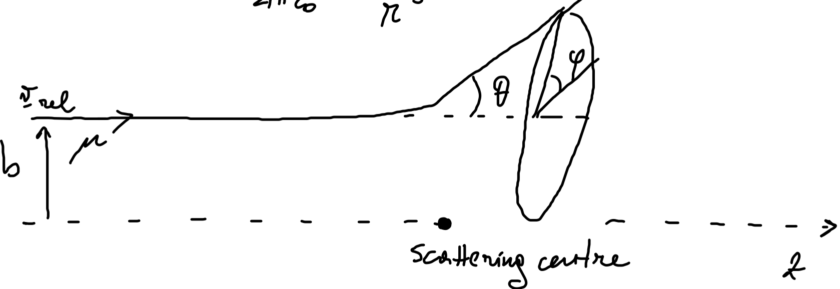
$$\frac{\partial f}{\partial t} = -\frac{\partial}{\partial \underline{v}}\left(\left\langle \frac{\Delta \underline{v}}{\Delta t} \right\rangle f(\underline{v}, t) \right) + \frac{1}{2} \frac{\partial}{\partial \underline{v}} \frac{\partial}{\partial \underline{v}} : \left(\left\langle \frac{\Delta \underline{v} \Delta \underline{v}}{\Delta t} \right\rangle f(\underline{v}, t) \right)$$

$$\left\langle \frac{\Delta \underline{v}}{\Delta t}_T \right\rangle = \int d^3 \Delta \underline{v} \; \frac{\Delta \underline{v}}{\Delta t} F(\underline{v}, \Delta \underline{v})$$

$$\left\langle \frac{\Delta \underline{v} \Delta \underline{v}}{\Delta t} \right\rangle = \int d^3 \Delta \underline{v} \; \frac{\Delta \underline{v} \Delta \underline{v}}{\Delta t} F(\underline{v}, \Delta \underline{v}) \qquad [\Delta \underline{v} \Delta \underline{v}]_{ij} = \Delta v_i \, \Delta v_j$$

$$\left\langle \frac{\Delta \underline{v}_{rel}}{\Delta t} \right\rangle \qquad \Delta \underline{v}_T = \frac{\mu}{m_T} \Delta \underline{v}_{rel}$$

$$\mu \dot{\underline{v}}_{rel} = \frac{1}{4\pi\varepsilon_0} \frac{q_T q_F}{r^3} \underline{r}$$



$$\underline{v}_{rel} = \underline{v}_T - \underline{v}_F$$

Specifying a collision $\begin{cases} b \\ \varphi \\ \underline{v}_F \text{ at given } \underline{v}_T \end{cases}$

$$\left\langle \frac{\Delta \underline{v}_{rel}}{\Delta t} \right\rangle$$

1) $\Delta \underline{v}_{rel}$ for a specific set of $b, \varphi, \underline{v}_F$

2) Average $\Delta \underline{v}_{rel}$ over all possible values of $b, \varphi, \underline{v}_F$

Before collision $\underline{v}_{rel,i} = v_{rel}\,\hat{\underline{z}}$

$$E = K_F + K_T = \frac{1}{2} m_T v_T^2 + \frac{1}{2} m_F v_F^2 = \frac{1}{2} M V_{cm}^2 + \frac{1}{2}\mu v_{rel}^2$$

$$\underline{V}_{cm} = \frac{m_T \underline{v}_T + m_F \underline{v}_F}{m_T + m_F}$$

$$\underline{v}_{rel} = \underline{v}_T - \underline{v}_F$$

In c.m. frame: $V_{cm} = 0 \Rightarrow E\Big|_{c.m.} = \frac{1}{2}\mu v_{rel}^2$

Elastic collision $\Rightarrow \frac{1}{2}\mu v_{rel}^2 = \text{const} \Rightarrow v_{rel} = \text{const}$

After collision:

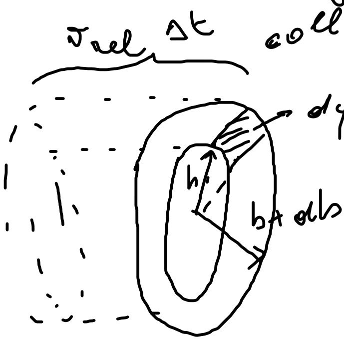
$$\underline{v}_{rel,\,after} = v_{rel}\cos\theta\,\hat{\underline{z}} + v_{rel}\sin\theta\left(\cos\varphi\,\hat{\underline{x}} + \sin\varphi\,\hat{\underline{y}}\right)$$

$$\cos\theta \approx 1-\frac{\theta^2}{2} \qquad \sin\theta \approx \theta$$

$$\Delta\underline{v}_{rel} = \underline{v}_{rel,\,after} - \underline{v}_{rel,\,i} = v_{rel}(\cos\theta - 1)\,\hat{\underline{z}} + v_{rel}\sin\theta\left(\cos\varphi\,\hat{\underline{x}} + \sin\varphi\,\hat{\underline{y}}\right)$$

$$\theta \ll 1 \quad \simeq \quad -v_{rel}\frac{\theta^2}{2}\,\hat{\underline{z}} + v_{rel}\,\theta\left(\cos\varphi\,\hat{\underline{x}} + \sin\varphi\,\hat{\underline{y}}\right)$$

(small angle collision)



In a time interval Δt

$$v_{rel}\,\Delta t \cdot (b\,d\varphi\,db) \cdot \left(\#\ \frac{\text{targets}}{\text{volume}}\ \text{at a given } v_F\right)$$

$$= v_{rel}\,\Delta t\, b\,d\varphi\,db\ f(\underline{v}_F)\,d^3\underline{v}_F$$

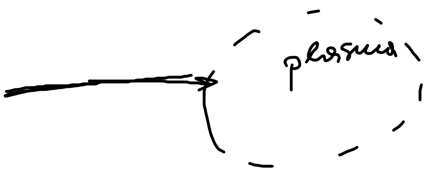
$$\left\langle \frac{\Delta\underline{v}_{rel}}{\Delta t} \right\rangle$$

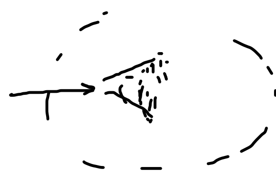
$$\left\langle \frac{\Delta \underline{v}_{rel}}{\Delta t} \right\rangle = \int_{\substack{\text{all} \\ b,\,\phi,\,\underline{v}_F}} \frac{\Delta \underline{v}_{rel}}{\Delta t}\bigg|_{b,\,\phi,\,\underline{v}_F} \cdot v_{rel}\, b\, db\, \Delta t\, d\varphi\, f_F(\underline{v}_F)\, d^3\underline{v}_F$$

$\left\langle \frac{\Delta \underline{v}_{rel}}{\Delta t} \right\rangle$ is a <u>friction term</u>

$$= \int_{\substack{\text{all} \\ b,\,\phi,\,\underline{v}_F}} d^3\underline{v}_F\, f_F(\underline{v}) \left(-v_{rel}\frac{\theta^2}{2}\hat{\underline{z}} + v_{rel}\,\theta\left(\cancel{\cos\varphi}\,\hat{\underline{x}} + \cancel{\sin\varphi}\,\hat{\underline{y}}\right)\right) v_{rel}\, b\, db\, d\varphi$$

$$\int_0^{2\pi} \cos\varphi\, d\varphi = \int_0^{2\pi} \sin\varphi\, d\varphi = 0$$





$$\int\limits_{\text{all } b,\, \underline{v}_F} d^3\underline{v}_F \, db \, f_F(\underline{v}_F) \left(-v_{rel} \frac{\theta^2}{\cancel{2}}\right) \hat{\underline{z}} \; v_{rel}\, b \cdot \underbrace{\cancel{2\pi}}_{\int_0^{2\pi} d\varphi}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{q_T q_F}{4\pi\varepsilon_0 b \mu v_{rel}^2} \underset{\theta \ll 1}{\Longrightarrow} \theta \approx \frac{q_T q_F}{2\pi \varepsilon_0 b \mu v_{rel}^2}$$

$$= \int d^3\underline{v}_F \, db \, f_F(\underline{v}_F) \left(-v_{rel}^2 \, \hat{\underline{z}}\right) \cancel{\pi} \underbrace{\frac{q_T^2 q_F^2}{\cancel{4}\pi^{\cancel{2}} \varepsilon_0^2 b^{\cancel{2}} \mu^2 v_{rel}^4}}\cdot \cancel{b}$$

$$b_{min} = b_{\pi/2}$$

$$b_{max} = \lambda_D$$

$$\simeq -\frac{q_T^2 q_F^2}{4\pi \varepsilon_0^2 \mu^2} \int\limits_{\text{all } \underline{v}_F} d^3\underline{v}_F \frac{f_F(\underline{v}_F)}{v_{rel}^2} \cdot \underbrace{\int_{b_{\pi/2}}^{\lambda_D} \frac{db}{b}}_{\to \ln\Lambda} \hat{\underline{z}} = -\frac{q_T^2 q_F^2}{4\pi\varepsilon_0^2 \mu^2} \cdot \ln\Lambda \cdot \int\limits_{\text{all } \underline{v}_F} d^3\underline{v}_F \cdot \frac{1}{v_{rel}^2} \cdot f_F(\underline{v}_F)\, \hat{\underline{z}}$$

$$\left\langle \frac{\Delta \underline{v}_T}{\Delta t} \right\rangle = \frac{\cancel{\mu}}{m_T} \cdot \left\langle \frac{\Delta \underline{v}_{rel}}{\Delta t} \right\rangle =$$

$$= -\frac{\cancel{\mu}}{m_T} \frac{q_T^2 q_F^2 \ln \Lambda}{4\pi\varepsilon_0^2 \mu^{\cancel{2}}} \int_{\text{all } \underline{v}_F} d^3\underline{v}_F \frac{f_F(\underline{v}_F)}{|\underline{v}_T - \underline{v}_F|^2} \hat{\underline{r}}$$

$$\hat{\underline{r}} = \frac{\underline{v}_T - \underline{v}_F}{|\underline{v}_T - \underline{v}_F|} \qquad \frac{\hat{\underline{r}}}{|\underline{v}_T - \underline{v}_F|^2} = \frac{\underline{v}_T - \underline{v}_F}{|\underline{v}_T - \underline{v}_F|^3} = -\frac{\partial}{\partial \underline{v}_T} \frac{1}{|\underline{v}_T - \underline{v}_F|}$$

$$\left\langle \frac{\Delta \underline{v}_T}{\Delta t} \right\rangle = \frac{q_F^2 q_T^2 \ln \Lambda}{4\pi\varepsilon_0^2 \mu m_T} \cdot \int d^3\underline{v}_F \frac{\partial}{\partial \underline{v}_T} \frac{1}{|\underline{v}_T - \underline{v}_F|} \cdot f_F(\underline{v}_F)$$

$$\left\langle \frac{\Delta \underline{v}_T}{\Delta t} \right\rangle = \frac{q_F^2 q_T^2 \ln\Lambda}{4\pi\varepsilon_0^2 \mu m_T} \frac{\partial}{\partial \underline{v}_T} \int d^3\underline{v}_F \frac{f_F(\underline{v}_F)}{|\underline{v}_T - \underline{v}_F|}$$

$$\left\langle \frac{\Delta \underline{v}_T \Delta \underline{v}_T}{\Delta t} \right\rangle$$