$$P(t) = \frac{\mu_0 q^2}{6\pi c} \dot{v}_0^2$$

$$\frac{d^2 W}{d\Omega\, d\omega} = \frac{\mu_0 q^2 \omega^2}{16\pi^3 c} \left| \int_{-\infty}^{+\infty} dt_r\, e^{i\omega\left(t_r - \frac{\hat{n}\cdot r_0(t_r)}{c}\right)} \left(\hat{n} \times (\hat{n} \times \dot{v}_0(t_r)\right) \right|^2$$

Larmor motion

$$\dot{v}_0 = \frac{F}{m} = \frac{q v_\perp B}{m} \qquad \dot{v}_0^2 \propto \frac{1}{m^2}$$

Neglect cyclotron emission from ion

$$P(t) = \frac{\mu_0 q^2}{6\pi c} \frac{q^2 v_\perp^2 B^2}{m^2}$$

$$W = P(t)\cdot T_L = \frac{\mu_0 q^{\cancel{4}\,3}}{\cancel{6\pi}\,3 c} \frac{v_\perp^2 B^{\cancel{2}\,1}}{m^{\cancel{2}}} \frac{\cancel{2\pi}}{\cancel{q B}} \cancel{m} = \frac{2\mu_0 q^3 B}{3c\, m^2} \underbrace{\frac{1}{2} m v_\perp^2}_{W_\perp} = W_\perp \cdot \frac{2\mu_0 q^3 B}{3c\, m^2}$$

Larmor period

10^{-12}

$12\ B \sim 1\ T$

Electrons: $$f_e(\underline{v}) = n_e \left(\frac{m_e}{2\pi T_e}\right)^{\frac{3}{2}} \exp\left(-\frac{m v^2}{2T_e}\right)$$

$$\frac{dP}{dV} = \int_{\text{all velocities}} \underbrace{P(t)}_{\text{depends on } v_\perp} \underbrace{f(\underline{v})\, d^3\underline{v}}$$

$$\frac{\#\text{ of electrons that have } v = v_\perp}{\text{volume}}$$

Use $(v_\parallel, v_\perp, \varphi)$

$0 < \varphi < 2\pi \qquad 0 < v_\perp < +\infty$

$-\infty < v_\parallel < +\infty$

$$d^3\underline{v} = d\varphi\, dv_\parallel\, v_\perp\, dv_\perp$$

$$\frac{dP}{dV} = \int \frac{\mu_0 q^4}{6\pi c} \frac{B^2}{m^2} v_\perp^2\, v_\perp\, d\varphi\, dv_\parallel\, dv_\perp\, n_e \left(\frac{m_e}{2\pi T_e}\right)^{\frac{3}{2}} \exp\left(-\frac{m v_\parallel^2}{2T_e}\right) \cdot \exp\left(-\frac{m v_\perp^2}{2T_e}\right)$$

Исп $\quad J_{2n+1} = \int_0^{+\infty} dx\, x^{2n+1} e^{-ax^2} = \frac{n!}{2} \frac{1}{a^{n+1}}$

$$\int_0^{+\infty} dv_\perp\, v_\perp^3 \exp\left(-\frac{mv_\perp^2}{2T_e}\right) = \frac{2T^2}{m^2}$$

$$n = 1$$

$$a = \frac{m}{2T_e}$$

$$\int_{-\infty}^{+\infty} dv_\parallel \exp\left(-\frac{mv_\parallel^2}{2T_e}\right) = \left(\frac{2\pi T_e}{m}\right)^{\frac{1}{2}}$$

$$\frac{dW}{dV} = \frac{\mu_0 q^4}{3\pi c m^3} n_e B^2 \cdot T$$

$$\frac{dW}{dV} \simeq 6.2 \cdot n_{e_{20}} B^2[T]\, T[keV] \quad kW/m^3$$

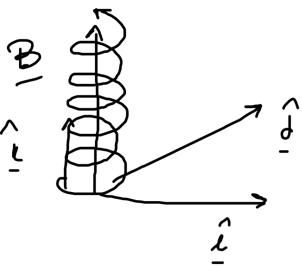
$$n_{e_{20}} = n_e / 10^{20}\, m^{-3}$$

$T \sim 10$ keV $\quad B \sim 1$ T $\quad n_{e_{20}} \sim 1 \quad \sim 0.1\, MW/m^3$

$dW/dV \sim 100\, kW/m^3$

Consider the Larmor motion

$$\underline{v}_0 = v_\perp \cos(\omega_L t_r)\,\hat{\underline{i}} + v_\perp \sin(\omega_L t_r)\,\hat{\underline{j}} + v_\parallel\,\hat{\underline{k}}$$

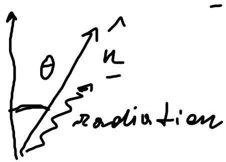


$$\underline{r}_0 = \frac{v_\perp}{\omega_L}\sin(\omega_L t_r)\,\hat{\underline{i}} - \frac{v_\perp}{\omega_L}\cos(\omega_L t_r)\,\hat{\underline{j}} + v_\parallel t_r\,\hat{\underline{k}}$$

$$\hat{\underline{n}} = \cos\theta\,\hat{\underline{k}} + \sin\theta\,\hat{\underline{i}}$$

$$\hat{\underline{n}} \times (\hat{\underline{n}} \times \underline{v}_0) = \dots = -v_\perp \sin(\omega_L t_r)\,\hat{\underline{j}} + \hat{\underline{i}}\left(\sin\theta\cos\theta\, v_\parallel - v_\perp \cos^2\theta \cos(\omega_L t_r)\right)$$

$$+ \hat{\underline{k}}\left(-v_\parallel \sin^2\theta + \sin\theta\cos\theta\cos(\omega_L t_r)\, v_\perp\right)$$



$$\beta_= = \frac{v_=}{c} \qquad \beta_\perp = v_\perp / c$$

$$i\omega\left(t_r - \frac{1}{c}\,\hat{\underline{n}}\cdot\underline{r}_0\right) =$$

$$= i\left(\omega t_r\left(1-\beta_{\parallel}\cos\theta\right) - \frac{\beta_\perp}{\omega_L}\sin(\omega_L t)\sin\theta\cdot\omega\right)$$

$$\underline{J} = \int dt_r\, e^{i\left[\omega t_r\left(1-\beta_{\parallel}\cos\theta\right) - \frac{\beta_\perp \omega}{\omega_L}\sin(\omega_L t_r)\sin\theta\right]} \cdot$$

$$\cdot\left[\hat{\underline{i}}\left(\sin\theta\cos\theta\, v_= - v_\perp\cos^2\theta\cos(\omega_L t_r)\right) - \hat{\underline{j}}\, v_\perp\sin(\omega_L t_r) + \right.$$

$$\left. + \hat{\underline{k}}\left(-v_{\parallel}\sin^2\theta + \sin\theta\cos\theta\cos(\omega_L t_r)v_\perp\right)\right]$$

Identity:

$$e^{-i\xi \sin\phi} = \sum_{m=-\infty}^{+\infty} e^{-im\phi} J_m(\xi) \qquad J_m : \text{Bessel function of order } m$$

$$\phi = \omega_L t \qquad \xi = \frac{\omega}{\omega_L} \beta_\perp \sin\theta$$

$$\frac{\partial}{\partial \xi}\left(e^{-i\xi \sin\phi}\right) = -i \sin\phi \, e^{-i\xi\sin\phi} = \sum_{m=-\infty}^{+\infty} e^{-im\phi} J_m'(\xi)$$

$$\frac{\partial}{\partial \phi}\left(e^{-i\xi \sin\phi}\right) = -i\xi \cos\phi \, e^{-i\xi\sin\phi} = \sum_{m=-\infty}^{+\infty} (-im) e^{-im\phi} J_m(\xi)$$

$$\mathcal{J} = \sum_{n=-\infty}^{+\infty} \hat{\underline{i}} \left[\left(\sin\theta\cos\theta\, v_{\parallel} - v_{\perp} \cos^2\theta \, \frac{n}{\xi} \right) J_n(\xi) \right] + \hat{\underline{j}} \, (-i)\, v_{\perp} J_n'(\xi)$$

$$+ \hat{\underline{k}} \left[\left(- v_{\parallel} \sin^2\theta + \sin\theta\cos\theta \, \frac{n}{\xi} \, v_{\perp} \right) J_n(\xi) \right] \cdot \int_{-\infty}^{+\infty} dt_r \, e^{i\left[(1-\beta_{\parallel}\cos\theta)\omega - n\omega_L\right] t_r}$$

Identity: $$\int_{-\infty}^{+\infty} e^{i\alpha t} \, dt = 2\pi \, \delta(\alpha)$$

$$\mathcal{J} \propto \delta\left((1-\beta_{\parallel}\cos\theta)\omega - n\omega_L \right)$$

$$\Rightarrow \quad \omega = \frac{n\,\omega_L}{\underbrace{1-\beta_{\parallel}\cos\theta}_{\text{Doppler}}} \qquad n = 1, 2, 3 \ldots$$

If $\theta = \pi/2$

$$\underline{\omega = n\,\omega_L}$$