I_T - C_T) = J' - J' C_T = J' - J' = 0$$

$$\Downarrow$$

$$\underset{(1\cdot 1)}{\hat{y}_{iT+1}} = \underset{1\cdot K}{x_{iT+1}} \underset{K\cdot 1}{\hat{\delta}_{GLS}} + \underset{1\cdot 1}{\frac{\sigma_\mu^2}{\sigma_1^2}} \underset{1\cdot T}{J'} \underset{T\cdot 1}{\hat{v}_i} = \sum_{t=1}^{T} \hat{v}_{it}$$

$$= x_{iT+1} \hat{\delta}_{GLS} + \boxed{\frac{\sigma_\mu^2}{\sigma_1^2} T \cdot \hat{v}_{i\cdot}}$$

N.B $\dfrac{\tilde{\sigma}^2_\mu}{\sigma^2_1} T \cdot \hat{v}_{i\cdot} \equiv \mu_i$

$$\boxed{\hat{\mu}_i = \frac{\hat{\sigma}^2_\mu}{\hat{\sigma}^2_1} T \cdot \hat{v}_{i\cdot}}$$

STIMATORE EFFETTI INDIVIDUALI RANDOM)

INTERPRETAZIONE DI $\hat{\mu}_i$

$$y_i = x_i\delta + \nu_i \quad , \quad \nu_i = \mu_i J + U_i$$

prima componente di ν_i

$$y_{iT+1} = x_{iT+1}\hat{\delta}_{GLS} + \boxed{Q' V^{-1} \hat{\nu}_i}$$

$$\Big[\underbrace{\frac{\sigma_\mu^2}{\sigma_1^2} J' C_T}_{\mu_i} + \underbrace{\frac{\sigma_\mu^2}{\sigma_u^2} J' D_T}_{v_i}\Big]\hat{v}_i$$

QUESTA ESPRESSIONE EVIDENZIA LA SCOMPOSIZIONE DEI RESIDUI $\hat{v}_i$ NELLA PARTE IMPUTABILE A μ_i E NELLA PARTE IMPUTABILE A v_i

TEST DI SIGNIFICATIVITÀ DEGLI EFFETTI RANDOM

(CENNI)

TEST DI BREUSCH-PAGAN (TIPO LM)
(BP)

$$BP \underset{H_0}{\overset{a}{\sim}} \chi^2_{(1)} ,$$

DOVE $\boxed{H_0 : \sigma^2_\mu = 0}$ (NO EFFETTI RANDOM)

vs

$H_1 : \sigma^2_\mu > 0$