$\underline{B} = 0$ weakly ionized plasma

$$mn\frac{d\underline{u}}{dt} = q\underline{E}n - \underline{\nabla}p - mn\nu\underline{u}$$

$n$: density $m$: mass $\underline{u}(\underline{x},t)$: average velocity

$q$: charge $\underline{E}$: (ambipolar) electric field $p = nk_BT$
pressure

$\nu$: collisional frequency

$$\frac{d}{dt}u_j(\underline{x},t) = \frac{\partial u_j}{\partial t} + \sum_{i=1}^{3}\frac{\partial u_j}{\partial x_i}\cdot\underbrace{\frac{\partial x_i}{\partial t}}_{u_i} = \frac{\partial u_j}{\partial t} + \left[(\underline{u}\cdot\underline{\nabla})\underline{u}\right]_j\cdot$$

Steady state conditions: $\frac{\partial u_j}{\partial t} = 0$

$$\|(\underline{u}\cdot\underline{\nabla})\,\underline{u}\| \approx \frac{u^2}{L}$$

$L$: size of the system

$$\underbrace{u\nu^{-1}}_{\text{path travelled in between collisions}} \ll L \Rightarrow \frac{u}{L} \ll \nu$$

$$\frac{u^2}{L} \ll u\cdot\nu \sim \frac{\partial u}{\partial t}$$

$$\|(\underline{u}\cdot\underline{\nabla})\,\underline{u}\| \ll \frac{\partial u}{\partial t}$$

$$0 \approx q\underline{E}\cancel{n} - \frac{\underline{\nabla}(nT)}{n} - m\cancel{n}\nu\underline{u}$$

$T$ is uniform

$$\underline{\nabla}(nT) = T\cdot\underline{\nabla}n$$

$$\underline{u} \approx \frac{q\underline{E}}{m\nu} - \frac{T}{mn\nu}\underline{\nabla}n \quad \text{(ions and electrons)}$$

Set $\underline{\Gamma}_{ion} = \underline{\Gamma}_{el}$ Hydrogen plasma $n_e = n_i = n$

$$\cancel{n}\,\underline{u}_i = \cancel{n}\,\underline{u}_e$$

$$\frac{q_i \underline{E}}{m_i \nu_i} - \frac{T_i}{m_i n \nu_i}\underline{\nabla} n = \frac{q_e \underline{E}}{m_e \nu_e} - \frac{T_e}{m_e n \nu_e}\underline{\nabla} n$$

$$\underline{E}\left(\frac{q_e}{m_e \nu_e} - \overbrace{\frac{q_i}{m_i \nu_i}}^{\sim \text{neglect}}\right) = -\frac{\underline{\nabla} n}{n}\left(\overbrace{\frac{T_i}{m_i \nu_i}}^{\sim \text{neglect}} - \frac{T_e}{m_e \nu_e}\right)$$

$$\frac{m_e \nu_e}{m_i \nu_i} = \frac{m_e}{m_i}\cdot\sqrt{\frac{m_i}{m_e}} = \sqrt{\frac{m_e}{m_i}} \Rightarrow m_e \nu_e \ll m_i \nu_i$$

$$\frac{\nu_e}{\nu_i} \sim \sqrt{\frac{m_i}{m_e}}$$

$$\frac{-e}{\cancel{m_e}\nu_e}\underline{E} = +\frac{\underline{\nabla} n}{n}\frac{T_e}{\cancel{m_e}\nu_e}\,; \qquad \underline{E}^{amb} = -\frac{\nabla n}{en}T_e$$

$$\underline{\Gamma} = n\,\underline{u}_i = \left[\frac{\cancel{e}}{m_i\nu_i}\left(-\frac{\underline{\nabla} n}{\cancel{e}n}T_e\right) - \frac{T_i}{m_i n\nu_i}\underline{\nabla} n\right]\cdot n$$

$$= \left[-\frac{\underline{\nabla} n}{n}\frac{1}{m_i\nu_i}(T_e+T_i)\right]\cdot n = -\underbrace{\frac{(T_e+T_i)}{m_i\nu_i}}\cdot\underline{\nabla} n$$

w/o $\underline{E}^{amb}$ $\qquad \underline{\Gamma} = -D_i\underline{\nabla} n \qquad D_i = \frac{T_i}{m_i\nu_i} \qquad D^{amb}$

$$\underline{\Gamma} = -D^{amb}\,\underline{\nabla} n$$

$$D_e = \frac{T_e}{m_e\nu_e} \qquad \frac{D_e}{D_{amb}} = \frac{T_e}{m_e\nu_e}\frac{m_i\nu_i}{T_e+T_i} \approx \frac{1}{2}\frac{m_i}{m_e}\sqrt{\frac{m_e}{m_i}} \approx \sqrt{\frac{m_i}{m_e}} \qquad D_e = \frac{T_e}{m_e\nu_e}$$