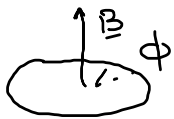
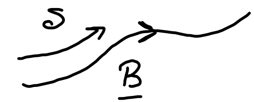
$$\mu = \frac{m v_\perp^2}{2B} \quad \text{is an adiabatic invariant}$$



Mirror machine

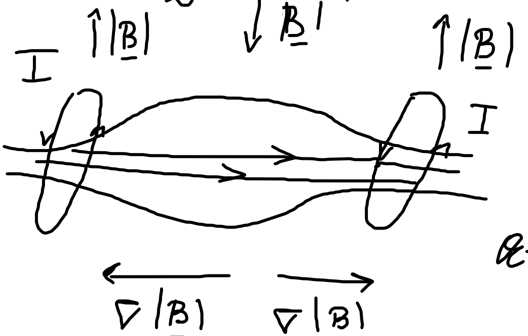
$$\underbrace{v_\parallel \cdot m \frac{d v_\parallel}{dt}}_{\frac{d}{dt}\left(\frac{1}{2} m v_\parallel^2\right)} = \left(-\mu \nabla |B|\right)_\parallel = \underbrace{-\mu \frac{\partial B}{\partial s} \cdot v_\parallel}_{-\mu \frac{\partial B}{\partial s}\frac{\partial s}{\partial t} \,=\, -\mu \frac{dB}{dt} \,=\, -\frac{d}{dt}(\mu \cdot B)} \qquad v_\parallel = \frac{\partial s}{\partial t}$$



$$\frac{d}{dt}\left(\frac{1}{2} m v_\parallel^2 + \mu B\right) = 0 \Rightarrow \frac{1}{2} m v_\parallel^2 + \mu B = \text{const}$$

$$\frac{1}{2}mv_{=}^2 + \mu B = \frac{1}{2}mv_{=}^2 + \frac{mv_\perp^2}{2\cancel{B}}\cdot\cancel{B} = E = \frac{1}{2}mv^2$$

$$\frac{1}{2}mv_{=}^2 + \mu B = E$$



Let's call 0 quantities where $B = B_{min}$

v_0 at $B = B_{min}$

$$E = \frac{1}{2}mv_{=0}^2 + \frac{mv_{\perp 0}^2}{2B_{min}}$$

$$\frac{1}{2}mv_{=}^2 = E - \mu B$$

It can be that $\begin{cases} E > \mu B_{max} \\ E < \mu B_{max} \end{cases}$

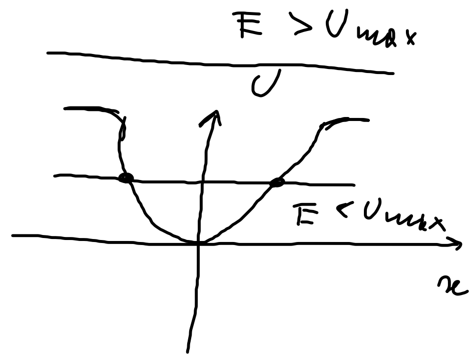
If $E > \mu B_{max}$ then $v_{\parallel} > 0$

If $E < \mu B_{max}$ at some place $\tilde{p}$

$$E = \mu \tilde{B} \Rightarrow v_{\parallel} = 0$$

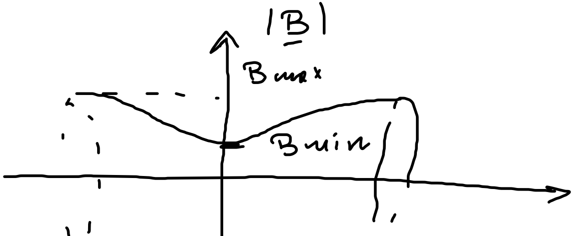
1D motion of a particle

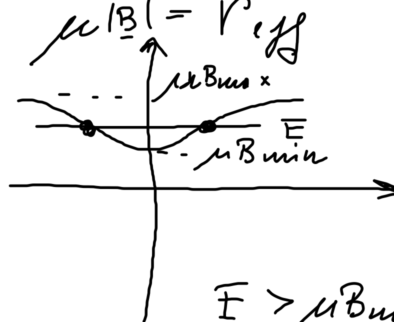
$$K + U = E$$



If $E > U_{max}$: $v > 0$

If $E < U_{max}$: $v = 0$ at some point



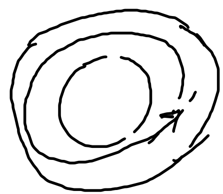


Trapped particle
$E < \mu B_{max}$

$E > \mu B_{max}$
unconfined particle

$E < \mu B_{max}$

$E > \mu B_{max}$

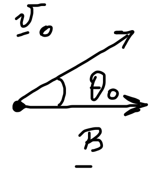


Confinement criterion

$$E < \mu B_{max}$$

$$\cancel{\tfrac{1}{2}} m v_0^2 < \frac{m v_{\perp 0}^2}{\cancel{2} B_{min}} \cdot B_{max}$$

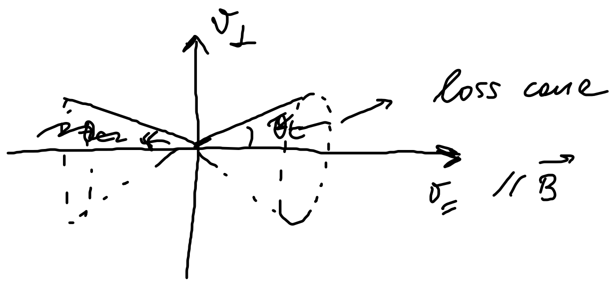
$$\cancel{m v_0^2} < \cancel{m v_0^2} \sin^2\theta_0 \frac{B_{max}}{B_{min}}$$



pitch angle θ_0

$$v_{\perp 0} = v_0 \sin\theta_0$$

$$\sin^2\theta_0 > \frac{B_{min}}{B_{max}} \qquad \sin^2\theta_C \overset{def}{=} \frac{B_{min}}{B_{max}}$$



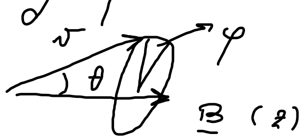
$$\theta_C = \arcsin\left(\sqrt{\frac{B_{min}}{B_{max}}}\right)$$

$$f_{\substack{\text{confined}\\ \text{particles}}} = \frac{\#\text{ part. outside the loss cone}}{\text{total }\#\text{ of part.}}$$

Plasma at eq. $\Rightarrow f_M(\underline{v})$ Maxwellian distr

$$f = \frac{\displaystyle\int_{\text{outside}} f_M(\underline{v})\, d^3\underline{v}}{\displaystyle\int_{\substack{\text{all}\\ \text{velocity space}}} f_M(\underline{v})\, d^3\underline{v}} = \frac{\cancel{\int_0^{2\pi} d\varphi} \int_{\theta_c}^{\pi-\theta_c} d\theta \sin\theta\, \cancel{\int_0^{+\infty} dv\, v^2 f_M(v)}}{\cancel{\int_0^{2\pi} d\varphi} \int_0^{\pi} d\theta \sin\theta\, \cancel{\int_0^{+\infty} dv\, v^2 f_M(v)}}$$

sph. coordinates



$$= \frac{-\cos\theta \Big|_{\theta_c}^{\pi-\theta_c}}{-\cos\theta \Big|_0^{\pi}} = \frac{-\left[-\cos\theta_c - \cos\theta_c\right]}{2} = \cos\theta_c$$

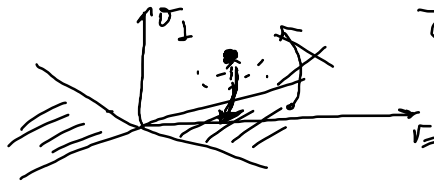
$$\frac{J}{J} = \cos\theta_c \Longrightarrow \frac{J}{J} = \sqrt{1-\sin^2\theta_c} = \sqrt{1-\frac{B_{min}}{B_{max}}}$$

$$\sin^2\theta_c = \frac{B_{min}}{B_{max}}$$

Eg.

$$B_{max} = 2B_{min} \qquad \frac{J}{J} = \sqrt{1-\frac{1}{2}} = \sqrt{\frac{1}{2}} \approx 70\%$$

<u>Collisions</u>



$\tau_{coll} \sim ms$ in $T \sim keV$

$T \sim 1\ keV \qquad L \sim 1\ m$

$v_H \sim 4 \cdot 10^5\ m/s$

$\Delta t \sim \frac{L}{v} \sim$ some μs