$$P(t) = \frac{\mu_0}{6\pi c} q^2 \dot{v}^2$$

$$F = \frac{q_1 q_2}{4\pi \varepsilon_0} \frac{1}{r^2} \Rightarrow a \propto \frac{1}{m}$$

$$a = \frac{q_1 q_2}{4\pi \varepsilon_0} \frac{1}{r^2(t)}$$

e - e

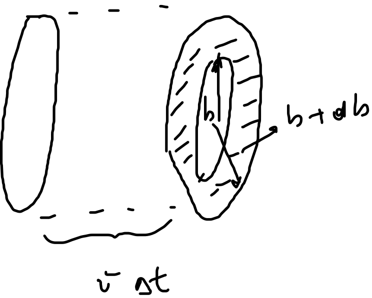
e - i

$$W = \int_{-\infty}^{+\infty} P(t)\, dt$$

$$W \approx P \cdot \Delta t$$

$$\Delta t \sim \frac{2b}{v}$$

$$W \approx \frac{\mu_0 e^2}{6\pi c} \frac{Z^2 e^4}{16\pi^2 \varepsilon_0^2} \frac{1}{b^{\cancel{4}\,3}} \frac{2\cancel{b}}{v}$$



$$\# \text{ collisions} =$$

$$= \underbrace{2\pi b\, db\, v \Delta t}_{\text{Volume where collisions take place}} \cdot n_i$$

$$\dot{W}_{TOT} = W \cdot \frac{\# \text{collisions}}{\text{time}} = 2\pi n_i v \frac{e^6 Z^2}{48\pi^3 c^3 \varepsilon_0^3 m_e^2} \frac{1}{v} \int_{b_{min}}^{+\infty} \frac{db}{b^3} \cdot b^2$$

$$-\frac{1}{b}\Big|_{b_{min}}^{+\infty} = \frac{2 m_e v}{\hbar}$$

$$\Delta x \Delta p \geq \hbar/2 \qquad \Delta p \sim m_e v \qquad \Delta x \sim \frac{\hbar}{2 m_e v}$$

$$\dot{W}_{TOT} = \frac{n_i v e^6 Z^2}{12\pi c^3 \varepsilon_0^3 m_e \hbar}$$

$$f_e(v) = n_e \left(\frac{m_e}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-\frac{m v^2}{2 T_e}\right)$$

$$\left.\frac{dW}{d\nu}\right|_{Brems} = \int \dot{W}_{TOT} \cdot \underset{at\ v}{\#electrons}/volume$$

$$= \int \dot{W}_{TOT}\, n_e \left(\frac{m_e}{2T_e\pi}\right)^{\frac{3}{2}} \exp\left(\frac{-mv^2}{2T_e}\right) d^3\underline{v} =$$

$$= \frac{n_i n_e e^6 Z^2 \cancel{4\pi}^{1}}{\cancel{12\pi}_{3} c^3 \varepsilon_0^3 m_e h} \underbrace{\int_0^{+\infty} dv\, v^3 \left(\frac{m_e}{2T_e\pi}\right)^{\frac{3}{2}} \exp\left(\frac{-mv^2}{2T_e}\right)}_{\frac{1}{2}\left(\frac{m_e}{2\pi T_e}\right)^{\frac{3}{2}}}$$

Full calculation

$$\frac{2^{\frac{1}{2}}}{3\pi^{3/2}} \longrightarrow \frac{2^{\frac{1}{2}}}{3\pi^{5/2}}$$

$$\left.\frac{dW}{d\nu}\right|_{Brem} = \frac{2^{\frac{1}{2}}}{6\pi^{3/2}} \frac{n_e n_i e^6 Z^2 T^{\frac{1}{2}}}{c^3 \varepsilon_0^3 m_e^{3/2} h}$$

$$\left.\frac{dW}{dV}\right|_{\text{Bremss}} = \sum_j \left.\frac{dW}{dV}\right|_j =$$

j : label for an ion species

$$= \sum_j C \cdot T^{\frac{1}{2}} n_e n_j \cdot Z_j^2 = C \cdot T^{\frac{1}{2}} n_e \sum_j n_j Z_j^2 = C \cdot T^{\frac{1}{2}} n_e^2 Z_{eff}$$

Define: $$Z_{eff} = \frac{\sum_j n_j Z_j^2}{\underbrace{\sum_j n_j Z_j}_{= n_e \text{ quasi-neutrality}}} = \frac{\sum_j n_j Z_j^2}{n_e}$$

Hydrogen plasma

$n_H = n_e \quad Z_H = 1 \Rightarrow Z_{eff} = 1$

$$\left.\frac{dW}{dV}\right|_{\text{Bremss}} = C_B \cdot Z_{eff} \cdot n_{20}^2 \cdot T[\text{keV}]^{\frac{1}{2}} \ \text{kW}/\text{m}^3 \qquad C_B = 5.35$$

$n_{20} = n_e / 10^{20}\,\text{m}^{-3}$

$n_{20} \sim 1$

$T_{\text{keV}} \sim 10 - 20$

$\left.\frac{dW}{dV}\right|_{\text{Bremss}} \sim$ tens kW/m^3