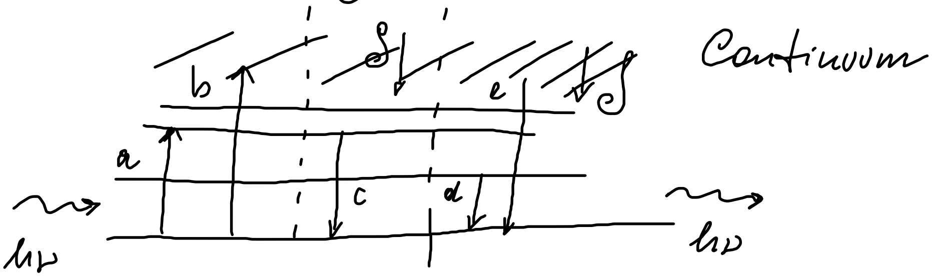
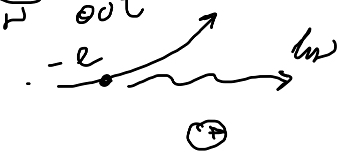
Radiation from plasmas

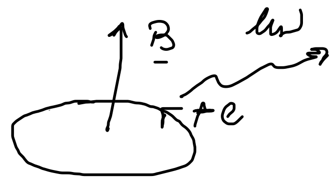


hν in	hν in	nothing in
nothing out	hν out	hν out



f : Bremsstrahlung

Cyclotron radiation



Emission of radiation from a free charge

$$\underline{\nabla} \cdot \underline{E} = \rho/\varepsilon_0 \qquad \underline{\nabla} \times \underline{E} = -\frac{\partial \underline{B}}{\partial t}$$

$$\underline{\nabla} \cdot \underline{B} = 0 \qquad \underline{\nabla} \times \underline{B} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}$$

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial}{\partial t}\underline{A} \qquad \underline{B} = \underline{\nabla} \times \underline{A}$$

$$\underline{\nabla} \times \underline{B} = \underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \underline{\nabla}(\underline{\nabla} \cdot \underline{A}) - \nabla^2 \underline{A} = \mu_0 \underline{j} + \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t} =$$

Ampere-Maxwell

$$= \mu_0 \underline{j} + \varepsilon_0 \mu_0 \frac{\partial}{\partial t}\left(-\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t}\right)$$

Lorentz gauge: $\underline{\nabla} \cdot \underline{A} + \varepsilon_0 \mu_0 \frac{\partial \phi}{\partial t} = 0 \implies -\nabla^2 \underline{A} = \mu_0 \underline{j} - \varepsilon_0 \mu_0 \partial^2 \underline{A}/\partial t^2$

$$\nabla^2 \underline{A} - \frac{1}{c^2}\frac{\partial^2 \underline{A}}{\partial t^2} = -\mu_0 \underline{j}$$

$$c \overset{def}{=} \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\underline{\nabla} \cdot \underline{E} = \underline{\nabla} \cdot \left(-\underline{\nabla}\phi - \frac{\partial}{\partial t}\underline{A} \right) = -\nabla^2\phi - \frac{\partial\, \underline{\nabla} \cdot \underline{A}}{\partial t} = -\nabla^2\phi + \varepsilon_0\mu_0 \frac{\partial^2 \phi}{\partial t^2} = \rho/\varepsilon_0$$

Lorentz gauge

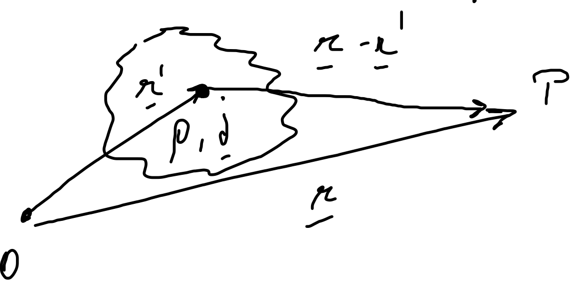
Poisson's theorem

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}$$

$$\underline{\nabla} \cdot \underline{A} = -\varepsilon_0\mu_0 \frac{\partial \phi}{\partial t}$$

$$\underline{A}(\underline{r},t) = \frac{\mu_0}{4\pi}\int_{\substack{\text{volume}\\ \text{where } \underline{j}\neq \underline{0}}} \frac{\underline{j}\left(\underline{r}', t - \frac{|\underline{r}-\underline{r}'|}{c}\right) d^3\underline{r}'}{|\underline{r}-\underline{r}'|}$$

$$\phi(\underline{r},t) = \frac{1}{4\pi\varepsilon_0}\int_{\substack{\text{Volume}\\ \text{where } \rho\neq 0}} \frac{\rho\left(\underline{r}', t - \frac{|\underline{r}-\underline{r}'|}{c}\right) d^3\underline{r}'}{|\underline{r}-\underline{r}'|}$$



$\frac{|\underline{r}-\underline{r}'|}{c}$: time it takes for a wave travelling at the speed c to move from the source to P

Free charge q with an arbitrary motion at $\underline{r}_0 = (x_0, y_0, z_0)$

$$\rho(\underline{r}', t) = q\,\delta(x'-x_0)\,\delta(y'-y_0)\,\delta(z'-z_0)$$

$$\underline{j}(\underline{r}', t) = q\,\underline{v}_0\,\delta(x'-x_0)\,\delta(y'-y_0)\,\delta(z'-z_0)$$

$$\phi(\underline{r}, t) = \frac{q}{4\pi\varepsilon_0}\int \frac{\delta\left(x' - x_0\left(t - \frac{|\underline{r}-\underline{r}'|}{c}\right)\right)\delta\left(y' - y_0\left(t - \frac{|\underline{r}-\underline{r}'|}{c}\right)\right)\delta\left(z' - z_0\left(t - \frac{|\underline{r}-\underline{r}'|}{c}\right)\right)}{|\underline{r}-\underline{r}'|}\,dx'\,dy'\,dz'$$

$$\begin{cases} X = x' - x_0\left(t - \frac{|\underline{r} - \underline{r}'|}{c}\right) \\ Y = y' - y_0\left(t - \frac{|\underline{r} - \underline{r}'|}{c}\right) \\ Z = z' - z_0\left(t - \frac{|\underline{r} - \underline{r}'|}{c}\right) \end{cases}$$

$$|\underline{r} - \underline{r}'| = \sqrt{(x - x'_{(t)})^2 + (y - y'_{(t)})^2 + (z - z'_{(t)})^2}$$

$$dX\,dY\,dZ = \left\| \frac{\partial(X,Y,Z)}{\partial(x',y',z')} \right\| dx'\,dy'\,dz'$$

$$\tau \overset{def}{=} t - \frac{|\underline{r} - \underline{r}'|}{c}$$

$$\frac{\partial X}{\partial x'} = 1 - \frac{\partial x_0}{\partial x'} = 1 - \frac{\partial x_0}{\partial \tau} \cdot \frac{\partial \tau}{\partial x'} = 1 - v_{0x}\left(-\frac{1}{c}\right)\frac{\partial}{\partial x'}\sqrt{(\quad)}$$

$$= 1 - \frac{v_{0x}}{c}\frac{(x - x')}{|\underline{r} - \underline{r}'|}$$

$$\frac{\partial X}{\partial y'} = -\frac{\partial x_0}{\partial \tau} \cdot \frac{\partial \tau}{\partial y'} = -\frac{v_{0x}}{c}\frac{(y - y')}{|\underline{r} - \underline{r}'|}$$

$$\frac{\partial X}{\partial z'} = -\frac{\partial x_0}{\partial \tau} \cdot \frac{\partial \tau}{\partial z'} = -\frac{v_{0x}}{c}\frac{(z - z')}{|\underline{r} - \underline{r}'|}$$

$$\frac{\partial Y}{\partial x'} = -\frac{v_{0y}}{c}\frac{(x-x')}{|\underline{r}-\underline{r}'|} \qquad \frac{\partial Y}{\partial y'} = 1 - \frac{v_{0y}}{c}\frac{(y-y')}{|\underline{r}-\underline{r}'|}$$

$$\frac{\partial Y}{\partial z'} = -\frac{v_{0y}}{c}\frac{(z-z')}{|\underline{r}-\underline{r}'|} \qquad \frac{\partial Z}{\partial x'} = -\frac{v_{0z}}{c}\frac{(x-x')}{|\underline{r}-\underline{r}'|}$$

$$\frac{\partial Z}{\partial y'} = -\frac{v_{0z}}{c}\frac{(y-y')}{|\underline{r}-\underline{r}'|} \qquad \frac{\partial Z}{\partial z'} = 1 - \frac{v_{0z}}{c}\frac{(z-z')}{|\underline{r}-\underline{r}'|}$$

$$\left\| \frac{\partial(X,Y,Z)}{\partial(x',y',z')} \right\| = \left\| \begin{matrix} 1 - \frac{v_{0x}}{c}\frac{(x-x')}{|\underline{r}-\underline{r}'|} & -\frac{v_{0x}}{c}\frac{(y-y')}{|\underline{r}-\underline{r}'|} & -\frac{v_{0x}}{c}\frac{(z-z')}{|\underline{r}-\underline{r}'|} \\ - & 1 - \frac{v_{0y}}{c}\frac{(y-y')}{|\underline{r}-\underline{r}'|} & - \\ - & - & 1 - \frac{v_{0z}}{c}\frac{(z-z')}{|\underline{r}-\underline{r}'|} \end{matrix} \right\| = \ldots =$$

$$= 1 - \frac{\underline{v}_0 \cdot (\underline{r}-\underline{r}')}{c\,|\underline{r}-\underline{r}'|}$$

$$\phi(\underline{r},t) = \frac{q}{4\pi\varepsilon_0} \int \frac{\delta(x)\,\delta(y)\,\delta(z)\; dx\,dy\,dz}{|\underline{r}-\underline{r}'| \left(1 - \frac{\underline{v}_0 \cdot (\underline{r}-\underline{r}')}{c\,|\underline{r}-\underline{r}'|} \right)}$$

If $x = 0$ then $x' = x_0\left(t - \frac{|\underline{r}-\underline{r}_0|}{c}\right)$

similarly for y' and z'

$$t_{ret} = t - \frac{|\underline{r}-\underline{r}_0|}{c}$$

$$\phi(\underline{r},t) = \frac{q}{4\pi\varepsilon_0} \frac{1}{|\underline{r}-\underline{r}_0(t_{ret})|} \cdot \frac{1}{1 - \frac{\underline{v}_0 \cdot (\underline{r}-\underline{r}_0(t_{ret}))}{c\,|\underline{r}-\underline{r}_0(t_{ret})|}}$$

If $v_0 \to 0$

$$\Rightarrow \phi(\underline{r},t) = \frac{q}{4\pi\varepsilon_0} \frac{1}{|\underline{r}-\underline{r}_0|}$$

$$\underline{A}(\underline{r},t) = \frac{\mu_0}{4\pi} \frac{\underline{v}_0(t_{ret})}{|\underline{r} - \underline{r}_0(t_{ret})|} \frac{q}{1 - \frac{\underline{v}_0 \cdot (\underline{r} - \underline{r}_0(t_{ret}))}{c\,|\underline{r} - \underline{r}_0(t_{ret})|}}$$

$\underline{B}$ $\underline{E}$