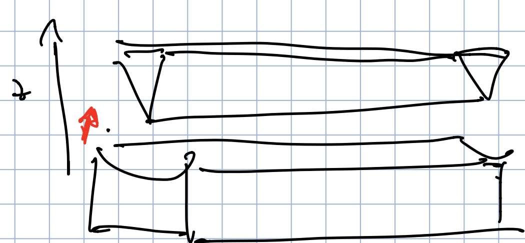
Gas di elettroni liberi

$$E_{n_x, n_y, n_z} = c\left(n_x^2 + n_y^2 + n_z^2\right) \;;\; c = \frac{\hbar^2}{2m_e}\frac{\pi^2}{L^2}$$

E_{n_x, n_y, n_z, m_s} ; $m_s = \pm \frac{1}{2}$

$$\begin{cases} \hat{S}_z \longrightarrow \hbar m_s \;;\; m_s = \pm \frac{1}{2} \\ \hat{S}^2 \to \hbar^2 s(s+1) \quad s = \frac{1}{2} \end{cases}$$

H



$$B = B(z) = B_0 \cdot z$$

$\vec{\mu}$ $\quad E = -\vec{\mu} \cdot \vec{B} = -\mu_z \cdot B_0 \cdot z$

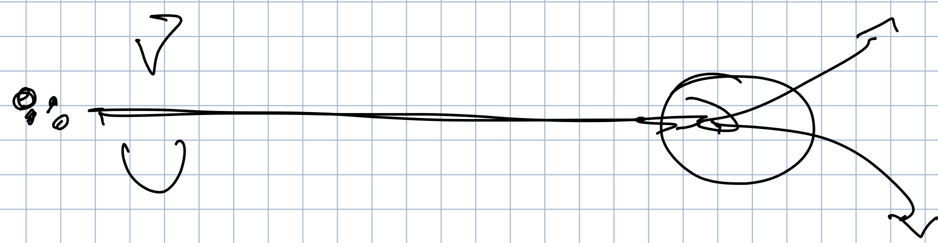
$$\hat{\vec{\mu}} = -\frac{e}{2m_e}\hat{\vec{L}} \qquad E = +\frac{e}{2m_e} L_z \cdot B_0 \cdot z$$

$$F_z = -\frac{\partial E}{\partial z} = \frac{e}{2m_e} L_z \cdot B_0$$

Atomi di H nel loro stato fondamentale $\psi_{100} \begin{cases} n = 1 \\ l = 0 \\ m = 0 \end{cases}$

$$F_z = \frac{e}{2m_e} L_z \cdot B_0$$

$$L_z \to \hbar m = 0$$



$F_z \sim L_z$ con 2 possibili autovalori ±

$$[\hat{L}_x, \hat{L}_y] = i\hbar \hat{L}_z$$

$$\begin{cases} L^2 \to \hbar^2 \ell(\ell+1) \\ L_z \to \hbar m_L \ ; \ m_L = \overbrace{\underbrace{-\ell,}_{\text{salti di } 1} \dots \ell}^{2\ell+1} \end{cases} \qquad [\hat{\vec{L}}^2, \hat{L}_z] = 0 \qquad Y_\ell^m(\theta, \varphi)$$

$$\hat{S}_x, \hat{S}_y, \hat{S}_z \ ; \ [\hat{S}_x, \hat{S}_y] = i\hbar \hat{S}_z$$

$$S^2 \to \hbar^2 s(s+1)$$

$$S_z \to \hbar m_s \ , \ m_s = -s, \dots s$$

$[\hat{S}^2, \hat{S}_z] \to$ autof. comuni $\chi^s_{m_s}$

$$\begin{cases} \hat{S}^2 \chi^s_{m_s} = \hbar^2 s(s+1) \chi^s_{m_s} \qquad s = \frac{1}{2} \\ \hat{L}_z \chi^s_{m_s} = \hbar m_s \chi^s_{m_s} \end{cases}$$

$\chi_{+\frac{1}{2}}$ $|\uparrow\rangle$; α $\rightarrow$ $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\chi_{-\frac{1}{2}}$ $|\downarrow\rangle$; β $\rightarrow$ $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\left| \begin{aligned} \hat{S}_z |\uparrow\rangle &= \frac{\hbar}{2} |\uparrow\rangle \\ \hat{S}_z |\downarrow\rangle &= \frac{\hbar}{2} |\downarrow\rangle \end{aligned} \right] \qquad \hat{S}_z = \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Idrogeno

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$E_n = -\frac{R\hbar c}{n^2} \qquad \psi_{n,l,m}(r,\theta,\varphi) \cdot \chi_{m_s}$$

spaziali

$n = 2$ $\quad l = \begin{cases} 0 \\ 1 \end{cases}$ $\quad m = \begin{matrix} \cancel{0} \\ -1 \\ 0 \\ 1 \end{matrix}$ $\quad$ deg $= 4 \times 2$ spin $= 8$

$\deg(n) \rightarrow 2n^2$

$n=2$ $\quad \uparrow\downarrow$ $l=0$ 2s $\quad \uparrow\downarrow \; \uparrow\downarrow \; \uparrow\downarrow$ $l=1$ 2p

$n=1$ $\quad \uparrow\downarrow$ $\quad l=0, m=0$ $\quad$ 1s

$$\varepsilon_{n_x, n_y, n_z, m_s} \rightarrow \varepsilon_\alpha$$

$$\left(n_{n_x, n_y, n_z, m_s} \right)_{FD} = \frac{1}{e^{\beta(\varepsilon_{n_x, n_y, n_z, m_s} - \mu)} + 1}$$

$$P_{n_x, n_y, n_z, m_s} = \frac{e^{-\beta \varepsilon_{n_x, n_y, n_z, m_s}}}{Z_I}$$

$$\varepsilon_{n_x, n_y, n_z, m_s} = C \cdot \left(\overbrace{n_x^2 + n_y^2 + n_z^2}^{\to n^2} \right)$$

& ∮ose H

$$P_{n, l, m, m_s} = \frac{e^{-\beta \varepsilon_{n, l, m, m_s}}}{\sum_{n, l, m, m_s} e^{-\beta \varepsilon_{n, l, m, m_s}}}$$

$$\varepsilon = -\frac{Rhc}{n^2}$$

$$P_n = \frac{e^{-\beta \varepsilon_n} \cdot \overbrace{g(n)}^{\to 2n^2}}{\sum_n e^{-\beta \varepsilon_n} g(n)}$$

$$\varepsilon = c\left(n_x^2 + n_y^2 + n_z^2\right) \quad ; \quad n_x = 1,2,3 \ldots \quad n_y = 1,2,3 \ldots \quad n_z = 1,2,3 \ldots$$

$P_{n_x, n_y, n_z, m_s} \rightarrow P_n \cdot g$ g ha un 2 legato allo spin

$n^2 = 3 \rightarrow$ deg. $= 1$ ($\times 2$ spin)

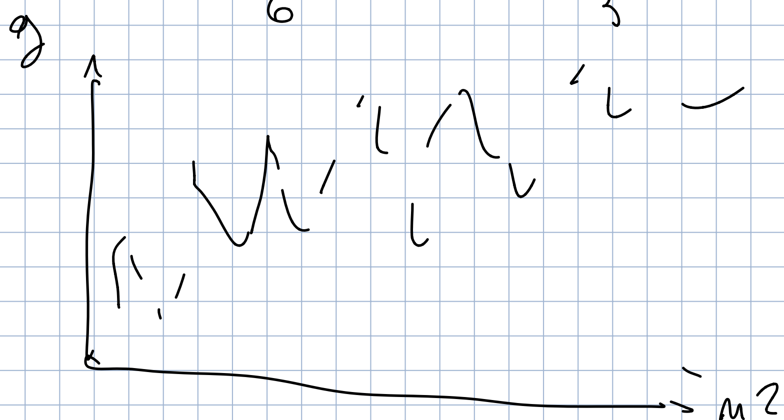
$n^2 = 4 \rightarrow \not\exists$

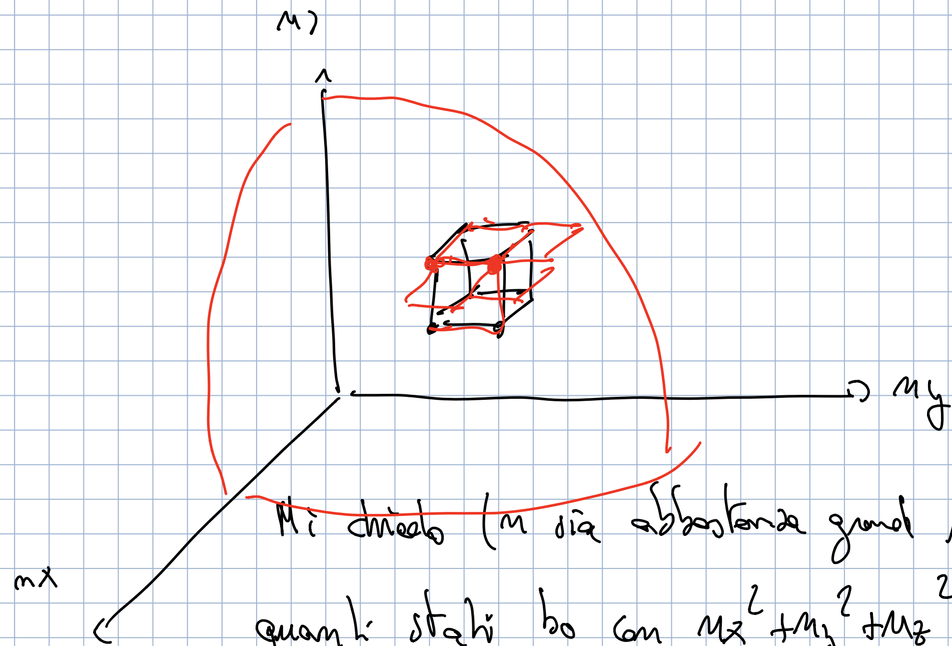
$n^2 = 5 \rightarrow \not\exists$

$n^2 = 6 \rightarrow (1,2,1), (2,1,1), (1,1,2)$ 3 ($\times 2$ spin)

$$n^2 = 86 \quad \underbrace{(81+4+1)}_{\substack{9\ 2\ 1 \\ 6}} + \underbrace{(25+25+36)}_{\substack{5\ 5\ 6 \\ 3}} + \left(\underset{\not\exists}{49}, \underset{6}{36}, \underset{1}{1}\right)$$

$$\begin{array}{r} 49 \\ 36 \\ \hline 85 \end{array}$$





Mi chiedo (n sia abbastanza grande)

quanti stati ho con $n_x^2 + n_y^2 + n_z^2 < n^2$

$$V_n = \frac{1}{8}\frac{4}{3}\pi n^3 \Big/ V_{cubetto} = 1 \simeq \text{Numero di punti}$$

con $n_x^2 + n_y^2 + n_z^2 < n^2$

Quanti stati ho nell'ottavo di sfera con raggio $(n+dn)$

$$V_{n+dn} = \frac{4}{6}\pi (n+dn)^3$$

$$\hookrightarrow n^3 + 3n^2 dn + \text{termini in } \cancel{dn^2} \text{ o } \cancel{dn^3}$$

$N(n) \cdot dn$ = numero di punti tali per cui (n_x, n_y, n_z) sono tali che $n_x^2 + n_y^2 + n_z^2 \in U_{dn}(n) = V_{n+dn} - V_n$

$$N(n)\,dn = \frac{3\pi}{6} n^2\,dn = \frac{\pi}{2} n^2\,dn \qquad \varepsilon = c\,n^2$$

$$n^2 = \frac{\varepsilon}{c}$$

$$\frac{\pi}{2} n^2\,dn = \frac{\pi}{2}\frac{\varepsilon}{c}\frac{1}{2\sqrt{\varepsilon}}\cdot\frac{1}{\sqrt{c}}\,d\varepsilon \qquad n = \sqrt{\frac{\varepsilon}{c}}$$

$$dn = \frac{1}{2\sqrt{\varepsilon}\cdot\sqrt{c}}\,d\varepsilon$$

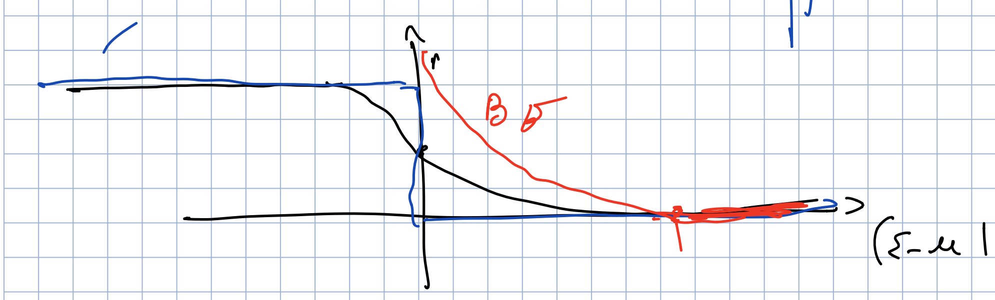
$$= \frac{\pi}{4}\frac{\sqrt{\varepsilon}}{c^{\frac{3}{2}}}\,d\varepsilon$$

→ numero di stati con energia compresa tra ε e $\varepsilon + d\varepsilon$

Anzi, se considero lo spin $g(\varepsilon) = \frac{\pi}{2}\frac{\sqrt{\varepsilon}}{c^{\frac{3}{2}}}\,d\varepsilon$

$$\left(f_{n_x,n_y,n_z,m_s}\right)_{FD} = \frac{1}{e^{\beta(\varepsilon_{n_x,n_y,n_z,m_s} - \mu)} + 1}$$

$$BE \qquad \frac{1}{e^{\beta(\varepsilon-\mu)} - 1}$$



$$\sum_{n_x,n_y,n_z,m_s} \left(f_{n_x,n_y,n_z,m_s}\right)_{FD} = \langle N \rangle = N$$

$$2\sum_n g(n) \left(f_n\right)_{FD}$$

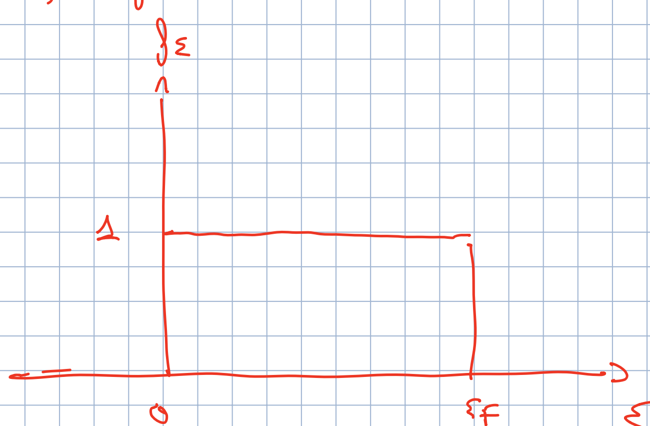
$$\frac{\pi}{2}\frac{\sqrt{\varepsilon}}{c^{3/2}} \qquad c = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}$$

$$\int_0^{+\infty} g(\varepsilon) \cdot f_{FD}(\varepsilon)\, d\varepsilon = \langle N \rangle$$

$$f_{FD} = \frac{1}{e^{\beta(\varepsilon-\mu)}+1} \; ; \; \mu = \mu(V,N,T)$$

Possiamo immaginare V fissato, N fissato $\Rightarrow \mu = \mu(T)$.

Definiamo $\varepsilon_F = \mu(0)$



$$N = \int_0^{+\infty} g(\varepsilon)\, f_{FD}(\varepsilon)\, d\varepsilon \overset{T\to 0}{\simeq} \int_0^{\varepsilon_F} g(\varepsilon)\, d\varepsilon$$

$$N = \int_0^{\varepsilon_F} \frac{\pi}{2}\frac{\sqrt{\varepsilon}}{C^{\frac{3}{2}}}\, d\varepsilon = \frac{\pi}{2}\frac{1}{C^{\frac{3}{2}}}\frac{2}{3}\varepsilon_F^{\frac{3}{2}}$$

$$N = \frac{\pi}{3\cdot C^{\frac{3}{2}}}\varepsilon_F^{\frac{3}{2}}$$

$$C = \frac{\hbar^2}{2m}\frac{\pi^2}{L^2}$$

$$C^{\frac{3}{2}} = \frac{\hbar^3\pi^3}{(2m)^{\frac{3}{2}} V}$$

$$N = \frac{\cancel{\pi}}{3\hbar^3\pi^{\cancel{3}2}}(2m)^{\frac{3}{2}}\cdot V\, \varepsilon_F^{\frac{3}{2}} \quad ; \quad \rho = \frac{N}{V}$$

$$\varepsilon_F^{\frac{3}{2}} = \frac{3\hbar^3\pi^2}{(2m)^{\frac{3}{2}}}\rho \quad ; \quad \varepsilon_F = \underbrace{(3\pi^2\rho)^{\frac{2}{3}}}_{k_F^2}\cdot\frac{\hbar^2}{2m_e}$$

$\rho_{\text{el. cond metallo}} \simeq 10^{23}/cm^3 \to$ tipiche $\varepsilon_F \simeq 5eV$

Se $\varepsilon_F \approx 5eV$ $\quad k_B \cdot T = 5eV \rightarrow T = 60'000\,K$

A temperature ragionevoli: $\mu \approx \varepsilon_F$ (non esattamente, ma

$f_{FD} \approx$ gradino, some correzioni).

Energia media a $T \approx 0$

$$\langle U(0) \rangle = \int_0^{\varepsilon_F} g(\varepsilon) \cdot \varepsilon \cdot d\varepsilon$$

$$= \frac{\pi}{2} \frac{1}{c^{\frac{3}{2}}} \int^{\varepsilon_F} \varepsilon^{\frac{3}{2}} = \frac{\pi}{2} \frac{1}{c^{\frac{3}{2}}} \cdot \frac{2}{5} \varepsilon_F^{\frac{5}{2}}$$

$$= \pi \frac{1}{c^{\frac{3}{2}}} \frac{1}{5} \varepsilon_F \cdot \varepsilon_F^{\frac{3}{2}} = \frac{3}{5} N \varepsilon_F$$

$$\Rightarrow \frac{U(0)}{N} = \frac{3}{5} \varepsilon_F$$

$$N = \frac{\pi}{3 \cdot c^{\frac{3}{2}}} \varepsilon_F^{\frac{3}{2}} \Rightarrow \varepsilon_F^{\frac{3}{2}} = \frac{3 N c^{\frac{3}{2}}}{\pi}$$