$\left\langle \frac{\Delta \underline{v}_T}{\Delta t} \right\rangle$ $\qquad$ $\left\langle \frac{\Delta \underline{v}_T \, \Delta \underline{v}_T}{\Delta t} \right\rangle = ?$

1) $\Delta \underline{v}_T$ at given $b, \phi, \underline{v}_F$

2) Average over all $b, \phi, \underline{v}_F$

$$\Delta \underline{v}_{rel} = - v_{rel} \frac{\theta^2}{2} \hat{\underline{z}} + v_{rel}\, \theta \left(\cos\varphi \, \hat{\underline{x}} + \sin\varphi \, \hat{\underline{y}} \right)$$

$$\xrightarrow[\underline{v}_{rel}]{} \hat{\underline{z}}$$

$$\Delta \underline{v}_{rel} \, \Delta \underline{v}_{rel} \longrightarrow \Delta \underline{v}_T \, \Delta \underline{v}_T$$

$$\Delta \underline{v}_T = \frac{\mu}{m_T} \Delta \underline{v}_{rel}$$

$$\left\langle \frac{\Delta \underline{v}_T \, \Delta \underline{v}_T}{\Delta t} \right\rangle = \frac{\mu^2}{m_T^2} \left\langle \frac{\Delta \underline{v}_{rel} \, \Delta \underline{v}_{rel}}{\Delta t} \right\rangle$$

$$\Delta \underline{v}_{rel}\, \Delta \underline{v}_{rel} = \begin{bmatrix} v_{rel}^2\, \theta^2 \underbrace{\cos^2\varphi}_{\pi} & \alpha\, \cancel{\sin\varphi\cos\varphi} & \alpha\, \cancel{\cos\varphi} \\ \alpha\, \cancel{\sin\varphi\cos\varphi} & v_{rel}^2\, \theta^2 \underbrace{\sin^2\varphi}_{\pi} & \alpha\, \cancel{\sin\varphi} \\ \alpha\, \cancel{\cos\varphi} & \alpha\, \cancel{\sin\varphi} & \begin{matrix} \alpha\, \theta^4 \\ 12 \\ 0 \end{matrix} \end{bmatrix}$$

$$\underbrace{v_{rel}\, \Delta t\, d\varphi\, b\, \overset{db}{\cancel{d\varphi}}\, f(\underline{v}_F)\, d^3 \underline{v}_F}_{\substack{\#\ \text{collisions} \\ \text{at } b,\ \varphi,\ \underline{v}_F}}$$

$$\int_0^{2\pi} \sin\varphi\, d\varphi = \int_0^{2\pi} \cos\varphi\, d\varphi = \int_0^{2\pi} \sin\varphi\cos\varphi\, d\varphi = 0$$

$$\int_0^{2\pi} \sin^2\varphi\, d\varphi = \int_0^{2\pi} \cos^2\varphi\, d\varphi = \pi$$

$$\frac{\langle \Delta \underline{v}_{rel}\, \Delta \underline{v}_{rel} \rangle}{\Delta t} = \int_{\underline{v}_F,\, b} d^3 \underline{v}_F\, db\; v_{rel}\, b\, f_F(\underline{v}_F)\; v_{rel}^2\, \theta^2\, \pi \begin{bmatrix} 1 & 0 & \\ 0 & 1 & 0 \end{bmatrix}$$

$$\theta \approx \frac{q_F q_T}{2\pi\varepsilon_0 b \,\cancel{\mu}\, v_{rel}^2}$$

$$\left\langle \frac{\Delta \underline{v}_{rel}\, \Delta \underline{v}_{rel}}{\Delta t} \right\rangle = \int_{\text{all } \underline{v}_F} d^3 \underline{v}_F\, \cancel{\pi v^3_{rel}}\, f_F(\underline{v}_F)\, \frac{q_F^2 q_T^2}{4\cancel{\pi^2}\varepsilon_0^2\, \cancel{\mu^2}\, v_{rel}^{\cancel{4}}} \underbrace{\int_{b_{\pi/2}}^{\lambda_D} db\, \frac{b}{b^{\cancel{2}}}}_{\ln\Lambda}\; \overbrace{\begin{bmatrix} 1 & 0 & \\ 0 & 1 & \\ & & 0 \end{bmatrix}}^{\hat{\underline{x}}\hat{\underline{x}} + \hat{\underline{y}}\hat{\underline{y}}}$$

$$= \frac{q_F^2 q_T^2}{4\pi\varepsilon_0^2\, \cancel{\mu}^2} \ln\Lambda \int_{\text{all } \underline{v}_F} d^3 \underline{v}_F\, f_F(\underline{v}_F)\, \frac{1}{v_{rel}}$$

$$\left\langle \frac{\Delta \underline{v}_T\, \Delta \underline{v}_T}{\Delta t} \right\rangle = \frac{q_F^2 q_T^2 \ln\Lambda}{4\pi\varepsilon_0^2 m_T^2} \int_{\text{all } \underline{v}_F} d^3 \underline{v}_F\, \frac{f_F(\underline{v}_F)}{|\underline{v}_T - \underline{v}_F|} \left(\hat{\underline{x}}\hat{\underline{x}} + \hat{\underline{y}}\hat{\underline{y}}\right)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \underbrace{\mathbb{1}}_{\text{Identity matrix}} - \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$= \mathbb{1} - \frac{\underline{v}_{rel}\,\underline{v}_{rel}}{v_{rel}^2} = \frac{v_{rel}^2\,\mathbb{1} - \underline{v}_{rel}\,\underline{v}_{rel}}{v_{rel}^2}$$

$$\left\langle \frac{\Delta \underline{v}_T\,\Delta \underline{v}_T}{\Delta t} \right\rangle = \frac{q_F^2 q_T^2 \ln\Lambda}{4\pi\varepsilon_0^2 m_T^2} \int_{\text{all } \underline{v}_F} d^3\underline{v}_F \, \frac{f_F(\underline{v}_F)}{|\underline{v}_T - \underline{v}_F|^3} \left(|\underline{v}_T - \underline{v}_F|^2 - (\underline{v}_T - \underline{v}_F)(\underline{v}_T - \underline{v}_F) \right)$$

$\underline{\nabla}(\underline{x})$: Gradient of $\underline{V}(\underline{x})$

$$\frac{\partial}{\partial \underline{x}} V(\underline{x}) : \left[\frac{\partial}{\partial \underline{x}} V(\underline{x}) \right]_{ij} \overset{\text{def}}{=} \frac{\partial}{\partial x_i} V_j(\underline{x})$$

$$\frac{\partial}{\partial \underline{v}_T} \frac{\partial |\underline{v}_T - \underline{v}_F|}{\partial \underline{v}_T} = \frac{v_{rel}^2 \mathbb{I} - \underline{v}_{rel} \underline{v}_{rel}}{v_{rel}^3}$$

$$\left\langle \frac{\Delta \underline{v}_T \Delta \underline{v}_T}{\Delta t} \right\rangle = \frac{q_F^2 q_T^2 \ln \Lambda}{4\pi \varepsilon_0^2 m_T^2} \frac{\partial}{\partial \underline{v}_T} \int d^3 \underline{v}_F \, f(\underline{v}_F) \frac{\partial}{\partial \underline{v}_T} |\underline{v}_T - \underline{v}_F|$$

Define the <u>Rosenbluth potentials</u>

$$g_F(\underline{v}) = \int d^3 \underline{v}' \, f_F(\underline{v}') \, |\underline{v} - \underline{v}'|$$

$$h_F(\underline{v}) = \frac{m_T}{\mu} \int \frac{f_F(\underline{v}')}{|\underline{v} - \underline{v}'|} \, d^3 \underline{v}'$$

$$\frac{\partial f}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda}{4\pi\varepsilon_0^2 m_T^2}\left[\overbrace{-\frac{\partial}{\partial \underline{v}}\cdot\left(f_T\frac{\partial h_F}{\partial \underline{v}}\right)}^{\text{friction term}} + \right.$$

$$\left. + \frac{1}{2}\frac{\partial}{\partial \underline{v}}\frac{\partial}{\partial \underline{v}} : \underbrace{\left(f_T\frac{\partial^2 g_F}{\partial \underline{v}\,\partial \underline{v}}\right)}_{\text{isotropy term}}\right]$$