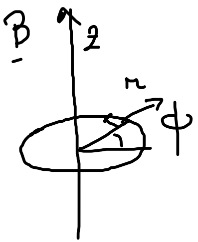
$$S = \oint_{\text{period}} P\,dq \qquad q: \text{periodic coordinate}$$

$$P = \frac{\partial \mathcal{L}}{\partial \dot q} \quad \text{associated can. mom.}$$



ϕ: gyroangle

$$S = \oint P_\phi \, d\phi$$

$$\mathcal{L} = \frac{1}{2} m v^2 + q \underline{v} \cdot \underline{A} - q\varphi = \frac{1}{2} m \left(v_R^2 + v_z^2 + v_\phi^2\right) + q\left(v_z A_z + v_R A_R + v_\phi A_\phi\right) - \cancel{q\varphi}$$

$$\left.\begin{aligned} &\underline{B} = \underline{\nabla} \times \underline{A} \neq \underline{0} \\ &\underline{E} = \underline{0} \Rightarrow \underline{E} = -\frac{\partial \underline{A}}{\partial t} - \underline{\nabla}\varphi \end{aligned}\right\} \text{set } \varphi = 0$$

$$v_\phi = r_L \dot\phi$$

$$\mathcal{L} = \frac{1}{2} m \left(v_r^2 + v_z^2 + r_L^2 \dot\phi^2 \right) + q \left(A_r v_r + A_z v_z + A_\phi r_L \dot\phi \right)$$

$$\phi \longrightarrow \dot\phi$$

$$P_\phi = \frac{\partial \mathcal{L}}{\partial \dot\phi} = \frac{1}{\cancel{2}} m r_L^2 \cancel{2} \dot\phi + q A_\phi r_L = \underset{\text{canon.}}{m r_L^2 \dot\phi + q A_\phi r_L}$$

ϕ comp. angular momentum

$$S = \int_0^{2\pi} P_\phi \, d\phi = \int_0^{2\pi} \left(m r_L^2 \dot\phi + q A_\phi r_L \right) d\phi$$

adiabatic motion

T_L

$$\int_0^{2\pi} m r_L^2 \dot\phi \, d\phi = \int_0^{2\pi} m r_L^2 \frac{d\phi}{dt} \frac{d\phi}{dt} dt = \int_0^{T_L} m r_L^2 (\dot\phi)^2 dt = \int_0^{T_L} m \cancel{r_L^2} \frac{v_\phi^2}{\cancel{r_L^2}} dt =$$

$$\dot\phi = \frac{v_\phi}{r_L}$$

$$= v_L^2 m \cdot \int_0^{T_L} dt = m v_L^2 T_L$$

$$\int_0^{2\pi} q r_L A_\phi \, d\phi \approx q r_L \int_0^{2\pi} A_\phi \, d\phi$$

adiabatic motion

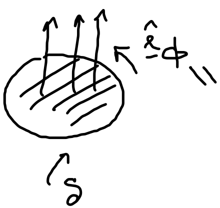
$$A_\phi = ?$$

Choose a ref. frame : z axis $\parallel \underline{B}$

$$\underline{B} = \underline{\nabla} \times \underline{A}$$

$$\int_S \underline{B}\cdot d\underline{S} = \int_S B\, dS \approx B \cdot \pi r_L^2$$

$$d\underline{\ell} = r_L \, d\phi \, \hat{\underline{e}}_\phi$$



$$\int_S (\underline{\nabla} \times \underline{A}) \cdot d\underline{S} = \oint \underline{A} \cdot d\underline{\ell} = \int (\underbrace{\underline{A} \cdot \hat{\underline{e}}_\phi}_{A_\phi}) r_L \, d\phi = \int_0^{2\pi} A_\phi r_L \, d\phi$$

Stokes theorem

Larmor circle

$$\approx A_\phi \cdot r_L \int_0^{2\pi} d\phi = 2\pi A_\phi r_L$$

$$B\cancel{\pi}\cancel{r}_L^2 \approx 2\cancel{\pi} A_\phi \cancel{r_L}$$

$$\Rightarrow A_\phi \underset{2\pi}{=} \frac{B}{2} r_L$$

$$q r_L \int_0^{2\pi} A_\phi \, d\phi = q r_L \int_0 \underbrace{\frac{B}{2} r_L} \, d\phi \approx q r_L^2 \frac{B}{\cancel{2}_1} \cdot \overset{1}{\cancel{2}}\pi$$

$$S = m v_L^2 T_L + q r_L^2 B\pi = m v_L^2 \frac{2\pi m}{qB} + q r_L^2 B\pi =$$

$$= \frac{m^2 v_L^2}{qB} 2\pi + \cancel{q} \frac{m^2 v_L^2}{q^{\cancel{2}} B^{\cancel{2}}} \cancel{B}\pi = (3\pi) \cdot \frac{m^2 v_L^2}{qB} =$$

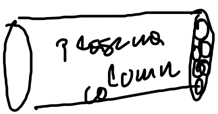
$$= \left(\frac{6\pi m}{q}\right) \cdot \frac{m v_L^2}{2B} = \frac{6\pi m}{q} \cdot \mu$$

If $\Delta t \gg T_L$ Δt: time over which a change of $\underline{B}$ occurs

then μ : magnetic momentum is an adiabatic invariant

<u>Frozen flux law</u>

$$\underset{\text{Larmor orbit}}{\Phi(\underline{B})} = B\cdot\pi\cdot r_L^2 = \pi \cancel{B}\frac{m^2 v_L^2}{q^2 B^{\cancel{2}}} = \frac{2\cdot m\cdot\pi}{q}\,\frac{m v_L^2}{2B} = \frac{2m\pi}{q}\cdot\mu$$



$\Phi = B\pi r_L^2$ say $B\uparrow$ slowly ($\Delta t \gg T$)

$\Phi \approx$ const $\Rightarrow r_L \downarrow \Rightarrow$ plasma column shrinks

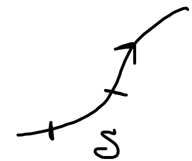
say $B\downarrow$ slowly $\Delta t \gg T \Rightarrow \Phi \approx$ const : $r_L \uparrow \Rightarrow$ plasma column becomes larger

Hydrogen ion $\quad B \approx 1\,T$

$$T = \frac{2\pi m}{qB} \approx \frac{6.3 \cdot 1.67 \cdot 10^{-27}}{1.6 \cdot 10^{-19} \cdot 1} \approx \frac{10}{1.6} \cdot \frac{10^{-27}}{10^{-19}} \approx 6 \cdot 10^{-8} \ll \mu s$$

Plasma confinement in a linear device

$$m \frac{dv_{\parallel}}{dt} = qE_{\parallel} - \mu \frac{\partial B}{\partial s} \qquad (\text{along } \vec{B})$$



$$\underline{F}_{\nabla|\underline{B}|} = -\mu \underline{\nabla} |\underline{B}|$$