# Chapter 1 - Introduction to plasma physics

## 1 Rutherford scattering

Consider a Rutherford scattering process between two particles having velocities $\mathbf{v_1}$, $\mathbf{v_2}$, charges $\mathrm{q}_1$, $\mathrm{q}_2$ and masses $m_1$,$m_2$, respectively.

a) Show that, in the centre of mass frame, the equation of motion is given by

$$\mu\ddot{\mathbf{r}} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}} \tag{1}$$

where $\mathbf{r}$ is the relative position vector.

b) Define the angular momentum $\mathbf{L} = \mu\mathbf{r} \times \dot{\mathbf{r}}$. By taking its derivative with respect to the time, show that this is a constant vector. In particular, note that this implies that the scattering process must occur in the plane where $\mathbf{r}$ and $\dot{\mathbf{r}}$ lie.

c) Using the geometry shown in 1, show that $|\mathbf{L}| = \mu b v_\infty = \mu r^2\dot{\phi}$, from which $\dot{\phi} = bv_\infty/r^2$.

d) Call $\mathbf{v_i}$ and $\mathbf{v_f}$ the initial and final velocity of the charged particle shown in the figure and note that $|\mathbf{v_i}| = |\mathbf{v_f}|$. From symmetry, the $x$ component of the velociy before and after scattering must be unchanged, whereas the $y$ component must flip. Starting from these considerations, evaluate the change of the $y$ velocity component before and after the collision ($\Delta v_y$) as a function of $v_{yi}$.

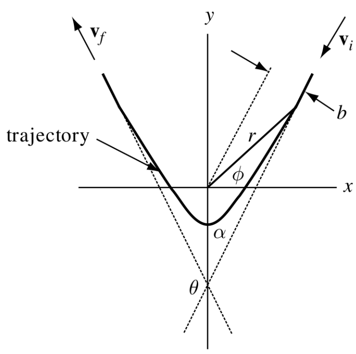


Figure 1: Scattering geometry in the centre of mass frame. The scattering centre is in the origin and $\theta$ denotes the scattering angle.

e) Starting from the $y$ component of equation 1, find a relation between $dv_y$ and $d\cos\phi$. (Hint: consider the conservation of the angular momentum to write $dt$ as a function of $d\phi$). Call $\phi_i$ and $\phi_f$ the initial and final values of $\phi$. By integrating $dv_y$, find $\Delta v_y$ for the whole collision and as a function of $\phi_i$ and $\phi_f$.

f) Using the figure, find the relation between the angles $\theta$ and $\alpha$ and between the angles $\phi_i$,$\phi_f$ and $\alpha$. By combining the different formulas for $\Delta v_y$ found above, show that

$$\tan\frac{\theta}{2} = \frac{q_1 q_2}{4\pi\epsilon_0 \mu b v_\infty^2} \tag{2}$$

and find an approximation for small angle collisions.

## 2 Debye shielding

We aim at solving the 3-dimensional Poisson equation for the electrostatic potential $\phi(\mathbf{r})$ of a point charge $q_T$ in a uniform plasma of ions and electrons

$$\nabla^2\phi - \frac{1}{\lambda_D^2}\phi = -\frac{q_T}{\epsilon_0}\delta(\mathbf{r}) \tag{3}$$

where $\lambda_D$ is the Debye length and $\delta$ is the Dirac-delta.

a) Starting from the known solution of the Poisson equation

$$\nabla^2\phi = -\frac{q_T}{\epsilon_0}\delta(\mathbf{r}) \tag{4}$$

of a charge $q_T$ in vacuum, which is

$$\phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{q_T}{r} \tag{5}$$

show that we can write

$$\delta(\mathbf{r}) = -\frac{1}{4\pi}\nabla^2(\frac{1}{r}) \tag{6}$$

b) Using this result in (3), evaluate the laplacian operator in spherical coordinates and find a solution for (3) of the type

$$\phi(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{g(r)}{r} \tag{7}$$

In particular, work out an equation for $g(r)$ e show that the solution is $g(r) = \exp(-r/\lambda_D)$.