$\langle \sigma v \rangle$: reactivity

$$\frac{\#\text{reactions}}{\text{time volume}} = \langle \sigma v \rangle \cdot n_d n_t$$

$d + t \rightarrow \alpha + n$

50:50 DT mixture

$n_d + n_t = n_e \Rightarrow n_d = n_t = n_e/2$

$$\frac{\#\text{reactions}}{\text{time volume}} = \frac{n_e^2}{4} \langle \sigma v \rangle$$

Gain / Heating		Cooling	
1) $d + t \rightarrow \alpha + n$	S_α	1) Emission of radiation (bremsstrahlung)	S_B
2) Auxiliary heating	S_h	2) Transport of heat	S_K

S_α: E_α: en. released by αs in each fusion reaction

$E_\alpha \approx 3.5$ MeV

$$S_\alpha = E_\alpha \cdot \left(\frac{\#\text{ reactions}}{\text{volume time}} \right) = E_\alpha \frac{n_e^2}{4} \langle \sigma v \rangle (T)$$

$p = nT$

$$p_{TOT} = p_e + p_d + p_t = n_e T + \underbrace{n_d}_{\frac{n_e}{2}} T + \underbrace{n_t}_{\frac{n_e}{2}} T = 2 n_e T$$

$$\Rightarrow n_e = \frac{p}{2T}$$

$$S_\alpha = E_\alpha \frac{p^2}{16 T^2} \langle \sigma v \rangle (T)$$

S_η:

$$S_B = c_B \cdot \underbrace{Z_{eff}}_{1}\, n^2 \cdot T^{\frac{1}{2}} = \frac{1}{4} \cdot c_B \frac{p^2}{T^{3/2}}$$

$$\uparrow \quad n_e = \frac{p}{2T}$$

$$S_K = \frac{3}{2} \frac{p}{\tau_E} \qquad \tau_E: \text{energy confinement time}$$

$$p \quad T \quad \tau_E$$

Practical units

$$S_\alpha = K_\alpha \frac{\langle \sigma v \rangle}{T^2} p^2 \quad MW/m^3 \qquad K_\alpha = 1.37$$

$$[T] = keV \qquad [p] = bar \qquad [\langle \sigma v \rangle] = 10^{-22}\, m^2/s \qquad [\tau_E] = s$$

$$S_B = k_B \frac{P^2}{T^{3/2}} \ MW/m^3 \qquad k_B = 0{,}052$$

$$S_K = k_K \frac{P}{\tau_E} \ MW/m^3 \qquad k_K = 0{,}15$$

Power balance:

heating = cooling

$$S_h + S_\alpha = S_K + S_B$$

→ Ideal → ignition

$S_h = 0 \quad S_h = 0$

1) Ideal ignition

$$S_\alpha \geq S_B$$

$$k_\alpha \frac{\langle \sigma v \rangle (T)}{T^{2}} P^2 \geq k_B P^2 / T^{3/2}$$

$$f(T) \geq k_B / k_\alpha$$

$$\frac{\langle \sigma v \rangle (T)}{T^{\frac{1}{2}}} \geq \frac{k_B}{k_\alpha}$$

$T_{min} = 4.4$ keV

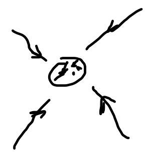
2) Ignition

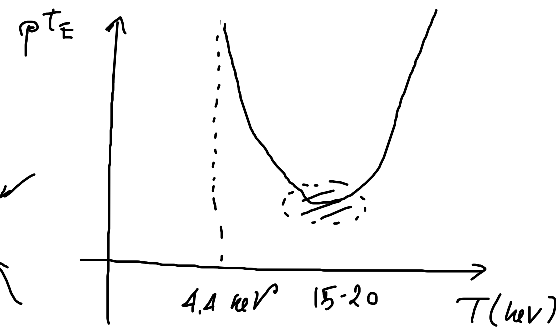
$$S_\alpha \geq S_B + S_K \qquad (S_h = 0)$$

$p\, T\, \tau_E$

$$K_\alpha \frac{\langle \sigma v \rangle}{T^2}\, p^{\cancel{2}} \geq K_B \frac{p^{\cancel{2}}}{T^{3/2}} + K_K \frac{\cancel{p}}{\tau_E}$$

$$p\tau_E \geq \frac{K_K T^2}{\underbrace{K_\alpha \langle \sigma v \rangle - K_B T^{\frac{1}{2}}}_{f(T)}}$$





Minimum $\quad T_{min} = 15$ keV

$(p\tau_E) = 8$ atm $\cdot$ s

$p = 2nT$

$p\tau_E T = 8\ \text{atm}\cdot s \cdot 15\ \text{keV}$

$2nT\tau_E T = 8\ \text{atm}\cdot s \cdot 15\ \text{keV}$

$$nT\tau_E = \frac{8\ \text{atm}\cdot s\cdot 15\ \text{keV}}{2\cdot 15\ \text{keV}} = \dots = 3\cdot 10^{21}\ \text{keV}\,m^{-3}\cdot s$$

Lawson criterion

$T \approx 15\ \text{keV}$

$n \approx 10^{20}\ m^{-3}$ $\quad \tau_E \approx s$

If

$S_h \neq 0$

$$f_\alpha = \frac{S_\alpha}{S_\alpha + S_h}$$

If $S_h = 0$ $\quad f_\alpha = 1$ (ignition)

If $S_h = S_\alpha$ $\quad f_\alpha = \frac{1}{2}$

If $f_\alpha \geq \frac{1}{2}$

ITER

$$S_h = \frac{1-f_\alpha}{f_\alpha} \cdot S_\alpha$$

$$S_\alpha + S_h \geq S_\kappa + S_B$$

$$S_\alpha + \frac{1-f_\alpha}{f_\alpha} S_\alpha \geq S_\kappa + S_B$$

$$\frac{S_\alpha}{f_\alpha} \geq S_\kappa + S_B ; \qquad S_\alpha \geq f_\alpha (S_\kappa + S_B)$$

$$p\tau_E \geq f_\alpha \cdot (p\tau_E)_I$$

$$Q = \frac{\text{net thermal power}}{\text{total input power}} = \frac{P_{out} - P_{in}}{P_{in}} \qquad Q = 1 \text{ breakeven}$$

$Q = \infty$ ignition

Factors that contribute to power out:

1) Bremsstrahlung

Cannot use: S_α

2) neutrons

3) Heat due to transport

$$S_\kappa + S_B = S_\alpha + S_h$$

$$\Rightarrow S_\kappa + S_B - S_h = S_\alpha$$

$$Q = \frac{P_{out}}{P_{in}} = \frac{\overbrace{4S_\alpha}^{\text{neutrons}} + S_B + S_\kappa - S_h}{S_h} = \frac{5S_\alpha}{\underbrace{S_h}_{S_\kappa + S_B - S_\alpha}} = \frac{P_{fusion}}{P_{heating}}$$

$$Q = \frac{5p\tau_E \quad K_\alpha \langle\sigma v\rangle}{K_\kappa T^2 - K_\alpha \langle\sigma v\rangle p\tau_E} = \frac{5p\tau_E}{(p\tau_E)_{Ignition} - p\tau_E}$$

If at ignition:

$p\tau_E = (p\tau_E)_I$

$Q = +\infty$

If not at ignition: $p\tau_E = f_\alpha \, (p\tau_E)_I$

$$Q = \frac{5 f_\alpha (p\tau_E)_I}{(p\tau_E)_I - f_\alpha (p\tau_E)_I} = \frac{5 f_\alpha}{1 - f_\alpha}$$

$$f_\alpha = \frac{Q}{Q+5}$$

JET DT $Q = 0.65$ $f_\alpha \approx 12\%$
(1997: transient)
16 MW fusion power
$\approx$ 100 ms

Divertor tokamak Test

ITER $Q = 10$ $f_\alpha = 67\%$
Start in ≈ 2027

DEMO $\approx$ '50 s $Q \approx 40$ $\approx 8/9$ % A

$Q = 10$ goal $\approx$ '30 s

$Q_E \approx 10$

$$Q_E = \frac{\text{Electrical power}}{\text{Heating power}}$$