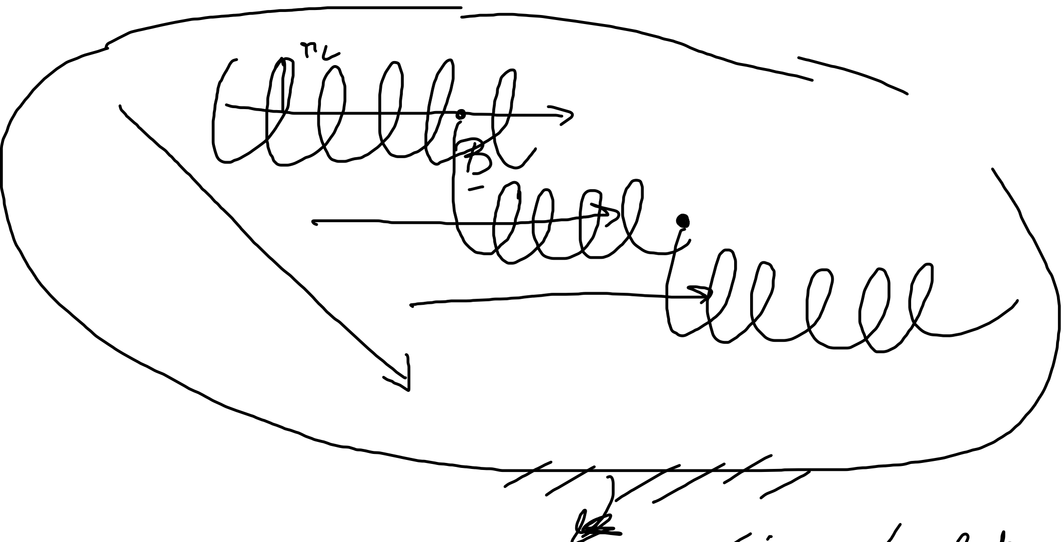
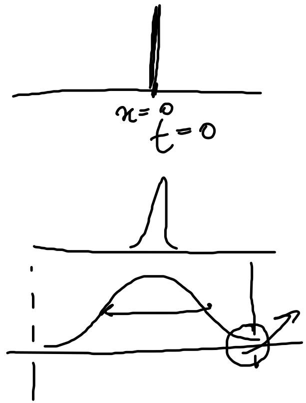


Weakly ionized plasma { ions / electrons
neutrals

n + i collisions are dominant
n + e

Fully " " ion el } Coulomb. coll. " "

1D model for diffusion as a random walk



Δt Δx

$\langle x \rangle = 0$

$\langle x^2 \rangle \neq 0$

Consider $t = N\Delta t$: r displacements to the right $\left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{N-r}$

$N-r$ " to the left $\binom{N}{r}$

$P = ?$ $P_{r,N-r} = \binom{N}{r} \left(\frac{1}{2}\right)^r \cdot \left(\frac{1}{2}\right)^{N-r}$

Total displ. $= \Delta x \cdot r - (N-r)\Delta x =$

$$= r\cdot\Delta x + r\cdot\Delta x - N\cdot\Delta x = (2r-N)\Delta x$$

$r = 0 \dots N$

$$\langle x\rangle = \sum_{\substack{\text{all}\\ \text{possible}\\ \text{displ.}}} \text{displ.} \times (\text{prob. assoc. to displ}) =$$

$$= \sum_{r=0}^{N} (2r-N)\cdot\Delta x \binom{N}{r} \frac{1}{2^r}\,\frac{1}{2^{N-r}}$$

$\frac{1}{2^N}$

$$= \frac{\Delta x}{2^N}\sum_{r=0}^{N}\binom{N}{r}(2r-N)$$

$$\sum_{r=0}^{N} \binom{N}{r} (2r-N)$$

Auxiliary function $F_N(y) \overset{def}{=} \dfrac{(1+y)^N}{2^N y^{N/2}} = \sum_{r=0}^{N} \dfrac{1}{2^N} \dfrac{1}{y^{N/2}} \cdot \binom{N}{r} y^r (1)^{N-r}$

$$= \frac{1}{2^N y^{N/2}} \sum_{r=0}^{N} \binom{N}{r} y^r$$

Newton's binomial expansion

$$(a+b)^N = \sum_{r=0}^{N} \binom{N}{r} a^r b^{N-r}$$

$$= \frac{1}{2^N} \sum_{r=0}^{N} \binom{N}{r} y^{r-\frac{N}{2}}$$

$$\left.\frac{d}{dy} F_N\right|_{y=1} = \left.\frac{1}{2^N} \sum_{r=0}^{N} \binom{N}{r} \left(r - \frac{N}{2}\right) y^{r-\frac{N}{2}-1}\right|_{y=1} = \frac{1}{2^N} \sum_{r=0}^{N} \binom{N}{r} \left(r - \frac{N}{2}\right)$$

$$= \frac{1}{2^{N+1}} \sum_{r=0}^{N} \binom{N}{r} (2r-N)$$

$$F_N(y) = \frac{(1+y)^N}{2^N y^{N/2}}$$

$$\frac{d}{dy} F_N(y)\Big|_{y=1} = \frac{1}{2^N} \frac{N(1+y)^{N-1} \cdot y^{N/2} - \frac{N}{2} y^{\frac{N}{2}-1} (1+y)^N}{y^N}\Big|_{y=1} =$$

$$= \frac{1}{2^N} \frac{N \cdot 2^{N-1} - \frac{N}{2} 2^N}{1} = \frac{1}{2^N} \frac{N}{2} (2^N - 2^N) = 0$$

$$\sum_{r=0}^{N} \binom{N}{r} (2r - N) = 0 \implies \langle x \rangle = 0$$

$$\langle x^2 \rangle = \sum_{\substack{\text{all} \\ \text{possible} \\ \text{tot. displ.}}} (\text{Tot. displ.})^2 \times (\text{Prob.})$$

$$= \sum_{r=0}^{N} (2r-N)^2 (\Delta x)^2 \binom{N}{r} \frac{1}{2^N}$$

$$\langle x^2 \rangle = \frac{(\Delta x)^2}{2^N} \sum_{r=0}^{N} \binom{N}{r} (2r-N)^2$$

$$= \frac{1}{2^N} \cdot \frac{1}{4} \sum_{r=0}^{N} \binom{N}{r} (2r-N)^2$$

$$y \frac{d}{dy}\left(y \frac{dF_N}{dy} \right)\Bigg|_{y=1} = \ldots = \frac{1}{2^N} y \sum_{r=0}^{N} \binom{N}{r} \left(r - \frac{N}{2}\right)^2 y^{r-\frac{N}{2}-1} \Bigg|_{y=1}$$

expanded version of F_N

$$= \frac{1}{2^N} \sum_{r=0}^{N} \binom{N}{r} \left(r - \frac{N}{2}\right)^2 \cdot 1 =$$

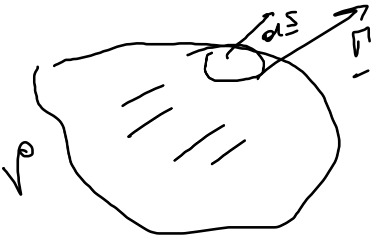
$$y \frac{d}{dy}\left(y \frac{dF_N}{dy}\right)\Bigg|_{y=1} = \frac{N^2}{4}$$

w/o expanding F_N

$$\cancel{\frac{N}{4}} = \frac{1}{2^N} \cancel{\frac{1}{4}} \underbrace{\sum_{r=0}^{N} \binom{N}{r} (2r-N)^2}_{\frac{\langle x^2 \rangle \cdot 2^N}{(\Delta x)^2}} \Rightarrow \langle x^2 \rangle = N(\Delta x)^2$$

$$t = N\Delta t \rightarrow N^2 = t/\Delta t$$

$$\langle x^2 \rangle \propto \sqrt{t}$$



$$\frac{dN}{dt} = -\int_{\text{surface}} \underline{\Gamma}\cdot d\underline{S}$$

$\underbrace{\underline{\Gamma}\cdot d\underline{S}}$ net number of particles that leave $d\underline{S}$

$\underline{\Gamma}$: flux of particles

n: density

N = # of particles in volume V

$$N = \int_{\text{volume}} n\, dV$$

$$\frac{d}{dt}\int_{\text{volume}} n\, dV = -\int_{\text{surface}} \underline{\Gamma}\cdot d\underline{S} \underset{\substack{\uparrow \\ \text{divergence theorem}}}{=} -\int_{\text{volume}} (\underline{\nabla}\cdot\underline{\Gamma})\, dV \Rightarrow \int_{\text{volume}} \left(\frac{dn}{dt} + \underline{\nabla}\cdot\underline{\Gamma}\right) dV = 0$$

$$\frac{dn}{dt} + \underline{\nabla}\cdot\underline{\Gamma} = 0$$

How do we relate

$\underline{\mu}$ to $\left\{ \begin{array}{l} \Delta x \\ \\ \Delta t \end{array} \right.$?

in a 1D random walk model