Properties of collisions in plasmas

1) Coulomb collisions

2) θ: scattering angle b: impact parameter

μ: reduced mass

v_{rel}: relative velocity

$$\tan\left(\frac{\theta}{2}\right) = \frac{q_1 q_2}{4\pi\varepsilon_0 b \mu v_{rel}^2}$$

$\theta < \frac{\pi}{2}$: small angle collisions

$\theta > \frac{\pi}{2}$: large " "

If $b \to 0 \Rightarrow \frac{\theta}{2} \to \frac{\pi}{2}$; $\theta \to \pi$

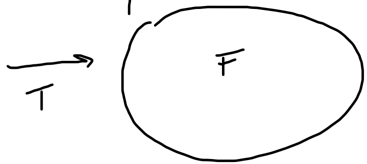
3)

$$\sigma_{\theta<\frac{\pi}{2}} \approx \underbrace{8\ln\Lambda}_{\substack{12 \\ 200}} \cdot \sigma_{\theta>\frac{\pi}{2}}$$

4) $\exists\ b_{max} \approx \lambda_D$ $\qquad \frac{q}{4\pi\varepsilon_0 r} e^{-r/\lambda_D}$

Test - field particle approach
(T) (F)

Bunch of test particles entering the plasma



Fokker - Planck equation

$f_T(\underline{v})$

$F(\underline{v}, \Delta\underline{v})$

↳ prob. ~~that~~ density after a collision

$$\int F(\underline{v}, \Delta\underline{v})\, d^3\Delta\underline{v} = 1$$

$\underline{v} \rightarrow \underline{v} + \Delta\underline{v}$

$t \qquad t + \Delta t$

Δt : average time between collisions

$f(\underline{v}, t)$?

$f(\underline{v} - \Delta\underline{v}, t - \Delta t)$

$$f(\underline{v}, t) = \int f(\underline{v} - \Delta\underline{v}, t - \Delta t)\, F(\underline{v} - \Delta\underline{v}, \Delta\underline{v})\, d^3 \Delta\underline{v}$$

$$\underline{v} - \Delta\underline{v} \xrightarrow[\Delta\underline{v}]{} \underline{v}$$

Prob.: $\underbrace{F(\underline{v} - \Delta\underline{v}, \Delta\underline{v})}_{\text{Prob. density}}\, \underbrace{d^3 \Delta\underline{v}}_{\text{volume element}}$

Assume: $|\Delta\underline{v}| \ll |\underline{v}|$

$|\Delta t| \ll |t|$

$$f(\underline{v}-\Delta\underline{v}, t-\Delta t)\, F(\underline{v}-\Delta\underline{v}, \Delta\underline{v})$$

$$\approx f(\underline{v},t)F(\underline{v},\Delta\underline{v}) - \Delta t \frac{\partial}{\partial t}\left[f(\underline{v},t)\,F(\underline{v},\Delta\underline{v})\right] - \Delta\underline{v}\cdot\frac{\partial}{\partial\underline{v}}\left(f(\underline{v},t)F(\underline{v},\Delta\underline{v})\right)$$

$$\underbrace{+\frac{1}{2}\Delta\underline{v}\,\Delta\underline{v} : \frac{\partial}{\partial\underline{v}}\frac{\partial}{\underline{v}}\left(f(\underline{v},t)\,F(\underline{v},\Delta\underline{v})\right)}_{2^{nd}\text{ order term}}$$

Taylor expansion

centred in $(\underline{v},t)$

stop: II order

$$\left[\frac{\partial}{\partial\underline{v}}\frac{\partial}{\partial\underline{v}}(fF)\right]_{ij} = \frac{\partial}{\partial v_i}\frac{\partial}{\partial v_j}(fF)$$

$$\Delta\underline{v}\,\Delta\underline{v} : \frac{\partial}{\partial\underline{v}}\frac{\partial}{\partial\underline{v}}\left[\quad\right] = \sum_{i=1}^{3}\sum_{j=1}^{3}\Delta v_i\,\Delta v_j\,\frac{\partial}{\partial v_i}\frac{\partial}{\partial v_j}(fF)$$

$$\int f(\underline{v},t)\, F(\underline{v},\Delta\underline{v})\, d^3\Delta\underline{v} = f(\underline{v},t) \int \underbrace{F(\underline{v},\Delta\underline{v})\, d^3\Delta\underline{v}}_{1}$$

$$= f(\underline{v},t)$$

$$-\int \Delta t \frac{\partial}{\partial t}\left(f(\underline{v},t)\, F(\underline{v},\Delta\underline{v})\right) d^3\Delta\underline{v} = -\Delta t \frac{\partial f}{\partial t} \int \underbrace{F(\underline{v},\Delta\underline{v})\, d^3\Delta\underline{v}}_{1} = -\Delta t \frac{\partial f}{\partial t}$$

$$-\int \Delta\underline{v} \cdot \frac{\partial}{\partial \underline{v}} (f F)\, d^3\Delta\underline{v} = -\frac{\partial}{\partial \underline{v}} \cdot \left(\langle \Delta\underline{v} \rangle f(\underline{v},t) \right)$$

$$\langle \Delta\underline{v} \rangle = \int \Delta\underline{v}\, F(\underline{v},\Delta\underline{v})\, d^3\Delta\underline{v}$$

$$\frac{1}{2}\int \Delta\underline{v}\,\Delta\underline{v} : \left(\frac{\partial}{\partial\underline{v}}\frac{\partial}{\partial\underline{v}}(fF)\right) d^3\Delta\underline{v} =$$

$$= \frac{1}{2}\frac{\partial}{\partial\underline{v}}\frac{\partial}{\partial\underline{v}} : \langle \Delta\underline{v}\,\Delta\underline{v}\rangle f(\underline{v},t)$$

$$\langle \Delta\underline{v}\;\Delta\underline{v}\rangle = \int d^3\Delta\underline{v}\;\underbrace{\Delta\underline{v}\;\Delta\underline{v}}_{\text{matrix}}\,F(\underline{v},\Delta\underline{v}) \qquad (\Delta\underline{v}\,\Delta\underline{v})_{ij} = \Delta v_i\,\Delta v_j$$

$$\cancel{f(\underline{v},t)} = \cancel{f(\underline{v},t)} - \Delta t\frac{\partial f}{\partial t} - \frac{\partial}{\partial\underline{v}}\cdot\left(\langle\Delta\underline{v}\rangle f(\underline{v},t)\right) + \frac{1}{2}\frac{\partial}{\partial\underline{v}}\frac{\partial}{\partial\underline{v}} : \langle\Delta\underline{v}\;\Delta\underline{v}\rangle f(\underline{v},t)$$

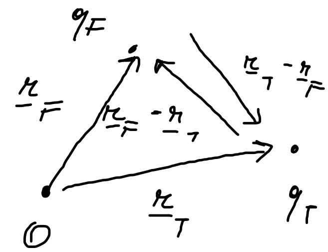
$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \underline{v}} \cdot \left(\left\langle \frac{\Delta \underline{v}}{\Delta t} \right\rangle f(\underline{v}, t) \right) + \frac{1}{2} \frac{\partial}{\partial \underline{v}} \frac{\partial}{\partial \underline{v}} : \left(\left\langle \frac{\Delta \underline{v} \Delta \underline{v}}{\Delta t} \right\rangle f(\underline{v}, t) \right)$$

Fokker-Planck equation

$$\left\langle \frac{\Delta \underline{v}}{\Delta t} \right\rangle \overset{\text{def}}{=} \int d^3 \Delta \underline{v} \; \Delta \underline{v} \, F(\underline{v}, \Delta \underline{v})$$

$$\left\langle \frac{\Delta \underline{v} \Delta \underline{v}}{\Delta t} \right\rangle \overset{\text{def}}{=} \int d^3 \Delta \underline{v} \; \Delta \underline{v} \, \Delta \underline{v} \, F(\underline{v}, \Delta \underline{v})$$

Laboratory frame



(1) $$m_T \ddot{\underline{r}}_T = \frac{1}{4\pi\varepsilon_0} \frac{q_T q_F}{|\underline{r}_T - \underline{r}_F|^3} (\underline{r}_T - \underline{r}_F)$$

(2) $$m_F \ddot{\underline{r}}_F = \frac{1}{4\pi\varepsilon_0} \frac{q_T q_F}{|\underline{r}_T - \underline{r}_F|^3} (\underline{r}_F - \underline{r}_T)$$

$$(1)+(2) \Rightarrow m_T \ddot{\underline{r}}_T + m_F \ddot{\underline{r}}_F = \vec{0} \qquad \underline{R} \overset{def}{=} \frac{m_T \underline{r}_T + m_F \underline{r}_F}{m_T + m_F}$$

$$\ddot{\underline{R}} = \vec{0} \Rightarrow \dot{\underline{R}} = \overrightarrow{const}$$

$$\underline{r} \overset{def}{=} \underline{r}_T - \underline{r}_F \Rightarrow \ddot{\underline{r}} = \ddot{\underline{r}}_T - \ddot{\underline{r}}_F = \left(\frac{1}{m_T} + \frac{1}{m_F}\right) \frac{1}{4\pi\varepsilon_0} \frac{q_T q_F}{|\underline{r}_T - \underline{r}_F|^3} \cdot (\underline{r}_T - \underline{r}_F)$$

$$\frac{1}{\mu} \overset{def}{=} \frac{1}{m_T} + \frac{1}{m_F}$$

reduced mass

$$\boxed{\Delta \underline{v}_T = \frac{\mu}{m_T} \Delta \underline{v}_{rel}}$$

$$\Rightarrow \quad \ddot{\underline{r}} = \frac{1}{\mu} \frac{1}{4\pi\varepsilon_0} \frac{1}{r^3} \cdot \underline{r} \quad \Rightarrow \quad \begin{cases} \mu \ddot{\underline{r}} = \dfrac{q_T q_F}{4\pi\varepsilon_0} \underline{r} / r^3 \\ \ddot{\underline{R}} = 0 \end{cases}$$

$$\Downarrow$$

$$\dot{\underline{R}} = \overrightarrow{const}$$

$$\begin{cases} \dot{\underline{r}}_T = \dot{\underline{R}} + \dfrac{\mu}{m_T} \dot{\underline{r}} \\ \dot{\underline{r}}_F = \dot{\underline{R}} - \dfrac{\mu}{m_F} \dot{\underline{r}} \end{cases}$$

$$\Uparrow$$

$$f(\underline{v}_T, t) \longrightarrow f(\underline{v}_{rel}, t)$$

$$\Delta \underline{v}_{rel} \; ? \; \Delta \underline{v}_T$$

$$\Delta \underline{v}_T = \cancel{\Delta \dot{\underline{R}}} + \frac{\mu}{m_T} \Delta \dot{\underline{r}}$$

$$\begin{cases} \underline{r} = \underline{r}_T - \underline{r}_F \\ \underline{R} = \dfrac{m_T \underline{r}_T + m_F \underline{r}_F}{m_T + m_F} \end{cases} \Rightarrow \begin{cases} \underline{r}_T = \underline{R} + \dfrac{\mu}{m_T} \underline{r} \\ \underline{r}_F = \underline{R} - \dfrac{\mu}{m_F} \underline{r} \end{cases}$$