1938:

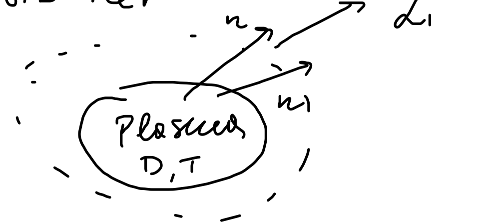
$$p + p \rightarrow d + e^+ + \nu_e$$

Fusion on the Earth

$$d + d \rightarrow \begin{cases} n + {}^3He & 50\% \quad Q = 3.27\ \text{MeV} \\ p + t & 50\% \quad Q = 4.03\ \text{MeV} \end{cases}$$

$\rightarrow$ $d + t \rightarrow \alpha + n$ $\qquad Q = 17.6\ \text{MeV}$

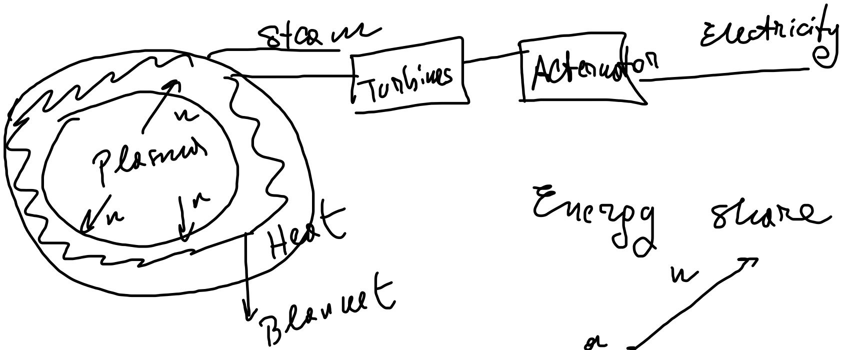
$d + {}^3He \rightarrow p + \alpha$ $\qquad Q = 18.3\ \text{MeV}$



d reservoir $\approx 10^6$ year

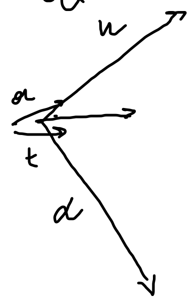
$${}^{1}_{0}n + {}^{6}_{3}Li \rightarrow {}^{3}_{1}t + {}^{4}_{2}He \qquad Q = 4.8\ \text{MeV}$$

$${}^{1}_{0}n + {}^{7}_{3}Li \rightarrow {}^{3}_{1}t + {}^{4}_{2}He + n \qquad Q = -2.5\ \text{MeV}$$



Energy share between α and n

Conserve linear momentum and energy



$n, \alpha \approx$ emitted back to back

$$\begin{cases} m_n v_{n_x} = m_\alpha v_{\alpha_x} \\ \frac{1}{2} m_n v_{n_x}^2 + \frac{1}{2} m_\alpha v_{\alpha_x}^2 = \underbrace{Q}_{\approx \text{MeV}} + \underbrace{K}_{\approx \text{keV}} \approx Q \end{cases}$$

v_α, v_n

$$E_\alpha = \frac{1}{2} m_\alpha v_\alpha^2 \qquad E_\alpha \approx \frac{m_n}{m_\alpha + m_n} \cdot Q \approx \frac{Q}{5} \approx 3.5\ \text{MeV}$$

$$E_n = \frac{1}{2} m_n v_n^2 \qquad E_n \approx \frac{m_\alpha}{m_n + m_\alpha} \cdot Q \approx \frac{4}{5} Q \approx 14\ \text{MeV}$$

<u>Fusion cross section?</u>

<u>Classical</u>

$$\sigma \approx \pi (r_d + r_t)^2 \approx 0.3 \cdot 10^{-28}\ m^2$$

$$r_d \approx A^{\frac{1}{3}} R_0 \qquad R_0 \approx 1.2\ \text{fm} \qquad \sigma \approx \text{barn}$$

$$r_t \approx A^{\frac{1}{3}} R_0$$

A: mass number

$A = 2$ (deuterium)

$A = 3$ (tritium)

$1\ \text{barn} = 10^{-28}\ m^2$

c.m. frame $E = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}\mu v_{rel}^2 = \frac{1}{2}\mu v_{rel}^2$

c.m. frame

$V_{cm} = 0$

$M = m_\alpha + m_t$

$\mu = \frac{m_\alpha m_t}{m_\alpha + m_t}$

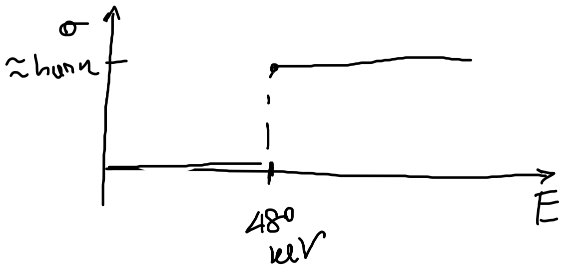
$\frac{1}{2}\mu v_{rel}^2 \geq \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d}$

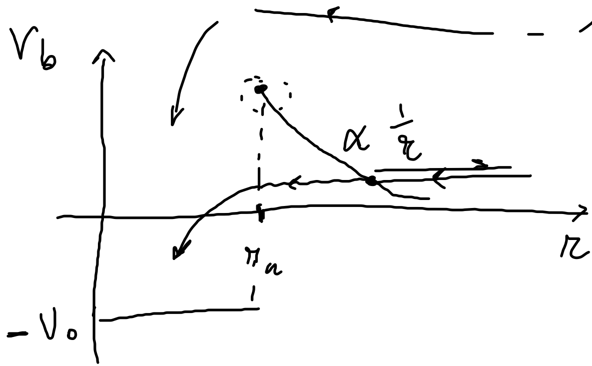
$d = r_\alpha + r_t$

$\frac{1}{2}\mu v_{rel}^2 \geq 480$ keV

1 eV → 12000 K

Classical picture



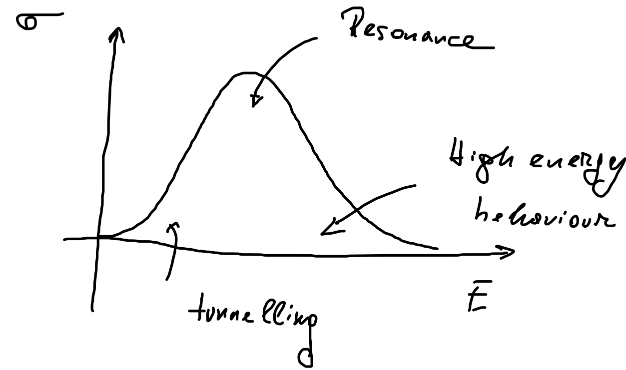


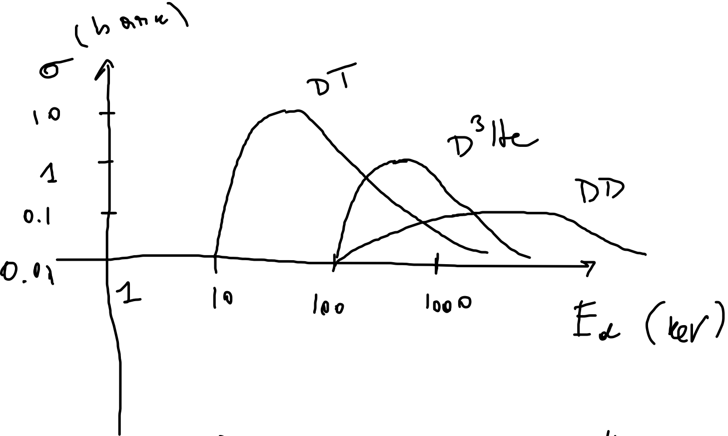
Quantum effects

1) Tunnelling

2) Resonances

3) Decrease of σ at high energies





$\nu = n_t \sigma v_{rel}$ $\qquad \dfrac{\#\ \text{reactions}}{\text{second} \quad \text{projectile}}$

n_p : projectile density. $\qquad n_p \nu = n_p n_t \sigma \underline{v_{rel}}$: $\dfrac{\#\ \text{fusion reactions}}{\text{volume} \quad \text{time}}$

at a given relative velocity

$$R = \sum_{\substack{\text{all} \\ \text{possible} \\ \text{values of } v_{rel}}} n_t n_p \sigma v_{rel} \left(\begin{array}{c} \# \text{ particles at} \\ \text{a specific } v_{rel} \end{array} \right)$$

$$= \sum_{\substack{\text{all} \\ v_{rel}}} n_t n_p \sigma v_{rel} \underbrace{f_\alpha(\underline{v}_\alpha)\, d^3\underline{v}_\alpha\, f_t(\underline{v}_t)\, d^3\underline{v}_t}_{|\underline{v}_\alpha - \underline{v}_t| = v_{rel}}$$

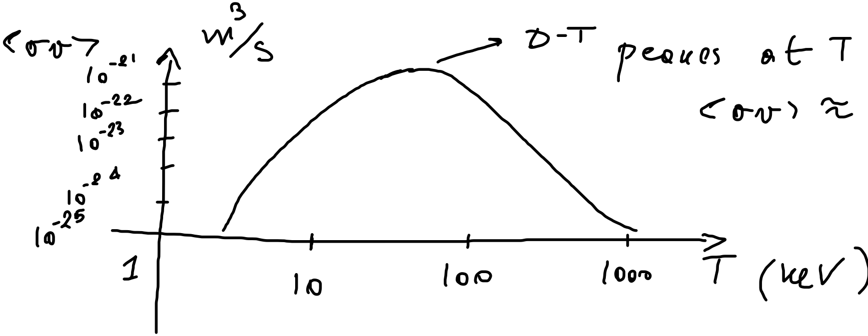
$$R = n_t n_p \underbrace{\langle \sigma v_{rel} \rangle}_{\text{Reactivity}}$$

$$\langle \sigma v_{rel} \rangle = \int d^3\underline{v}_\alpha\, d^3\underline{v}_t\, \sigma(v_{rel})\, v_{rel}\, f_\alpha(\underline{v}_\alpha)\, f_t(\underline{v}_t)$$

Thermal plasmas

f_α, f_t: Maxwellian $\rightarrow T$

$$\langle \sigma v_{rel} \rangle = \langle \sigma v_{rel} \rangle (T)$$



D-T peaks at $T \approx 70$ keV

$\langle \sigma v \rangle \approx 10^{-21}\ m^3/s$

$R = n_t n_d \langle \sigma v_{rel} \rangle (T)$

$n_d + n_t = n_e \; ; \quad n_t = n_e - n_d$

n_e is fixed

Maximize: $n_t n_d = n_d (n_e - n_d)$

$$\frac{\partial}{\partial n_d}\left[n_d (n_e - n_d) \right] = 0; \quad n_e - n_d - n_d = 0;$$

$$2 n_d = n_e; \quad n_d = \frac{n_e}{2}$$

$$n_t = n_e - n_d = \frac{n_e}{2} \qquad X[D] = \frac{n_d}{n_e} = X[T] = \frac{n_t}{n_e} = 50\%$$