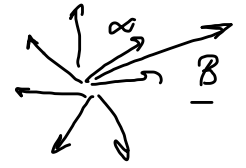


$E_\alpha \sim 3.5 \text{ MeV}$
 $E_n \sim 14 \text{ MeV}$
 $T \sim 10 \div 20 \text{ MeV}$

$$\frac{\partial g_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \kappa \Lambda}{4\pi \epsilon_0^2 m_T} \left[-\frac{\partial}{\partial \underline{r}} \cdot \left(\underline{g}_T \frac{\partial h_F}{\partial \underline{r}} \right) + \frac{1}{2} \frac{\partial}{\partial \underline{r}} \frac{\partial}{\partial \underline{r}} : \left(\underline{g}_T \frac{\partial^2 \underline{g}_F}{\partial \underline{r} \partial \underline{r}} \right) \right]$$

$$\underline{g}_F(\underline{r}) = \int |\underline{r} - \underline{r}'| \underline{g}_F(\underline{r}') d^3 \underline{r}'$$

$$\underline{h}_F(\underline{r}) = \frac{m_T}{\mu} \int \frac{\underline{g}_F(\underline{r}') d^3 \underline{r}'}{|\underline{r} - \underline{r}'|}$$



$E_\alpha = 3.5 \text{ MeV} \Rightarrow v_\alpha \approx 7 \cdot 10^6 \text{ m/s}$ $T \approx 10 \text{ MeV}$

$v_{He} \approx 6 \cdot 10^7 \text{ m/s}$

$\kappa = v / v_{th}$
 $\kappa_e = \frac{v_\alpha}{v_{the}} \ll 1$
 $\kappa_i = \frac{v_\alpha}{v_{thi}} \gg 1$

$v_{thi} < v_\alpha < v_{the}$

$v_{thi} \approx 10^6 \text{ m/s}$

$$h_F = \frac{m_T}{\mu} \frac{n_F}{\sigma} \text{erf} \left(\frac{\sigma}{\sigma_{th}} \right) \quad \left(\text{assume } \sigma \text{ is Maxwellian} \right)$$

$$\frac{\partial h_F}{\partial \sigma} = \frac{\partial}{\partial \sigma} h_F \hat{\sigma}$$

$$\frac{\partial}{\partial \sigma} h_F = \frac{m_T}{\mu} n_F \frac{\partial}{\partial \sigma} \left(\frac{1}{\sigma} \text{erf} \left(\frac{\sigma}{\sigma_{th}} \right) \right)$$

$$S.L. = \frac{d}{dx} \left(\frac{\text{erf}(x)}{x} \right)$$

$$x = \sigma / \sigma_{th}$$

$$\frac{\partial}{\partial \sigma} = \frac{\partial}{\partial x} \cdot \frac{\partial x}{\partial \sigma} = \frac{1}{\sigma_{th}} \frac{\partial}{\partial x}$$

$$\sigma_{th} \cdot x = \sigma \Rightarrow \frac{1}{\sigma} = \frac{1}{x \cdot \sigma_{th}}$$

$$\frac{1}{\sigma} = \frac{1}{\sigma_{th}} \cdot \frac{1}{x}$$

$$\frac{\partial}{\partial \sigma} h_F = \frac{m_T}{\mu} n_F \frac{1}{\sigma_{th}^2} \frac{\partial}{\partial x} \left(\frac{1}{x} \text{erf}(x) \right)$$

$$\alpha_i \gg 1$$

$$\alpha_e \ll 1$$

$$\frac{d}{d\alpha_i} \left(\frac{1}{\alpha_i} \text{erf}(\alpha_i) \right) \approx -\frac{1}{\alpha_i^2}$$

$$\frac{d}{d\alpha_e} \left(\frac{1}{\alpha_e} \text{erf}(\alpha_e) \right) \approx \frac{-4}{3\sqrt{\pi}} \alpha_e$$

$$\frac{\partial f_T}{\partial t} \approx \frac{Z_T^2 e^4 \ln \Lambda n_e}{4\pi \epsilon_0^2 m_T} \frac{\partial}{\partial v} \cdot \left[\left(\frac{Z_i}{\mu_i} \frac{1}{v^{3/2}} \cdot \frac{v_{hi}^2}{v^2} + \frac{4}{3\sqrt{\pi}} \frac{v}{v_{he}^3} \frac{1}{\mu_e} \right) \cdot f_T \right] \hat{v}$$

$$\frac{\partial f_T}{\partial t} \approx \frac{Z_i Z_T^2 e^4 \ln \Lambda n_e}{4\pi \epsilon_0^2 m_T \mu_i} \frac{\partial}{\partial v} \cdot \left[\frac{1}{v^2} f_T \left(1 + \frac{v^3}{v_{ce}^3} \right) \hat{v} \right]$$

$$v_{ce}^3 = \frac{3\sqrt{\pi} Z_i v_{he}^3 \mu_e}{\mu_i}$$

$$\frac{\partial}{\partial v} \cdot [] \Big|_{\text{sph. coordinates}} = \frac{1}{v^2} \frac{\partial}{\partial v} \left[v^2 [] \right]$$

$$\frac{\partial f_T}{\partial t} = \frac{n_c z_i z_T^2 e^4 \ln \Lambda}{4\pi \epsilon_0^2 m_T \mu_i} \underbrace{\frac{1}{v^2} \frac{\partial}{\partial v} \left[\left(1 + \frac{v^3}{v_{ch}^3}\right) f_T \right]}_{S.D.} + S(v)$$

$d+1t \rightarrow d+n$

S.D.

$$S(v) = \frac{S_0}{4\pi v^2} \delta(v-v_0) \quad v_0 = \left(\frac{2E_0}{m\alpha}\right)^{\frac{1}{2}}$$

$$S_0 = \frac{\# \text{ of alphas}}{\text{time} \cdot \text{volume}}$$



$$n = \int f(v) d^3 v$$

$$\frac{\partial}{\partial t} \underbrace{\int f_T d^3 v}_{\frac{\partial n}{\partial t}} = \left(\right) \int d^3 v \frac{1}{v^2} \frac{\partial}{\partial v} \left(\left(1 + \frac{v^3}{v_{ch}^3}\right) f_T \right) + \frac{S_0}{4\pi} \int d^3 v \frac{1}{v^2} \delta(v-v_0)$$

$d^3 v = 4\pi v^2 dv$

$$\frac{\partial n}{\partial t} = \left(\int dV \frac{\partial}{\partial t} \left[\left(1 + \frac{v^2}{v_{cn}^2} \right) \cdot f \right] + \frac{S_0}{4\pi} \int dV 4\pi \delta(v-v_0) \right) = S_0$$

$$\int \left(1 + \frac{v^2}{v_{cn}^2} \right) \Big|_0^{+\infty}$$

$$\frac{\partial n}{\partial t} = S_0$$

Added a loss term

→ it acts once particles are thermalized
 ↘ has the same strength as the source term

$$\mathcal{L}(v) = - \frac{S_0}{4\pi v^2} \delta(v - v_{th})$$

$\frac{\partial f_r}{\partial t} \approx 0$ (steady state)

$$\frac{n_e z_i z_T^2 e^4 k_B \Lambda}{4\pi \epsilon_0^2 m_T \mu_i} \frac{1}{r^2} \frac{\partial}{\partial r} \left[\left(1 + \frac{v^3}{v_{cn}^3}\right) f_T \right] = \frac{S_0}{4\pi r^2} \left[\delta(r - r_{in}) - \delta(r - r_0) \right]$$

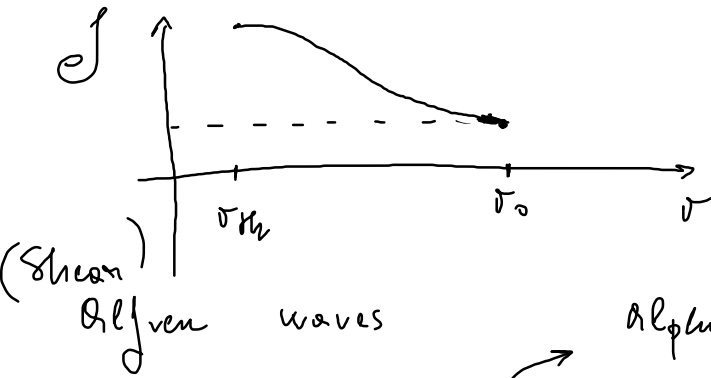
$$\int_0^r \frac{\partial}{\partial r} \left[\left(1 + \frac{v^3}{v_{cn}^3}\right) f_T \right] dr = \frac{S_0}{4\pi} \int_0^r dr \left[\delta(r - r_{in}) - \delta(r - r_0) \right]$$

$$\int_0^r \dots \quad r > r_0 : 0$$

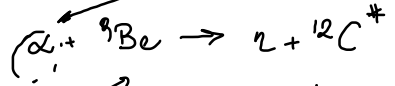
$$\int_0^r \dots \quad r < r_{in} : 0$$

$$r_{in} < r < r_0 \quad \int = 1$$

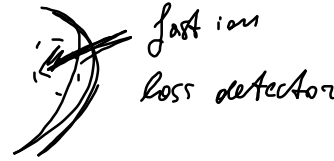
$$\left(1 + \frac{v^3}{v_{cn}^3}\right) f_r(r) = \frac{S_0}{4\pi} \Rightarrow f_r(r) \propto \frac{1}{1 + \frac{v^3}{v_{cn}^3}}$$



Nuclear emission



natural occurring isotope



Alpha part.

excite Alf. waves

Alfven waves

Necessary condition:

$$\omega = n\omega_\phi + p\omega_b$$

ω → wave frequency ω_ϕ → toroidal precession freq. ω_b → bounce freq.