

Principles of Corporate Finance

Written exam (proctored online) - Sept 10, 2020

THE EXAM LASTS 1 HOUR

THOSE WHO HAVE PRESENTED IN CLASS MUST ANSWER THE 2 NUMERICAL QUESTIONS.

ALL OTHERS HAVE 15 MINUTES MORE AND MUST ANSWER ALSO THE OPEN QUESTION.

Question 1 (numerical)

Consider an Entrepreneur who seeks funding for a risky project requiring $I = 50$ as investment at $t = 0$ and returning a cash flow $X = \{40, 100\}$ at $t = 2$. At $t = 1$ E can choose (not being observed) between two projects: project H has a greater success probability $p_H = 0.8$, while project L has a smaller probability $p_L = 0.3$ of success. However project L guarantees to E a private benefit $B = 15$.

1. Check that the NPV when E chooses H is positive.
2. Consider financing the project by issuing a stock leaving a proportion $\beta \in (0, 1)$ to investors: will E choose H ?
3. Consider financing the project with risky debt, i.e. a debt contract with face value $40 < D < 100$: will E choose H ?

Question 2 (numerical)

E owns liquidity A and seeks external funding for an investment that requires $I = 50$ at $t = 0$ and that returns $X = \{10, 100\}$ at $t = 2$. E can choose between two projects: a good project H and a bad project L . The success probability is $\Pr X = 100 = p$; project H has a greater success probability $p_H = 0.8$, while project L has $p_L = 0.3$. However project L guarantees to E a private benefit $B = 40$.

1. Compute the NPV of the project H .
2. E raises $(I - A)$ by issuing a bond that repays a face value R_u to investors. Write the incentive constraint for E to choose project H and compute his maximum pledgeable income (constraint on R_u).
3. Write the investors' rationality constraint and find the minimum value R_u , assuming that E chooses project H . Find the minimum threshold for A , call it \bar{A} , for which E manages to raise external financing.
4. The bank monitors at cost $c = 15$, reducing as a consequence the private benefit from $B = 40$ to $b = 20$. Assume an E who is credit rationed by investors, i.e. with $A < \bar{A}$. E asks funding exclusively to a bank and promises to repay R_m at $t = 2$. Which is the minimum threshold for A , call it \underline{A} , to obtain a loan from the bank?
5. Assume now funds A are uniformly distributed between 0 and 100. Compute the percentage of firms that are credit rationed, those that are financed by financial markets, those financed by the banks and those that self-finance the investment.

Question 3*

Define the "arms' length" finance. Which are the main differences with respect to the other types of external finance?

Solutions for the numerical questions

Question 1

1.

$$NPV_H = 100(.8) + 40(0.2) - 50 = 38 > 0$$

2. Suppose there is an investor considering to fund $I = 50$ in exchange for an equity stake β . Let's check whether E is willing to choose project H. Given the investor holds β stake, E works iff

$$(1 - \beta)(.8 \times 100 + .2 \times 40) \geq (1 - \beta)(.2 \times 100 + .8 \times 40) + 15$$

or

$$(1 - \beta)(50 - 20) \geq 15 \Leftrightarrow (1 - \beta) \geq 15/30 = .5$$

Given this limit on β can the investor hope to recoup his investment of 50?

$$\beta(.8 \times 100 + .2 \times 40) - 50 \Leftrightarrow \beta \geq 50/88 = 0.56$$

Since the two dis-equalities cannot be fulfilled contemporaneously, E cannot get financed by equity.

3. When financed by risky debt, i.e. debt with face value $D \in (40, 100)$, E works hard iff:

$$.8 \times (100 - D) \geq .3 \times (100 - D) + 15$$

or

$$.5 \times (100 - D) \geq 15 \Leftrightarrow D \leq 70$$

If the creditor expects E to work hard, she is willing to finance him iff

$$.8 \times D + .2 \times 40 \geq 50 \Leftrightarrow D \geq 42/0.8 = 52.5$$

Thus, any face value $D \in [52.5, 70]$ allows the investor to break even, while preserving E's incentives. Hence, E can get financed by debt.

Question 2

1.

$$NPV_H = 100(.8) + 10(0.2) - 50 = 32 > 0$$

2. E will choose project H whenever

$$.8 \times (100 - R_u) \geq .3 \times (100 - R_u) + 40 \Leftrightarrow R_u \leq 20$$

3. Bondholders will finance E if and only if

$$.8 \times R_u + .2 \times 10 \geq 50 - A \Leftrightarrow R_u \geq \frac{48 - A}{0.8}$$

Combining the two inequalities, we have that

$$\frac{48 - A}{0.8} \leq 20$$

Hence the minimum level of A fulfilling the above condition is

$$\bar{A} = 32$$

4. When the bank finances him, E will choose project H if and only if

$$.8 \times (100 - R_m) \geq .3 \times (100 - R_m) + 20 \Leftrightarrow R_m \leq 60$$

The bank will finance E if and only if

$$.8 \times R_m + .2 \times 10 - 15 \geq 50 - A \Leftrightarrow R_m \geq \frac{63 - A}{0.8}$$

Combining the two inequalities, we have that

$$\frac{63 - A}{0.8} \leq 60$$

Hence the minimum level of A fulfilling the above condition is

$$\underline{A} = 15$$

5. With a uniform distribution between 0 and 100, we have that:

- 50% self-finance their investment
- 18% finance by issuing bonds in financial markets
- 17% are financed by banks
- 15% are credit rationed