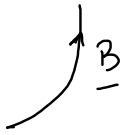
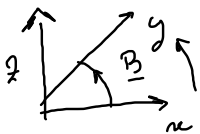
$$\underline{F} = -\mu \underline{\nabla} |\underline{B}|$$

$$\underline{F} = -(\hat{\underline{B}} \cdot \underline{\nabla}) \hat{\underline{B}}\, m v^2$$



Centrifugal force?



$$\hat{\underline{B}} = \hat{\underline{\phi}} = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}$$

$$\hat{\underline{r}} = \frac{\frac{\partial \vec{x}}{\partial r}}{\left\| \frac{\partial \vec{x}}{\partial r} \right\|} = \frac{\begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}}{\sqrt{\cos^2\phi + \sin^2\phi}} = \begin{pmatrix} \cos\phi \\ \sin\phi \\ 0 \end{pmatrix}$$

$$\begin{cases} x = r\cos\phi \\ y = r\sin\phi \\ z = w \end{cases}$$

$$\hat{\underline{\phi}} = \frac{\frac{\partial \vec{x}}{\partial \phi}}{\left\| \frac{\partial \vec{x}}{\partial \phi} \right\|} = \frac{\begin{pmatrix} -r\sin\phi \\ r\cos\phi \\ 0 \end{pmatrix}}{\sqrt{r^2\sin^2\phi + r^2\cos^2\phi}} = \frac{1}{r} \begin{pmatrix} -r\sin\phi \\ r\cos\phi \end{pmatrix} = \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix}$$

$$(\hat{\underline{B}} \cdot \underline{\nabla}) \hat{\underline{B}} = \left(\frac{1}{r}\frac{\partial}{\partial \phi}\right) \begin{pmatrix} -\sin\phi \\ \cos\phi \end{pmatrix} = \frac{1}{r} \begin{pmatrix} -\cos\phi \\ -\sin\phi \end{pmatrix} = -\frac{\hat{\underline{r}}}{r}$$

ϕ component of $\underline{\nabla}$

$$\underline{F} = -m v_{\parallel gc}^2 (\hat{\underline{B}} \cdot \underline{\nabla}) \hat{\underline{B}} = m \frac{v_{\parallel gc}^2}{R} \hat{\underline{R}}$$

Centrifugal force

Summary –

$\underline{B}(\underline{x})$ $\underline{E}(\underline{x})$ when uniformity is small on { scale length of r_L ; time interval of T_L

$\underline{x} = \underline{x}_{gc} + \underline{r}_L$

$\underline{x}_{gc}$ motion of the g.c.

$\underline{r}_L$ fast gyromotion around $\underline{x}_{gc}$

G.C. is subject to drifts

$\underline{v}_{D,\perp} = \frac{\underline{F} \times \underline{B}}{qB^2}$ + parallel motion $\underline{F}_{\parallel}$

Contributions to $\underline{F}$

1) $\underline{E}$ 2) $\underline{\nabla}|\underline{B}|$ 3) Centrifugal force

$$\underline{F} = q\underline{E}(\underline{x}_{gc}) - \mu \underline{\nabla}|\underline{B}|\Big|_{\underline{x}_{gc}} + m v_{\parallel gc}^2 \hat{\underline{r}}$$

(1)

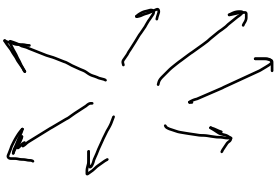
$$\underline{v}_D = \underline{v}_{\underline{E}_\perp} + \underline{v}_{\underline{\nabla}|\underline{B}|} + \underline{v}_{Centrifugal}$$

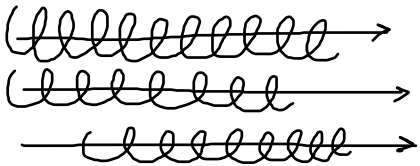
(∥)

$$m \frac{d v_\parallel}{dt} = q E_\parallel - (\mu \underline{\nabla}|\underline{B}|)_\parallel$$

Consequences of the $\nabla|B|$ drift for plasma confinement

$B = 0$ $\quad T \sim keV$ $\quad B \neq 0$

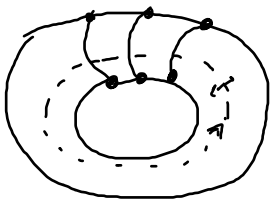




Confinement $\approx \eta_L$ $\perp$ to B

$\Rightarrow$ No conf. along B

Closed field lines

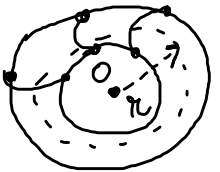


Good confinement expected !

1) $\underline{B}$ is non uniform in a toroid

2) $\Rightarrow$ Drifts due to $\nabla|\underline{B}|$ + curv. force

1)

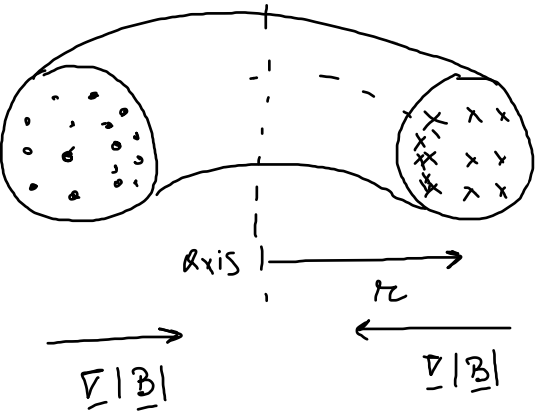


Magnetic field lines are circles around the toroid

Ampere theorem; choose a circle of radius r as a path

$$\oint_C \underline{B} \cdot d\underline{l} = \mu_0 \cdot I_{enclosed}$$

$$2\pi r \cdot B = \mu_0 \cdot N \cdot I; \quad B(r) = \frac{\mu_0 N I}{2\pi r}$$



$B \propto \frac{1}{r}$

$$\underline{v}_D = -\frac{\mu \nabla|B| \times \underline{B}}{qB^2} \propto -\frac{\nabla|B| \times \underline{B}}{q}$$

$$\propto \frac{\hat{r} \times \hat{\phi}}{q} \propto \frac{\hat{z}}{q}$$

Ions upward
Electrons downward

ions ↑
$\underline{E}$ ↓
electrons ↓

$$\underline{v}_{E \times B} = \frac{\underline{E} \times \underline{B}}{B^2}$$

→ drift towards the wall of the device

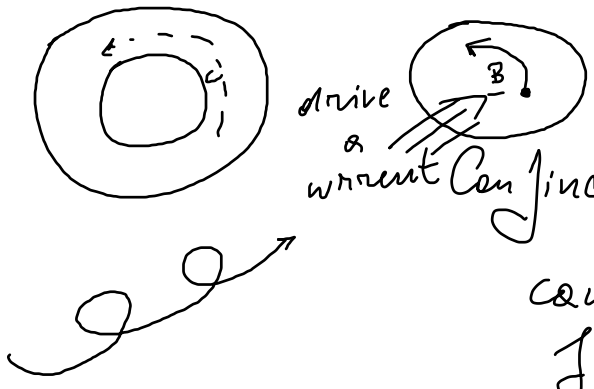
charge independent

No confinement!

Solution:

Add a Poloidal component to $\underline{B}$

$$\underline{v}_{-\underline{E}\times\underline{B}} = \frac{\underline{E}\times\underline{B}}{qB^2}$$



How do you apply a twist to the field line?