Non uniform $\underline{E}$ and $\underline{B}$
(weakly)

Larmor motion $\underline{x}_{gc}$

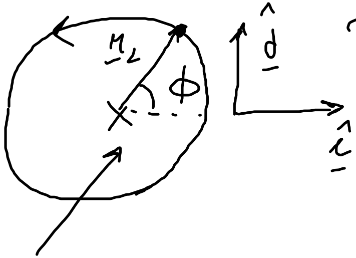
$\underline{x}_{gc}$ moves under the action of $\underline{F} = \underline{F}_{\parallel} + \underline{F}_{\perp}$

$$\underline{F}_{\parallel} = q\underline{E}_{\parallel} + q \langle \underline{v}_L \times (\underline{r}_L \cdot \underline{\nabla}) \underline{B} \rangle_{\parallel}$$

$$\underline{F}_{\perp} = q\underline{E}_{\perp} + q \langle \underline{v}_L \times (\underline{r}_L \cdot \underline{\nabla}) \underline{B} \rangle_{\perp} - (\hat{\underline{B}} \cdot \underline{\nabla}) \hat{\underline{B}} \, m v_{gc}^2$$

Focus on $q \langle \underline{v}_L \times (\underline{r}_L \cdot \underline{\nabla}) \hat{\underline{B}} \rangle$?

$q > 0$



Reference frame with z axis $//$ to $\underline{B}$ at the gyrocentre position

$\underline{B}$ entering the board

$$\underline{x} = \underline{x}_{gc} + \underline{r}_L$$

$$\phi = \omega_L t \quad \text{(gyro-angle)}$$

$$\underline{r}_L = r_L\left(\cos\underbrace{\phi}_{\omega_L t} \hat{\underline{i}} + \sin\underbrace{\phi}_{\omega_L t} \hat{\underline{j}}\right) \qquad \underline{v}_L = \frac{d\underline{r}_L}{dt} = -r_L\omega_L\left[\sin(\omega_L t)\,\hat{\underline{i}} + \cos(\omega_L t)\,\hat{\underline{j}}\right]$$

$$r_L^2 q \left\langle + \underbrace{\omega_L \left(-\sin(\omega_L t)\, \hat{\underline{i}} + \cos(\omega_L t)\, \hat{\underline{j}} \right)}_{\underline{v}_L} \times \right.$$

$$\langle \sin(\omega_L t) \rangle = 0$$
$$\langle \cos(\omega_L t) \rangle = 0$$
$$\langle \sin^2 \rangle = \frac{1}{2}$$
$$\langle \cos^2 \rangle = \frac{1}{2}$$
$$\langle \sin(\,)\cos(\,) \rangle = 0$$

$$\left. \left(\cos(\omega_L t) \frac{\partial}{\partial x} + \sin(\omega_L t) \frac{\partial}{\partial y} \right) \left[B_x \hat{\underline{i}} + B_y \hat{\underline{j}} + B_z \hat{\underline{k}} \right] \right\rangle$$

$$= q \omega_L r_L^2 \left\langle \left(-\sin(\omega_L t)\, \hat{\underline{i}} + \cos(\omega_L t)\, \hat{\underline{j}} \right) \times \right.$$

$$\left[\cos(\omega_L t) \left(\frac{\partial B_x}{\partial x} \hat{\underline{i}} + \frac{\partial B_y}{\partial x} \hat{\underline{j}} + \frac{\partial B_z}{\partial x} \hat{\underline{k}} \right) + \sin(\omega_L t) \left(\frac{\partial B_x}{\partial y} \hat{\underline{i}} + \frac{\partial B_y}{\partial y} \hat{\underline{j}} + \frac{\partial B_z}{\partial y} \hat{\underline{i}} \right) \right]$$

$$= \frac{q \omega_L r_L^2}{2} \left[-\frac{\partial B_x}{\partial x} \hat{\underline{k}} + \frac{\partial B_z}{\partial x} \hat{\underline{i}} - \frac{\partial B_y}{\partial y} \hat{\underline{k}} + \frac{\partial B_z}{\partial y} \hat{\underline{j}} \right]$$

$$= \frac{q\omega_L r_L^2}{2}\left[-\frac{\partial B_y}{\partial y}\hat{\underline{k}} + \frac{\partial B_z}{\partial y}\hat{\underline{j}} - \frac{\partial B_x}{\partial x}\hat{\underline{k}} + \frac{\partial B_z}{\partial x}\hat{\underline{i}}\right]$$

$$\underline{\nabla}\cdot\underline{B} = 0 \Rightarrow \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

$$-\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y} = \frac{\partial B_z}{\partial z}$$

$$= \frac{q\omega r_L^2}{2}\left[+\frac{\partial B_z}{\partial z}\hat{\underline{k}} + \frac{\partial B_z}{\partial y}\hat{\underline{j}} + \frac{\partial B_z}{\partial x}\hat{\underline{i}}\right] = -\frac{q\omega_L r_L^2}{2}\underline{\nabla}|\underline{B}|$$

At $\underline{x} = \underline{x}_{gc}$ $\quad \underline{B} = -B_z\hat{\underline{k}}$ $\qquad$ Let's call

$$\mu = +\frac{q\omega_L r_L^2}{2}$$

$$|\underline{B}|\Big|_{\underline{x}=\underline{x}_{gc}} = B_z$$

$$\mu = \frac{q\,\omega_L r_L^2}{2} =$$

magnetic moment

$$= \frac{q \frac{\cancel{q}B}{m} \frac{m^{\cancel{2}} v_L^2}{\cancel{q^2} B^{\cancel{2}}}}{2} = \frac{m v_\perp^2}{2B}$$

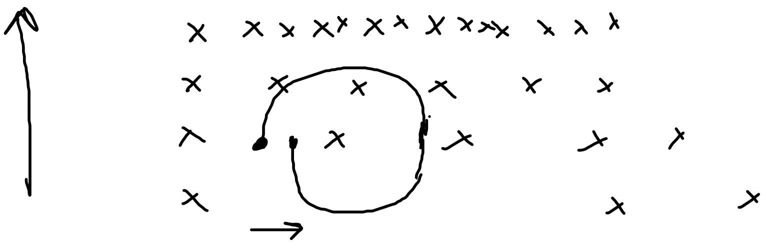
$$\underline{F}_\perp = -\mu \, \underline{\nabla} |\underline{B}|$$

$$\underline{v}_D = \frac{\underline{F}_\perp \times \underline{B}}{qB^2} = -\mu \frac{\underline{\nabla} |\underline{B}| \times \underline{B}}{qB^2}$$

Drift $\perp$ $\underline{B}$ and $\underline{\nabla}|\underline{B}|$

Why do we expect a drift if $\underline{\nabla}|\underline{B}| \neq 0$?

$\underline{\nabla}|\underline{B}|$



$r_L = \dfrac{m v_\perp}{qB}$

upper half $\quad B > B_0 \Longrightarrow r_L > r_{L_0}$

$B_0 \rightarrow$ average value of B

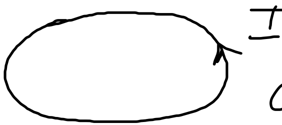
lower half $\quad B < B_0 \Longrightarrow r_L < r_{L_0}$

$$\underline{F}_\perp = -\underbrace{\frac{m v_\perp^2}{2B}}_{\mu} \underline{\nabla}|\underline{B}|$$

$\nabla|\underline{B}| \times \underline{B}$ is to the left

$\Rightarrow \quad -\nabla|\underline{B}| \times \underline{B}$ is to the right

Electromagnetism



Coil

$m = I \cdot A$

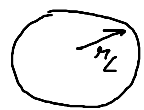
$m \ ? \ \mu$

$\mu = \dfrac{m v_\perp^2}{2B}$

$I = \dfrac{q}{T_L} = \dfrac{q}{2\pi}\,\omega_L = \dfrac{q}{2\pi}\,\dfrac{qB}{m}$

current carried by an ion in a Larmor motion

$A = \pi r_L^2 = \pi \dfrac{m^2 v_\perp^2}{q^2 B^2}$



$A \cdot I =$

$= \dfrac{\cancel{q}}{2\cancel{\pi}}\,\dfrac{\cancel{q}\cancel{B}}{\cancel{m}} \cdot \cancel{\pi}\,\dfrac{m^{\cancel{2}} v_\perp^2}{\cancel{q^2}B^{\cancel{2}}}$

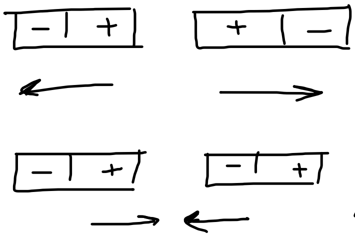
$= \dfrac{m v_\perp^2}{2B} = \mu$

Note Oersted experiment

Current carrying coil $\Longleftrightarrow$ magnet

$$\underline{F} = -\mu \underline{\nabla} |\underline{B}|$$

$$\perp : \quad \underline{v}_D = -\mu \frac{\underline{\nabla} |\underline{B}| \times \underline{B}}{q B^2}$$



Repulsion

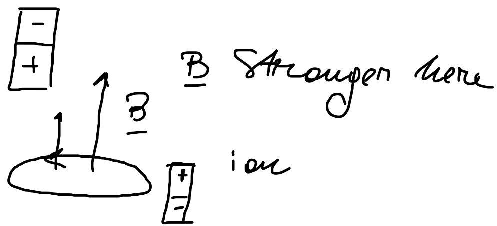
Attraction

$$\parallel : \quad F = -\mu (\underline{\nabla} \underline{B})_\parallel$$

Friction

(same pole magnets repel each other)

$$\underline{F} = -\mu \, \underline{\nabla} \underline{B}$$



$$\boxed{(\hat{\underline{B}} \cdot \underline{\nabla}) \hat{\underline{B}} \; m v_{gc}^2 = ?}$$