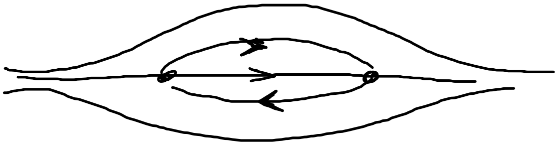
Mirror machine



Confined particles

Periodic motion

$\Downarrow$

2^{nd} adiabatic invariant

s coordinate

$\Downarrow$

$$P_{\parallel} = mv_{\parallel} + qA_{\parallel} \neq mv_{\parallel}$$

$\oint_{orbit} P_{\parallel}\, ds$ 2^{nd} adiabatic invariant

When can we assume $A_{\parallel}=0$? $(\underline{\nabla} \times \underline{B})_{\parallel} = \mu_0 j_{\parallel}$ $\underline{j}$: current density

$$\underline{B} = \underline{\nabla} \times \underline{A}$$

$$[\underline{\nabla} \times (\underline{\nabla} \times \underline{A})]_{\parallel} = \mu_0 j_{\parallel}$$

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \underline{\nabla}(\underline{\nabla}\cdot \underline{A}) - \nabla^2 \underline{A}$$

$$\underbrace{\cancel{\underline{\nabla}(\underline{\nabla}\cdot\underline{A})}} - \nabla_\parallel^2 \underline{A} = \mu_0 j_\parallel \qquad \nabla^2 A_\parallel = -\mu_0 j_\parallel$$

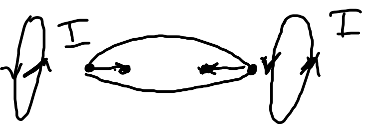
Coulomb gauge : $\underline{\nabla}\cdot\underline{A} = 0$

If $j_\parallel = 0$ (mirror)

$\nabla^2 A_\parallel = 0$ satisfied if $A_\parallel = 0$

$$P_\parallel = m v_\parallel$$

$$J = \int_{\substack{\text{closed}\\ \text{orbit}}} m v_\parallel \, ds \sim m v_\parallel \cdot \underbrace{L}_{\text{orbit size}}$$



$\dfrac{B_{min}}{B_{max}}$ $\quad V_{eff} = \mu B$

Say that we increase I in the coils slowly

$\Rightarrow L \downarrow$

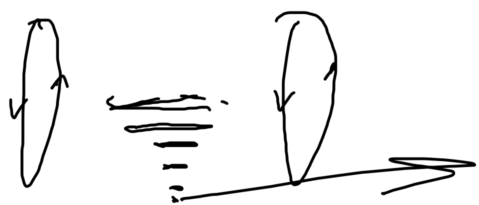
at slower than T_{bounce}

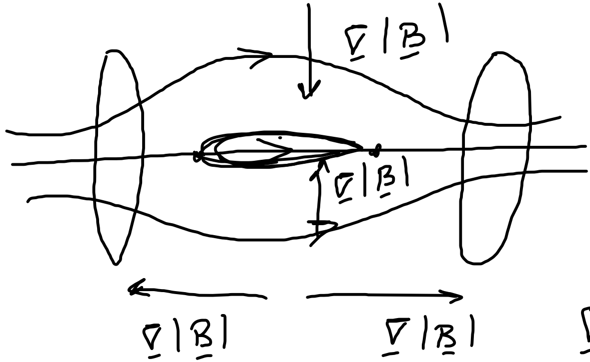
$\Rightarrow v_{\parallel} \uparrow$ $\quad \sin\theta = \dfrac{v_{\perp 0}}{v_0}$

If $\sin\theta < \sin\theta_C \Rightarrow$ particle is unconfined

$\Rightarrow$ at some point, when $v_{\parallel}$ is large enough, the particle becomes unconfined.

Fermi acceleration





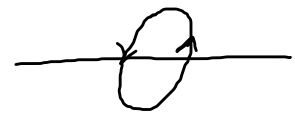
$\nabla|\underline{B}| \neq 0$ radially

$$\underline{v}_D = \frac{-\mu \nabla|\underline{B}| \times \underline{B}}{qB^2}$$

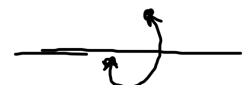
$$\nabla|\underline{B}| \propto \hat{\underline{e}}_r \qquad \underline{B} \propto \hat{\underline{z}}$$

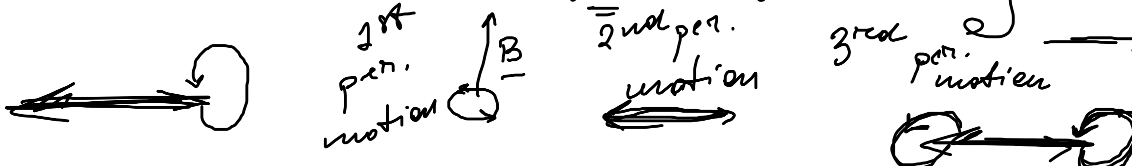
$$\underline{v}_D \propto -\hat{\underline{e}}_r \times \hat{\underline{z}} \propto \hat{\underline{e}}_\theta$$

Two types of particles → axis encircling



→ non axis encircling





3rd adiabatic invariant

$$\delta \underline{r}_j = \int_0^{\tau} \underline{v}\, dt \qquad \tau : \text{bounce period}$$

position of the turning point after 1 bounce

$$\langle \underline{v} \rangle = \frac{1}{\tau} \delta \underline{r}_j \qquad \underline{P} = m \langle \underline{v} \rangle + q \underline{A}$$

(canonical momentum)

$$S = \sum_j \int \left(m \langle \underline{v} \rangle + q \underline{A} \right) \cdot d\underline{r}_j \simeq \int \left(m \langle \underline{v} \rangle + q \underline{A} \right) \cdot d\underline{r}_{\text{bounce}}$$

j = index over the bounces

$$\frac{\int m\langle \underline{v}\rangle \cdot d\underline{r}}{\int q\underline{A}\cdot d\underline{r}} = \frac{m r_L \cdot v_\perp}{q\int_{\text{orbit}} \underline{B}\cdot d\underline{S}} \sim \frac{m r_L v_\perp}{q \pi r^2 B} \sim \frac{r_L^2}{r^2} \ll 1$$

described by T.P.

$$S_3 \simeq \int q\underline{A}\cdot d\underline{r} \propto \Phi_{\text{orbit}}(\underline{B})$$

$$\oint_{\text{circuit}} \underline{A}\cdot d\underline{l} = \int_S \underline{B}\cdot d\underline{S} = \int_S (\nabla\times\underline{A})\cdot d\underline{S} = \oint \underline{A}\cdot d\underline{l}$$

Stokes theorem

$$v_{dr} \sim \frac{-\mu \frac{B}{r}\cdot \cancel{B}}{q B^{\cancel{2}}} = \frac{-\mu}{r q} = -\frac{m v_\perp^2}{qBr} \sim r_L \cdot \frac{v_\perp}{r}$$

$$r_L = \frac{m v_\perp}{qB}$$

$$\int m v_{dr}\, dr \sim m v_{dr}\cdot r \sim m r_L v_\perp$$

3 adiabatic invariants

Periodic motions	Larmor motion	Motion between the T.P.	Motion of the T.P. around the axis
adiabatic invariants	μ	$S_2 \simeq \int_{\text{orbit between T.P.}} v_\parallel \, ds$	$S_3 \simeq \Phi(\underline{B})_{\text{T.P. orbit}}$
Time scales	T_L $\ll$	τ_{bounce} $\ll$	$\tau_{T.P.}$