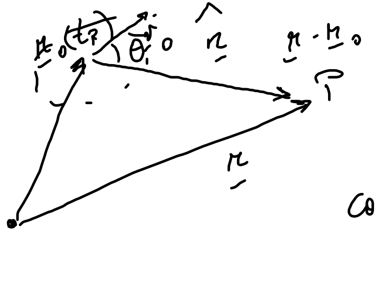
$$\underline{B}^{(R)} \approx \frac{\mu_0}{4\pi c} \frac{q}{|\underline{r} - \underline{r}_0(t_R)|} \dot{\underline{v}}_0 \times \hat{\underline{n}}$$

$$\underline{E}^{(R)} \approx \frac{\mu_0 q}{4\pi |\underline{r} - \underline{r}_0(t_R)|} (\dot{\underline{v}}_0 \times \hat{\underline{n}}) \times \hat{\underline{n}} = (\underline{B}^{(R)} \times \hat{\underline{n}}) \cdot c$$

$$\underline{S} = \frac{1}{\mu_0} (\underline{E}^{(R)} \times \underline{B}^{(R)}) \approx \frac{\mu_0^2 q^2}{[4\pi |\underline{r} - \underline{r}_0(t_R)|]^2} \frac{1}{\mu_0 c} (\dot{\underline{v}}_0 \times \hat{\underline{n}}) \times [(\dot{\underline{v}}_0 \times \hat{\underline{n}}) \times \hat{\underline{n}}]$$

$$(\underline{C} \times \underline{B}) \times \underline{A} = (\underline{A} \cdot \underline{C}) \underline{B} - (\underline{A} \cdot \underline{B}) \underline{C} = \left(\frac{\mu_0 q}{4\pi |\underline{r} - \underline{r}_0(t_R)|} \right)^2 \cdot \frac{1}{\mu_0 c} |\dot{\underline{v}}_0 \times \hat{\underline{n}}|^2 \hat{\underline{n}}$$

$$\underline{A} = \underline{C} = \dot{\underline{v}}_0 \times \hat{\underline{n}} \qquad \underline{B} = \hat{\underline{n}}$$



$$d \overset{\text{def}}{=} |\underline{r} - \underline{r}_0|$$

$$|\dot{\underline{v}}_0 \times \hat{\underline{n}}|^2 = \dot{v}_0^2 \sin^2\theta$$

$$\cos\theta = \frac{\dot{\underline{v}}_0 \cdot \hat{\underline{n}}}{\dot{v}_0}$$

$$P(t) = \int \underline{S} \cdot d\underline{A} = \int \left(\frac{\mu_0 q}{4\pi \cancel{d}}\right)^2 \frac{1}{\mu_0 c} \dot{v}_0^2 \sin^2\theta \, \cancel{d}^2 \sin\theta \, d\theta \, d\varphi$$

$$= \frac{\mu_0 q^2}{16\pi^2 c} \dot{v}_0^2 \cdot 2\pi \int_0^{\pi} d\theta \, \sin^3\theta$$

$$\int_0^{\pi} \sin^3\theta \, d\theta = \frac{2}{3} \int_0^{\pi} \sin\theta \, d\theta = 4/3$$

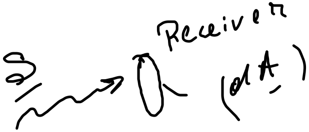
$$\int_0^{\pi} \sin^{(n)}\theta \, d\theta = \frac{n-1}{n} \int_0^{\pi} \sin^{(n-2)}\theta \, d\theta \qquad n \geq 2$$

$$\boxed{P(t) = \frac{\mu_0 q^2 \dot{v}_0^2}{6\pi c}}$$

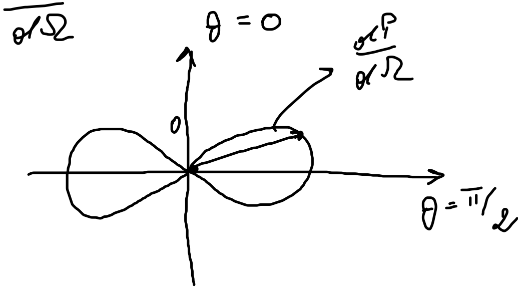
(Total power radiated at any angle)

$$P(t) = \int_{4\pi} \frac{dP}{d\Omega} \cdot d\Omega = \int \underline{S} \cdot d\underline{A} \qquad dA = d^2 \underbrace{\sin\theta \, d\theta \, d\varphi}_{d\Omega}$$

$$= \int |\underline{S}| \, d^2 \, d\Omega \Rightarrow \frac{dP}{d\Omega} = |\underline{S}| \cdot d^2$$



$$\boxed{\frac{dP}{d\Omega} = \frac{\mu_0 q^2}{16\pi^2 c} \dot{v}_0^2 \sin^2\theta}$$



$$W = \int_{-\infty}^{+\infty} P(t)\,dt \qquad \frac{dW}{d\Omega} = \int_{-\infty}^{+\infty} \frac{dP(t)}{d\Omega}\,dt$$

$$\hat{\underline{n}} \times \underline{E}^{(R)} = \hat{\underline{n}} \times c\left(\underline{B}^{(R)} \times \hat{\underline{n}}\right) = c\left[\cancel{\left(-\hat{\underline{n}} \cdot \underset{\underset{\dot{\underline{v}}_0 \times \hat{\underline{n}}}{\wr}}{\underline{B}^{(R)}}\right)} \cdot \hat{\underline{n}} + \underline{B}^{(R)}\right] = c\underline{B}^{(R)}$$

$$\Rightarrow \underline{B}^{(R)} = \frac{1}{c}\left(\hat{\underline{n}} \times \underline{E}^{(R)}\right)$$

$$\underline{S} = \frac{1}{\mu_0}\left(\underline{E}^{(R)} \times \underline{B}^{(R)}\right) = \frac{1}{\mu_0 c}\left(\underline{E}^{(R)} \times \left(\hat{\underline{n}} \times \underline{E}^{(R)}\right)\right) = \frac{1}{\mu_0 c}\left|\underline{E}^{(R)}\right|^2$$

$$\frac{dP}{d\Omega} = \frac{d^2}{\mu_0 c}\left|\underline{E}^{(R)}\right|^2 \qquad \frac{dW}{d\Omega} = \frac{d^2}{\mu_0 c}\int_{-\infty}^{+\infty}\left|\underline{E}^{(R)}\right|^2 dt =$$

$$= \frac{\mu_0 q^2}{16\pi^2 c}\int_{-\infty}^{+\infty} dt\,\left|\hat{\underline{n}} \times \left(\hat{\underline{n}} \times \dot{\underline{v}}_0\right)\right|^2$$

$$\frac{dW}{d\Omega} = \frac{\mu_0 q^2}{16\pi^2 c} \int_{-\infty}^{+\infty} dt \, \left| \underline{\hat{n}} \times (\underline{\hat{n}} \times \underline{\dot{v}}_0) \right|^2$$

$$\underline{A}(t) \stackrel{def}{=} \sqrt{\frac{\mu_0}{c}} \frac{q}{4\pi} \underline{\hat{n}} \times (\underline{\hat{n}} \times \underline{\dot{v}}_0)$$

$$\frac{dW}{d\Omega} = \int_{-\infty}^{+\infty} |\underline{A}(t)|^2 \, dt \stackrel{def.}{=} \int_0^{+\infty} \frac{d^2 W}{d\Omega \, d\omega} d\omega$$

$\to$ freq distribution of energy

Parseval theorem

$\underline{\tilde{A}}(\omega)$ is the Fourier transform of $\underline{A}(t)$

$$\underline{\tilde{A}}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt \, e^{i\omega t} \underline{A}(t)$$

$$\int_{-\infty}^{+\infty} |\underline{A}(t)|^2 dt = \int_{-\infty}^{+\infty} d\omega \, |\underline{\tilde{A}}(\omega)|^2$$

$\underset{\sim}{A}(t)$ is a real function $\Rightarrow$ $\underset{\sim}{A}(t) = \underset{\sim}{A}^*(t)$

$$\tilde{\underset{\sim}{A}}(-\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt\, e^{-i\omega t}\, \underset{\sim}{A}(t) = \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, A^*(t) \right]^*$$

$$= \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dt\, e^{i\omega t}\, \underset{\sim}{A}(t) \right]^* = \tilde{\underset{\sim}{A}}^*(\omega)$$

$$\Rightarrow \tilde{\underset{\sim}{A}}(-\omega) = \tilde{\underset{\sim}{A}}^*(\omega) \Rightarrow |\tilde{\underset{\sim}{A}}(-\omega)|^2 = |\underset{\sim}{A}^*(\omega)|^2 = |\underset{\sim}{A}(\omega)|^2$$

$$\int_{-\infty}^{+\infty} d\omega\, |\tilde{\underset{\sim}{A}}(\omega)|^2 = \int_{-\infty}^{0} d\omega\, |\tilde{\underset{\sim}{A}}(\omega)|^2 + \int_{0}^{+\infty} d\omega\, |\tilde{\underset{\sim}{A}}(\omega)|^2$$

$\omega' = -\omega$

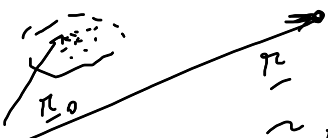
$$= -\int_{+\infty}^{0} d\omega'\, |\tilde{\underset{\sim}{A}}(-\omega')|^2 + // = 2\int_{0}^{+\infty} |\tilde{\underset{\sim}{A}}(\omega)|^2\, d\omega$$

$$\frac{dW}{d\Omega} = \int_0^{+\infty} \frac{d^2W}{d\Omega\, d\omega}\, d\omega = 2\int_0^{+\infty} d\omega \, |\tilde{\underline{A}}(\omega)|^2$$

$$\Rightarrow \frac{d^2W}{d\Omega\, d\omega} = 2\,|\tilde{\underline{A}}(\omega)|^2$$

$$\frac{d^2W}{d\Omega\, d\omega} = \cancel{2}\, \frac{\mu_0}{c}\, \frac{q^2}{16\pi^2}\, \frac{1}{\cancel{2}\pi} \left| \int_{-\infty}^{+\infty} dt\, e^{i\omega t} \left(\hat{\underline{r}} \times (\hat{\underline{r}} \times \dot{\underline{v}}_0(t_r))\right) \right|^2$$

$$t_r = t - \frac{|\underline{r} - \underline{r}_0(t_r)|}{c} \Rightarrow t = t_r + \frac{|\underline{r} - \underline{r}_0(t_r)|}{c} \qquad dt \simeq dt_r \text{ if } \frac{v_r}{c} \ll 1$$



$$r \gg r_0$$

$$|\underline{r} - \underline{r}_0| = (r^2 + r_0^2 - 2\,\underline{r}\cdot\underline{r}_0)^{\frac{1}{2}} = r\left(1 + \frac{r_0^2}{r^2} - 2\,\frac{\underline{r}\cdot\underline{r}_0}{r^2}\right)^{\frac{1}{2}}$$

$$\simeq r\left(1 + O\left(\left(\frac{r_0}{r}\right)^2\right) - 2\cdot \frac{(\underline{r} - \underline{r}_0)\cdot \underline{r}_0}{|\underline{r} - \underline{r}_0|\cdot r}\right)^{\frac{1}{2}} \simeq r - \hat{\underline{r}}\cdot \underline{r}_0$$

$$\frac{d^2W}{d\Omega\, d\omega} = \frac{\mu_0 q^2}{16\pi^3 c}\left| \int_{-\infty}^{+\infty} dt_r\, e^{i\omega\left(t_r + \frac{r}{c} - \frac{\hat{\underline{n}}\cdot\underline{r}_0}{c}\right)} \underbrace{\frac{d}{dt_r}\left(\hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0)\right)}_{} \right|^2$$

$$= \hat{\underline{n}} \times (\hat{\underline{n}} \times \dot{\underline{v}}_0)$$

$$\frac{d}{dt_r}\left[\hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0)\right] = \cancel{\frac{d\hat{\underline{n}}}{dt_r} \times (\hat{\underline{n}} \times \underline{v}_0)} + \hat{\underline{n}} \times \left(\cancel{\frac{d\hat{\underline{n}}}{dt_r}} \times \underline{v}_0\right)$$

$$+ \hat{\underline{n}} \times (\hat{\underline{n}} \times \dot{\underline{v}}_0)$$

Integrate by parts:

$$f = e^{i\omega\left(t_r - \frac{\hat{\underline{n}}\cdot\underline{r}_0}{c}\right)}$$

$$g' = \frac{d}{dt_r}\left(\hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0)\right)$$

$$\int f g'\, dx = fg - \int f' g\, dx$$

$$\int_{-\infty}^{+\infty} dt_r \, e^{i\omega\left(t_r - \frac{\hat{\underline{n}}\cdot \underline{r}_0}{c}\right)} \cdot \frac{d}{dt_r}\left(\hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0)\right) =$$

$$= \cancel{e^{i\omega\left(t_r - \frac{\hat{\underline{n}}\cdot \underline{r}_0}{c}\right)} \hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0)}\Bigg|_{-\infty}^{+\infty} - i\omega \int_{-\infty}^{+\infty} dt_r \, e^{i\omega\left(t_r - \frac{\hat{\underline{n}}\cdot \underline{r}_0}{c}\right)} \left(\hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0)\right)$$

$$\frac{d^2 W}{d\omega \, d\Omega} = \frac{\mu_0 q^2 \omega^2}{16\pi^3 c} \left| \int_{-\infty}^{+\infty} dt_r \, e^{i\omega\left(t_r - \frac{\hat{\underline{n}}\cdot \underline{r}_0(t_r)}{c}\right)} \hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0(t_r)) \right|^2$$