$$\frac{\partial f}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda}{4\pi\varepsilon_0^2 m_T^2}\left[-\frac{\partial}{\partial \underline{v}}\cdot\left(f_T \frac{\partial h_F}{\partial \underline{v}}\right) + \right.$$

$$\left. + \frac{1}{2}\frac{\partial \partial}{\partial \underline{v}\,\partial \underline{v}} : \left(f_T \frac{\partial^2 g_F}{\partial \underline{v}\,\partial \underline{v}} \right)\right]$$

Rosenbluth potentials

$$g_F(\underline{v}) = \int |\underline{v} - \underline{v}'| f_F(\underline{v}')\, d^3\underline{v}'$$

$$h_F(\underline{v}) = \frac{m_T}{\mu} \int \frac{f_F(\underline{v}')}{|\underline{v} - \underline{v}'|}\, d^3\underline{v}'$$

Average velocity

$$\underline{u} = \frac{\int \underline{v}\, f(\underline{v},t)\, d^3\underline{v}}{\int f(\underline{v},t)\, d^3\underline{v}}$$

$$\frac{\int \frac{\partial f}{\partial t} \underline{v} \, d^3\underline{v}}{\int f \, d^3\underline{v}} = \frac{\frac{\partial}{\partial t}\int f \underline{v} \, d^3\underline{v}}{\int f \, d^3\underline{v}} = \frac{\partial \underline{u}}{\partial t}$$

$$-\int \underline{v} \frac{\partial}{\partial \underline{v}} \cdot \left(f_T \frac{\partial h_F}{\partial \underline{v}} \right) d^3\underline{v} = \int f_T \frac{\partial h_F}{\partial \underline{v}} \, d^3\underline{v}$$

(x)

$$-\int v_x \frac{\partial}{\partial v_x}\left(f_T \frac{\partial h_F}{\partial v_x}\right) dv_x \, dv_y \, dv_z \; \underbrace{- \int v_x \frac{\partial}{\partial v_y}\left(f_T \frac{\partial h_F}{\partial v_y}\right) dv_x \, dv_y \, dv_z}_{=0} \; +$$

$$\cancel{-\int v_x \frac{\partial}{\partial v_z}\left(f_T \frac{\partial h_F}{\partial v_z}\right) dv_x \, dv_y \, dv_z} = 0$$

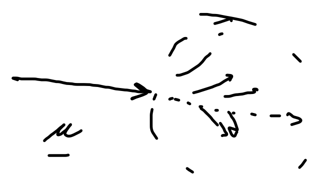
$$-\int dv_x \, v_x \int dv_z \underbrace{\cancel{\int dv_y \frac{\partial}{\partial v_y}\left(f_T \frac{\partial h_F}{\partial v_y}\right)}}_{\cancel{f_T \frac{\partial h_F}{\partial v_y}\Big|_{-\infty}^{+\infty}}} = 0$$

$$-\int dv_y \int dv_z \int dv_x \; v_x \frac{\partial}{\partial v_x}\left(f_T \frac{\partial h_F}{\partial v_x}\right) =$$

$f \qquad g'$

$$= -\int dv_y \int dv_z \left[\cancel{v_x \cdot f_T \frac{\partial h_F}{\partial v_x}}\right|_{-\infty}^{+\infty} - \int dv_x \, f_T \frac{\partial h_F}{\partial v_x} = \int d^3\underline{v} \; f_T \frac{\partial h_F}{\partial \underline{v}}\Bigg|_x$$

$$\int \underline{v} \; \frac{\partial}{\partial \underline{v}} \frac{\partial}{\partial \underline{v}} : \left(f_T \frac{\partial^2 g_F}{\partial \underline{v}\,\partial \underline{v}}\right) d^3\underline{v} = 0$$



$$\frac{\partial \underline{u}}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda}{4\pi\varepsilon_0^2 n_T m_T^2} \int f_T \frac{\partial h_F}{\partial \underline{v}} \, d^3 \underline{v}_F$$

Assume:

$$f_F(\underline{v}) = n_F \left(\frac{m_F}{2\pi T_F}\right)^{3/2} \exp\left(-\frac{m_F v_F^2}{2T_F}\right)$$

$$h_F(\underline{v}) = \frac{m_T}{\mu} n_F \left(\frac{m_F}{2\pi T_F}\right)^{3/2} \int \frac{\exp\left(-\frac{m_F v'^2}{2T_F}\right)}{|\underline{v} - \underline{v}'|} \, d^3 \underline{v}'$$

$$= \dots = \frac{m_T}{\mu} \frac{n_F}{v} \operatorname{erf}\left(\sqrt{\frac{m_F}{2T_F}}\, v\right)$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x dy \, \exp(-y^2) \qquad \operatorname{erf}(+\infty) = 1$$

Beam of test particles

$$f_T = n_T \, \delta(\underline{v} - \underline{u}_0)$$

Speed: $|\underline{u}_0|$

Direction: $\dfrac{\underline{u}_0}{|\underline{u}_0|}$

$\underline{u}_0$

$$\frac{\partial \underline{u}}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln\Lambda \, n_T}{4\pi\varepsilon_0^2 n_T m_T^2} \frac{m_T}{\mu} n_F \cdot \left[\underbrace{\frac{\partial}{\partial \underline{v}} \left(\frac{1}{v} \operatorname{erf}\left(\sqrt{\frac{m_F}{2T_F}}\, v \right) \right)}_{= \hat{\underline{u}}} \right]_{\underline{v} = \underline{u}_0}$$

$$\mu \to \mu_i, \quad \mu \to \mu_e$$

$$\frac{\partial}{\partial \underline{v}} (\;) = \frac{\partial}{\partial v} (\;) \, \hat{\underline{u}}$$

$$\frac{m_T}{\mu_{i,e}} = \frac{\cancel{m_T}}{\dfrac{\cancel{m_T}\, m_{i,e}}{m_T + m_{i,e}}} = 1 + \frac{m_T}{m_{i,e}}$$

$$n_F \to n_i, \quad n_F \to n_e$$

Quasi-neutrality

$$Z n_i \cancel{e} - n_e \cancel{e} = 0$$

$$n_i = n_e / Z$$

$$\frac{\partial u_T}{\partial t} = \frac{e^2 q_T^2 \ln\Lambda \, n_e}{4\pi\varepsilon_0^2 m_T} \cdot \Bigg[\underbrace{Z \left(1 + \frac{m_T}{m_i}\right) \frac{d}{du} \left(\frac{\operatorname{erf}\left(\sqrt{\frac{m_i}{2T_i}}\, u\right)}{u} \right)}_{\text{friction with ions}}$$

$$+ \underbrace{\left(1 + \frac{m_T}{m_e}\right) \frac{d}{du} \left(\frac{\operatorname{erf}\left(\sqrt{\frac{m_e}{2T_e}}\, u\right)}{u} \right)}_{\text{friction with electrons}} \Bigg]$$

$$\frac{d}{dx}\left(\frac{\operatorname{erf}(x)}{x}\right) < 0$$