Lecture Notes
The financial structure of the firm

Outline

- Standard financial contracts (equity and debt)
- Modigliani-Miller Theorem
- Debt and Incentives
- Debt and Risk-Shifting
- Debt Overhang Problem

Standard financial contracts (equity and debt)

- Assume that a firm can be fully described by its cash flow $X \in ([0, \infty)$ (random variable at date $t = 0$).
- The cash flow captures the future flow of revenues from all the current and past productive projects of the firm.
- The cash flow is observable by third parties and reflects the true economic state of the firm (accounting manipulation is ruled out).
- Standard financial contracts are then a specific transformation of the cash flow.

A firm with debt

- The firm at an initial date $t = 0$ has issued debt with "face value" D.
- This means that at a future date, $T > 0$ it promises to return D to its creditors.
- If the firm does not repay its creditors at date T, i.e. it defaults on its promises, bankruptcy occurs.
- Creditors do not own the firm, but can seize the assets of the company in case of default.
- The shareholders are owners of the firm and are protected by limited liability.

Value of debt and equity

- If $X \geq D$ then the creditors receive D and the shareholders are residual claimant, thus earn $(X - D)$.
- If $X < D$ the entrepreneur defaults: the firm is liquidated and all the cash flow, X, goes to creditors; shareholders earn 0 in case of default (because of limited liability).
- The value of equity is given by the transformation of the cash flow $\max\{X - D, 0\}$.
- The value of debt is instead $\min\{X, D\}$.

Option Analogy

Equity pays: $\max\{X - D, 0\} = (X - D)^+$ which is equivalent to

- Call option on X with strike price D.

  Equity gets the upside, but does not bear the full downside.

Risky debt with face value D pays: $\min\{D, X\} = D - (X - D)^+$ which is equivalent to:

- Risk-free bond with face value D and
- Written put on X with strike price D.

Like option value, the value of equity increases in risk, while the value of debt decreases in risk.

Real-life complications

- Junior (subordinated) vs. senior debt.
- Ordinary vs. preferred stocks.
- Convertible debt (option).
- Debt with collateral.

Modigliani-Miller Theorem

Irrelevance of the financial structure:
The amount of debt issued by does not affect the expected value of a firm.

- Assume firm's cash flow X is distributed according to a continuous probability function $g(x)$ with support $[0, \infty)$.
- The firm's expected value is given by the expected value of debt plus the expected value of equity, that is

$$\begin{aligned} V &= V_E + V_D = E(\max\{X - D, 0\}) + E(\min\{D, X\}) & (1) \\ &= \int_D^{\infty} (x - D) g(x) dx + \int_D^{\infty} D g(x) dx + \int_0^D x g(x) dx & (2) \\ &= \int_0^{\infty} x g(x) dx = E(X) & (3) \end{aligned}$$

- Note that V does not depend upon D: hence the financial structure of a firm (the mix of Debt and Equity) does not affect the value of the firm.

Special case - Bernoullian cash flow

- Assume that the cash flow can take only two values: $\{X^-, X^+\}$.
- The financial contract repays R^+ if X^+ and R^- if X^-.
- Limited liability: $R^+ \leq X^+$ and $R^- \leq X^-$.
- Risky Debt interpretation: $R^+ = D$ and $R^- = X^-$ with $X^- < D < X^+$.
- Equity interpretation: $R^+ = \beta X^+$ and $R^- = \beta X^-$
  - E's (inside) equity stake: $\frac{X^+ - R^+}{X^+} = 1 - \beta$,
  - investor's (outside) equity stake: $\frac{R^+}{X^+} = \beta$.

Debt and incentives

The corporate finance literature provides reasons why risky debt may be beneficial or costly.

Model

Two dates ($t = 1, 2$), no discounting ($r = 0$), everybody is risk-neutral.

Pennyless entrepreneur E (borrower, insider) has a project that requires funding I.

- Moral hazard: The expected return (cash flow) depends on E's behaviour: $e = \{e_L, e_H\}$
  - If undertaken project generates at $t = 2$ cash flow $X \in \{X^-, X^+\}$.
  - The success probability with working (e_H) is p_H.
  - The success probability with shirking (e_L) is p_L where $\Delta_p = p_H - p_L > 0$.
  - Shirking results in (non-transferable) private benefits B to E.

- Contract (Loan agreement):
  - In exchange for contributing I at $t = 1$, investor is promised a repayment repayments at $t = 2$

$$R^- \text{ if } X = X^- \quad \text{and} \quad R^+ \text{ if } X = X^+$$

    where $\Delta_R = R^+ - R^-$ denotes the "repayment spread"
    and $R^i \leq X^i$ (limited liability)
  - Competitive capital markets.

Assumptions:

- Project has a positive NPV if E works: $X^- + (p_L + \Delta_p)\,\Delta_X - I > 0$

- Exerting the effort is efficient: $\Delta_p \Delta_X > B$

First Best: Modigliani-Miller

Financing choices are irrelevant in the absence of moral hazard (if $B = 0$, or e_H is contractible)

Risky Debt: Suppose E chooses to raise I with risky debt with face value D:

$$R^- = X^- \qquad R^+ = D$$

Equity: E sells a fraction β of existing shares:

$$R^- = \beta X^- \qquad R^+ = \beta X^+$$

Optimality of Debt

Irrelevance of financing choice breaks down, once effort is costly and non-contractible.

At $t = 1$, the entrepreneur chooses $e = e_H$

$$p_H(X^+ - R^+) + (1 - p_H)(X^- - R^-) \geq p_L(X^+ - R^+) + (1 - p_L)(X^- - R^-) + B$$

The incentive compatibility constraint (IC) can be rewritten as

$$\Delta_p \left(\Delta_X - \Delta_R\right) > B \quad \text{or} \quad \Delta_R \leq \Delta_R^{\max} \equiv \Delta_X - B/\Delta_p$$

We need to solve the following:

$$\begin{array}{l} \min\limits_{R^-, R^+} \Delta_R \\ R^- \leq X^- \qquad R^+ \leq X^+ \\ I = R^- + p_H \Delta_R \end{array}$$

Debt: $\Delta_R = \frac{I - X^-}{p_H}$

Equity $\Delta_R = \frac{I - \beta X^-}{p_H}$

- Debt financing is the best way to tackle the entrepreneur's effort provision problem.
- Irrespective of financing, E receives the investment's entire NPV (because capital markets are competitive).
- All projects that can be financed with equity can also be debt-financed but the reverse is not true.

Intuition: The optimal (debt) contract maximizes the fraction of the return from effort that accrues to the entrepreneur. Hence, it maximizes his incentive to exert effort.

Remarks:

Jensen and Meckling (1976) show that debt dominates equity. The above argument establishes that debt also dominates all other possible contracts.

Debt as incentive contract has two specific features:

Being residual claimant exposes entrepreneur to substantial risk. Risk-neutrality is required to avoid trade-off between incentives and insurance.

Full incentives: Agent receives entire income at the margin, only for incomes above repayment level.

Without limited liability of the agent, the moral hazard problem becomes trivial. Given agent is risk neutral and has unlimited liability, solution to the problem is fixed repayment to lender and borrower becomes residual claimant.

Numerical example 1

Consider the following firm/project: $I = 60$ and

e_L and $B = 10$	Cash flow
$1 - p_L = 0.8$	40
$p_L = 0.2$	90

e_H	Cash flow
$1 - p_H = 0.2$	40
$p_H = 0.8$	90

1. Check that the NPV when E chooses e_H is positive.
2. Consider financing the project by issuing a stock which leaves a proportion $\beta \in (0, 1)$ to investors: will E choose e_H?
3. Consider financing the project with risky debt, i.e. a debt contract with face value $40 < D < 90$: will E choose e_H?

We now analyse the costs of debt: Debt overhang and risk-shifting.

Debt and risk-shifting

Question:

Suppose a levered firm is choosing between two projects with equal NPV, one of which is riskier than the other. Are equity- and debt holders indifferent between the two? If not, do they prefer the same project?

Model

Two dates ($t = 1, 2$), no discounting ($r = 0$), everybody is risk-neutral.

Entrepreneur E (borrower, insider) requires funding I.

For simplicity, E is penniless.

- Project choice:
  - When funded, E chooses between two mutually exclusive projects, Project A and B

	$\Pr\left[\mathbf{X} = \mathbf{2\hat{X}}\right]$	$\Pr\left[\mathbf{X} = \mathbf{\hat{X}}\right]$	$\Pr\left[\mathbf{X} = \mathbf{0}\right]$
Project A:	θ_1	$1 - (\theta_1 + \theta_2)$	θ_2
Project B:	$\theta_1 + \Delta_1$	$1 - (\theta_1 + \theta_2)$ $-(\Delta_1 + \Delta_2)$	$\theta_2 + \Delta_2$

- Project cash flows realize at date $t = 2$.

Assumptions:

- $0 < \Delta_1 < \Delta_2$ and $\theta_1 + \Delta_1 + \theta_2 + \Delta_2 < 1$
- Project A has a positive NPV:

$$\theta_1 \times 2\hat{X} + (1 - \theta_1 - \theta_2) \times \hat{X} + 0 = (1 + \theta_1 - \theta_2)\hat{X} > I$$

First Best

Project B's NPV is

$$(1 + \theta_1 + \Delta_1 - \theta_2 - \Delta_2)\hat{X} - I$$

which is less than NPV of Project A, the difference being:

$$(\Delta_2 - \Delta_1)\hat{X} > 0$$

Hence, I should be used for Project A.

Debt Finance

Suppose that I is raised in debt with face value D paying $\min\{X, D\}$.

Assume (for simplicity) that $(1 - \theta_2)\hat{X} < I$.

Hence, Dhas to be larger than $\hat{X}$, and E gets a positive payoff only if $X = 2\hat{X}$.

With project A, E gets

$$\theta_1(2\hat{X} - D)$$

while with Project B, she gets

$$(\theta_1 + \Delta_1)(2\hat{X} - D)$$

Once I has been raised, E picks Project B, as it is more likely to generate $X = 2\hat{X}$.

Because of limited liability, E does not internalize loss in low state, but maximizes returns in the state, where she is residual claimant.

Ultimately, the cost is borne by E: Investors anticipate E's choice and demand a higher face value of debt. That is, $\tilde{K}$ is such that

$$(\theta_1 + \Delta_1)\tilde{D} + (1 - \theta_1 - \Delta_1 - \theta_2 - \Delta_2)\hat{X} = I$$

If NPV of Project B < 0, no funding even though there is a project (Project A) with a positive NPV (market break down).

Equity Finance

Suppose I has been raised in equity. The sold fraction β entitles investors to βX.

Once I is raised and invested, E receives a fraction $1 - \beta$ of the cash flow: She choose the project which maximizes the expected cash flow:

$$(1 - \beta)(1 + \theta_1 - \theta_2)\hat{X} > (1 - \beta)(1 + \theta_1 + \Delta_1 - \theta_2 - \Delta_2)\hat{X}$$

Hence, she undertakes Project A.

Equity finance does not induce any distortion in the investment decisions.

Intuition

Debt Finance:

Levered equity's payoff is convex in cash flows, inducing equityholders to take excessive risk.

The difference between Project A and Project B can be decomposed into two parts:

- An increase in θ_1 and θ_2 by Δ_1 which preserves the mean $\Delta_1 2\hat{X} - 2\Delta_1\hat{X} + \Delta_1 0 = 0$ but increases the variance.
- An increase in θ_2 by $\Delta_2 - \Delta_1$ which decreases the mean $-(\Delta_2 - \Delta_1)\hat{X} + (\Delta_2 - \Delta_1)0 < 0$
- E is willing to incur cost of lower mean in order to increase the variance of the cash flow (Risk-shifting or asset substitution problem).

Note: Asset substitution requires risky debt, otherwise E internalizes all costs and all benefits. That is, her decisions will be value maximizing.

Equity Finance:

E and the investors have the same claims. Hence, there is no conflict about project choice.

Linearity of claims: No risk-shifting

Numerical example 2

E needs $I = 4.2$ to undertake a productive project. Once financed, E can choose between $\{A, B\}$, i.e. two risky projects with the following stochastic structure:

	$2\hat{X} = 10$		$\hat{X} = 5$		$X = 0$
A	0.2		0.6		0.2
B	0.3		0.2		0.5

1. Compute the NPV of the two projects and define in which of the two it is profitable to invest.
2. Assume E issues risky debt with face value $5 < D < 10$: which project will be selected?
3. Compute the rational face value D for investors.
4. assume instead E issues equity by promising a proportion β of X to investors: which project will be selected?
5. Compute the rational proportion β for investors.

Implications

- More debt when there is less risk-shifting potential:
  - Regulated public utilities with less managerial discretion.
  - Firms in mature industries with few growth opportunities.
- Risk shifting incentives are higher when the firm is in financial distress since this is when limited liability becomes important (Gambling for Resurrection).
  - For instance, manager may delay filing for bankruptcy so as to keep the option value of equity alive.

Mitigating Asset Substitution:

- **Short-term debt** allows to renegotiate lending rate to adjust for higher risk, lessening the incentives to switch or choose high risk projects.
- **Covenants** to debt contract, e.g. prohibit investments into new, unrelated lines of business.
- **Convertible Debt** gives debt-holders the option to become equityholders. Since they are bound to exercise this option when returns are high, E is not the sole residual claimant when returns are high. Anticipating such conversion, she abstains from risk-shifting and takes project with higher NPV.
  - Highly leveraged, high-growth firms are most likely to issue convertible bonds. These firms exhibit high bankruptcy risk, and hence high potential for asset substitution.

Remarks

Like Jensen and Meckling (1976), we analyzed effort-provision and risk-shifting problems separately. Therefore,it is merely a conjecture that a mix of debt and equity is optimal.

Combining insights from risk-shifting and effort provision problem:

- Under-provision of effort is increasing in the amount of outside equity, while asset substitution is increasing in the amount of debt.
- Capital structure is such that the sum of these agency costs are minimized. Hence, the optimal capital structure likely to be a mix of debt and equity.

Debt overhang problem

- Myers (1977) shows why firms (may) limit borrowing, even when
  - Tax-advantage to corporate borrowing
  - Capital markets are strictly perfect, efficient and complete
- (Too much) risky debt induces a suboptimal investment strategy.
- That is, some positive NPV projects are forgone.
- Basic insight: Because debt is senior and risky, it captures (a too large) part of the surplus generated by the investment.
- This discourages junior claimants (equity) from contributing capital.
- Too much debt reduces ex-ante value of firm.

Model

Two dates ($t = 1, 2$), no discounting, everybody is risk neutral.

At $t = 1$, E is running a firm with:

- **Assets in place:**
  - Generate $X \in \{X^-, X^+\}$ at $t = 2$
  - with $\Delta_X = X^+ - X^- > 0$ and $p = \Pr\{X = X^+\}$
- **Existing investors:**
  - Hold claims on X with promised repayments at $t = 2$
  - More precisely, they hold debt with face value D so that

$$R^- = \min\{D, X^-\} \quad \text{and} \quad R^+ = \min\{D, X^+\}$$

- **Investment opportunity:**
  - At $t = 1$, E considers undertaking a project
  - Investing I increases p to $p + \Delta_p$
  - This investment has a positive NPV:

$$\Delta_p \Delta_X - I > 0$$

Underinvestment

The existing debt-holders' payoff is: $R^- + p\Delta_R$

If I is invested, their payoff increases by

$$\Delta_p \Delta_R$$

and E's payoff increases by

$$\underbrace{(\Delta_p \Delta_X - I)}_{\text{project's NPV}} \quad - \quad \underbrace{\Delta_p \Delta_R}_{\substack{\text{part accruing to} \\ \text{existing debt-holders}}}$$

Existing debt acts as a tax on investment. Hence, E invests iff

$$\Delta_p \Delta_X - I > \Delta_p \Delta_R$$

With risky debt ($\Delta_R > 0$), the investment policy is distorted towards underinvestment. Indeed, projects such that

$$\Delta_p \Delta_R > \Delta_p \Delta_X - I > 0$$

should be undertaken, but are rejected.

Intuition:

- E would incur the entire investment cost, but gets only part of the return.
- Holders of existing risky claim do not contribute to the funding, but their claim becomes less risky. They get additional $\Delta_p \Delta_R$.
- This transfer may prevent optimal investment policy.

Remarks:

Debt overhang can only materializes if debt is risky.

With risk-free debt ($\Delta_R = 0$), there is no transfer and hence no distortion in the investment decision.

Equally crucial is that cash flow of the new investment cannot be contracted upon separately from that of the assets in place.

Otherwise, project can be undertaken in separate entity (firm) with no existing debtholders.

Assuming that investing raises $\Pr\left[X = X^{+}\right]$ is a mere modelling trick. The analysis also applies when the new and old projects are unrelated and only the sum of cash flows from both projects is verifiable.

Can Outside Financing Help?

With competitive capital markets, E needs to promise new investors a repayment with PV$= I$.

Thus, her decision problem is unchanged. That is, no investment if

$$\Delta_p \Delta_X - \Delta_p \Delta_R < I$$

Renegotiation?

Consider a project such that

$$\Delta_p \Delta_R > \Delta_p \Delta_X - I > 0$$

If E does not invest, her payoff is

$$p\left(X^{+} - D\right)$$

and the debt-holders' payoff is

$$X^{-} + p\left(D - X^{-}\right)$$

Debt forgiveness does not make creditors worse off.

Indeed, conditionally on the investment being made, the debtholders accept to reduce the face value of the debt from D to at most d such that

$$X^{-} + (p + \Delta_p)\left(d - X^{-}\right) = X^{-} + p\left(D - X^{-}\right)$$

That is,

$$d = \frac{pD + \Delta_p X^{-}}{p + \Delta_p}$$

The debt-holders' payoff is unchanged but now E invests because she receives the project's entire NPV:

$$\begin{aligned}
&= (p + \Delta_p)(X^{-} - d) - I \\
&= (p + \Delta_p)X^{+} - \left(pD + \Delta_p X^{-}\right) - I \\
&= \underbrace{p(X^{+} - D)}_{\substack{E\text{'s payoff} \\ \text{absent investment}}} + \underbrace{\Delta_p \Delta_X - I}_{\substack{\text{new investment's} \\ \text{NPV}>0}}
\end{aligned}$$

Intuition: Since the debtholders' payoff is unchanged, E receives the investment's entire NPV, which is positive. Hence, she undertakes it.

- The reasoning does not rely on E receiving the entire surplus:
  - How the surplus is shared depends on the parties' bargaining position.

Remark: Unconditional debt forgiveness does not necessarily work in the sense of ensuring that a positive NPV project is undertaken. If E cannot commit to invest in exchange for debt forgiveness, she undertakes the project iff

$$\Delta_p \Delta_X - \Delta_p \Delta_{\hat{R}} > I$$

Since $d - X^- > 0$ (otherwise the renegotiated debt would not be risky thereby eliminating the debt overhang problem), E may not want to invest, even though the investment has a positive NPV:

$$\Delta_p(X^+ - X^-) > I > \Delta_p(X^+ - d)$$

Numerical example 3

Consider a firm that in its past has issued debt with face value $D = 50$. This debt was issued to invest in a productive project that generates a payoff of 100 or 20 with equal probability. Firm has an investment opportunity: Its costs are 10 and the additional payoff is 15 with certainty. Assume that E has enough liquidity to finance this new investment opportunity.

1. Will E invest in this new opportunity?
2. What if the face value of the past debt was renegotiated down to $d < D$?

Essence of the Overhang Problem

Existing risky claims reduces the incentives to invest.

Crucial assumptions:

- Claims cannot be (perfectly) renegotiated.
- Cash flow from new investment cannot be separated from cash flow of assets in place.

Question: What makes it a debt overhang problem? That is, what is the (implicitly assumed) difference between debt and equity?

Unlike equity, debt is senior relative to new equity issued to finance investment.

That is, debtholders cannot be forced to give up anything if new securities are issued.

Main insights:

- Risky debt makes equityholders reluctant to finance some NPV > 0 projects because they would incur the full costs but receive only part of the returns.
- In principle, the claimholders could renegotiate to mitigate - if not avoid - the inefficiency. Thus, debt overhang problem is largely a renegotiation problem.

Implications

- Ex-ante: Firms with large discretionary investments or more prone to encounter financial distress should have:
  - Less debt
    - Empirical studies find that leverage is negatively correlated with growth
  - Debt structure which is more easily renegotiated
- Ex-post: Solutions should focus on renegotiation problems
- Rationale for project financing: It avoids tax on new project by existing investors (debtholders).

Financing During Distress

- Firms near (in) financial distress find it difficult (impossible) to raise new funds, even for positive NPV investments.

- Note: Similar problem for countries

Ex-Post Ways to Mitigate Overhang Problem:

- Issue more senior claims
  - Avoids the “tax”. But it may not be feasible (covenants).
  - If new senior claims can be issued,
    * Existing debtholders may lose
    * Even negative NPV projects might be undertaken
- Debt overhang and associated bargaining problems are recognized in Bankruptcy Law and in workouts.
  - Chapter 11 allows for new debt senior to existing non-secured debt (**Debtor-in-Possession**)

Ex-Ante Ways to Mitigate Overhang Problem:

- Less debt
- Project financing: No tax on new project by existing investors
- Debt structure which is more easily renegotiated
  - Bank rather than public debt,
  - Fewer rather than many banks
  - Investor with long-term interests

Lecture Notes
The role of banks versus bond-holders

Outline

- Why do banks and bond-holder cohexist?
- Banks vs. Arms' length creditors

Why do banks and bond-holders cohexist? (Holmstrom & Tirole, QJE 1997)

Three type of agents: entrepreneurs (f), banks (m) and uninformed bond-holders (u).

Three dates ($t = 0, 1, 2$) (alternative gross return $\gamma \geq 1$).

E has a productive project:

- At $t = 0$: financial contract is issued
  - E needs $I > 0$
  - internal funds $A \geq 0$.
- At $t = 1$: moral hazard
  - E is crucial to the project
  - but he may choose between exerting an effort or shirking, i.e. $\{e_H, e_L\}$
  - associated to shirking there is a private benefit B
- At $t = 2$: project returns
  - $X \in \{0, X^+\}$.
  - probability of success is $\Pr[X = X^+] = p$.
  - When exerting effort e_H the probability of success increases, i.e. $p_H > p_L$ with $\Delta p = p_H - p_L$.
- Firms differ for the amount of internal funds A (only source of heterogeneity): A is distributed according to the probability function $h(A)$ with $A \in [0, \infty)$.

- Banks, with own capital K_m (exogenous), lend money to firms. Banks might monitor firms: at a private cost c might reduce the private benefit of entrepreneurs from B to $b < B$. Monitoring is efficient if $c < B - b$.
- Uninformed bond-holders are not capable of monitoring.

Assumption: *The surplus of the project when E exerts effort, i.e. project H, is positive($p_H X^+ - \gamma I > 0$), while the surplus of project L is negative($p_L X^+ + B - \gamma I < 0$). Investing in project H is efficient:*

$$\Delta p X^+ > B$$

Direct credit: One possible financial strategy for E is to borrow from bond-holders the amount $I_u = (I - A)$ in exchange of the promise to repay D_u if the project returns X^+, 0 otherwise (due to limited liability). Given that project L are characterized by a negative surplus, investors are not willing to fund project L:

$$(IC_f) \qquad p_H(X^+ - D_u) \geq p_L(X^+ - D_u) + B$$

$$\Leftrightarrow D_u \leq \underbrace{X^+ - B/\Delta p}_{\substack{\text{maximum} \\ \text{pledgeable income} \\ \text{to investors}}}$$

Rationality condition by investors, together with condition (IC_f), sets an upper limit to I_u:

$$p_H D_u \geq \gamma I_u \Leftrightarrow I_u = (I - A) \leq \frac{p_H}{\gamma}\left[X^+ - \frac{B}{\Delta p}\right]$$

Thus, to be funded by bond-holders, there is a minimum threshold on internal funds:

$$A \geq \underset{+}{\overline{A}(\gamma)} \equiv I - \frac{p_H}{\gamma}\left[X^+ - \frac{B}{\Delta p}\right] \tag{4}$$

Bank credit: an alternative financial strategy is to apply for a bank loan I_m (and I_u from bond-holders) promising to repay D_m to the bank (respectively D_u to bond-holders) whenever the project returns X^+, 0 otherwise.

If the bank monitors the project, the incentive compatible constraint for E is:

$$(IC_f) \qquad p_H(X^+ - D_u - D_m) \geq p_L(X^+ - D_u - D_m) + b$$

$$\Leftrightarrow D_m \leq \underbrace{X^+ - D_u - b/\Delta p}_{\substack{\text{maximum pledgeable} \\ \text{income to the bank}}}$$

We must check that the bank will monitor, otherwise E will shirk and choose project L:

$$(IC_m) \qquad p_H D_m - c \geq p_L D_m \Leftrightarrow D_m \geq \frac{c}{\Delta p}$$

Notice that combining together the two conditions (IC_f) and (IC_m) we obtain the maximum pledgeable income for uninformed bond-holders once E is rewarded for giving up the private benefit b and bank for the cost of monitoring c:

$$D_u \leq X^+ - \frac{(b+c)}{\Delta p}$$

The gross return on bank credit is $\beta = \frac{p_H D_m}{I_m}$. When E promises to repay D_m, the bank lends $\frac{p_H D_m}{\beta}$. Bank credit is more expensive than direct credit, due to the cost of monitoring, since banks want to be rewarded for the monitoring cost. This result can be derived by looking at the condition for which the bank lends money instead of holding public bonds on financial markets

$$\beta I_m - c \geq \gamma I_m$$

from which it follows that $\beta - \gamma \geq \frac{c}{I_m} > 0$. As a consequence E wants to minimize bank credit, hence applies for $D_m = \frac{c}{\Delta p}$. He borrows $I_m = \frac{p_H c}{\beta \Delta p}$ from the bank and I_u directly from bond-holders.

The project, once it obtains bank credit, receives direct credit $I_u \geq I - A - I_m$ if and only if the investors' rationality condition is fulfilled, i.e.:

$$(IR_u) \qquad \gamma(I - A - I_m) \leq p_H \left(X^+ - \frac{(b+c)}{\Delta p} \right)$$

We derive the threshold on internal funds to obtain bank credit:

$$A \geq \underline{A}(\underset{+}{\gamma}, \underset{+}{\beta}) \equiv I - I_m(\beta) - \frac{p_H}{\gamma} \left(X^+ - \frac{b+c}{\Delta p} \right) \tag{5}$$

Equilibrium on credit markets:

There are two markets: direct credit and bank credit market. The aggregate demand in each type of credit market can be derived from the conditions (4) and (5).

Proposition 1 *At the equilibrium, firms with internal funds $A \geq \bar{A}(\gamma)$ issue bonds directly on financial markets. Firms with less internal funds $A \in [\underline{A}(\beta,\gamma), \bar{A}(\gamma)]$ get funded by applying also for bank loans, while poorer firms, in terms of own funds, are denied credit (rationing).*

Numerical example 4

E owns liquidity A and seeks external funding for an investment that requires $I = 85$ at $t = 0$ and that returns $X = \{50, 100\}$ at $t = 2$. E can choose between two projects: a good project H and a bad project L. The success probability is $\Pr\{X = 100\} = p$; project H has a greater success probability $p_H = 0.8$, while project L has $p_L = 0.6$. However project L guarantees to E a private benefit $B = 10$:

1. Compute the NPV of the project H.
2. E raises $(I - A)$ by issuing a bond that repays a face value R_u to investors. Write the incentive constraint for E to choose project H and compute his maximum pledgeable income (constraint on R_u).
3. Write the investors' rationality constraint and find the minimum value R_u, assuming that E chooses project H. Find the minimum threshold for A for which E manages to raise external financing.
4. The bank monitors at cost $c = 2$, reducing as a consequence the private benefit from $B = 10$ to $b = 5$. Assume an E who is credit rationed. E asks funding exclusively to a bank and promises to repay R_m at $t = 2$. Which is the minimum threshold for A to obtain a loan from the bank?
5. Assume now own funds A are uniformly distributed between 0 and 100, that is $A \sim h(A) = \frac{1}{100}$ on the interval $[0, 100]$, where $h(A)$ is the density function. Compute the percentage of firms that are credit rationed, those that are financed by financial markets, those financed by the banks and those that self-finance the investment.

Banks vs. Arms' length creditors (Rajan, JF 1992)

Three dates ($t = 0, 1, 2$) (alternative gross return $\gamma = 1$).

Entrepreneur E has a productive project:

- At $t = 0$: financial contract is issued;
  - E needs $I > 0$
  - E has no internal funds, i.e. $A = 0$.
  - capital markets are competitive.
- At $t = 1/2$: moral hazard.
  - E might exert a continuous effort $e \in [0, 1]$
  - private cost of effort is e
- At $t = 1$: productive project might be prematurely liquidated. One of the two possible states $\{G, B\}$ realizes and it is observed by banks and E.
  - However liquidation is costly, since $L < I$.
- At $t = 2$: the project returns; its revenue is contingent upon the realization of one of the two possible states G (Good) or B (Bad).
  - The probability of the occurrence of state G is $q(e)$, while $(1 - q(e))$ is the probability that state B occurs.
  - in state G (Good) the project returns X (with probability 1);
  - in state B (Bad) the project returns X with probability p, 0 otherwise.

Assumption 1: Banks are informed creditors, that is they observe the true state $\{G, B\}$, while bond-holders are uninformed creditors hence they do not observe it.

Assumption 2: $X > I > L \geq pX$

Assumption 3: $q'(e) > 0$, $q''(e) < 0$. A greater effort increases the probability of occurrence of state G, although at a decreasing pace.

First best: E has enough internal funds to finance the project (self-finance).

The liquidation decision at $t = 1$ is up to E:

when state G occurs, given that $X > L$ he will continue the project: his expected income is $q(e)(X - I)$;

when state B occurs, since $pX < L$ he will liquidate the project: his expected income is $[1 - q(e)](L - I)$

$\Rightarrow$ liquidation occurs only in state B.

At $t = 1/2$ the optimal effort maximizes E's expected profits, i.e.:

$$\pi(e) = q(e)(X - I) + (1 - q(e))(L - I) - e$$

thus

$$q'(e^*) = \frac{1}{X - L}$$

Direct credit (uninformed credit) - Arms' length (A)

E issues at $t = 0$ a financial contract that, in exchange for I, promises to repay $D \leq X$ at $t = 2$ when the project returns X.

In this case, since the creditor is an "arms' lenght" creditor, hence does not observe the state, he will never liquidate the project.

The decision about liquidation at $t = 1$ rests with E:

If state G occurs, E's payoff is $X - D$ when continuing, while it is 0 when he stops the project;

if state B occurs, E's payoff is $p(X - D)$ when continuing, while 0 if he stops it.

$\Rightarrow$ E will always decide for continuation.

At $t = 1/2$ the optimal effort of E maximizes:

$$\pi(e) = q(e)(X - D) + (1 - q(e))p(X - D) - e$$

that is, for given D, it is

$$q'(e^A) = \frac{1}{(1 - p)(X - D)}$$

At $t = 0$ the individual rationality condition for investors requires that the face value D fulfills:

$$q(e^A)D^A + (1 - q(e^A))pD^A = I$$

that is:

$$D^A = \frac{I}{q^A + (1 - q^A)p}$$

where $q^A = q(e^A)$. It is possible to show that

$$q'(e^A) = \frac{1}{(1-p)(X - D^A)} > \frac{1}{X - L}$$

Notice that funding the project with external finance reduces his incentive relatively to the First Best, that is $e^A < e^*$. The reason is that E pays the full cost of his effort, but the marginal revenue of this effort partially accrues to the external creditor.

Bank finance (B)

When E is granted a loan by the bank, he promises to repay D whenever the project succeeds and it is not liquidated.

At $t = 1$ the liquidation decision stands with the bank, given that she observes the state G or B. E would choose always to continue, instead.

If the state is G, bank's payoff is D, while if the project is liquidated, the recovery value is L: since $D > I$ and $I > L$, it follows that $D > L$): bank does not liquidate the project;

If the state is B, the bank's expected payoff is pD, while if liquidated the bank recovers L: since $L > pX > pD$) the bank liquidates the project.

$\Rightarrow$ The bank liquidates the project in state B.

At $t = 1/2$, the optimal effort maximizes profits, that is

$$\pi(e) = q(e)(X - D) + (1 - q(e)) \times 0 - e$$

it follows, for a given D, that

$$q'(e^B) = \frac{1}{X - D} > \frac{1}{X - L}$$

At $t = 0$ the individual rationality condition the bank requires

$$q(e^B)D^B + (1 - q(e^B))L = I$$

from which

$$D^B = \frac{I - (1 - q^B)L}{q^B}$$

where $q^B = q(e^B)$. Once we substitute the optimal effort,

$$q'(e^B) = \frac{1}{q^B(X - L) + (L - I)} > \frac{1}{X - L}$$

also in this case $e^B < e^*$ for the same reason as before.

Choice between bank credit (B) and direct credit (A)

Substituting the optimal effort e^A into the profit

$$\pi(e^A) = q^A(X - D^A) + (1 - q^A)p(X - D^A) - e^A$$

and the face value D^A from the individual rationality condition

$$q^A D^A + (1 - q^A)pD^A = I$$

we derive the equilibrium profit in the case of direct credit:

$$\pi^A = q^A X + (1 - q^A)pX - I - e^A$$

Substituting the optimal effort e^B into the profit

$$\pi(e^B) = q(e^B)(X - D^B) - e^B$$

and the face value D^B from the individual rationality condition

$$q^B D^B + (1 - q^B)L = I$$

we derive the equilibrium profit in the case of bank credit:

$$\pi^B = q^B X + (1 - q^B)L - I - e^B$$

If the equilibrium efforts were the same in both cases, $q^A = q^B$, given that $L > pX$, bank credit would dominate direct credit, since continuation in state B is sub-optimal.

However the two type of creditors have a different impact on E's incentive:

- if $q^B > q^A$ (the threat of liquidation in B increases E's effort): then bank credit is superior;
- if $q^B < q^A$(the threat of liquidation demotivates E who reduces his effort due to the fear of being expropriated): direct credit, that is a more passive creditor, is optimal.

To conclude, **banks are tougher creditors compared to bond-holders** since they tend to liquidate projects more often: however if this demotivates E, who will exert a lower effort, direct credit is preferable.

Numerical example 5

An E, without own liquidity ($A = 0$), needs an investment of $I = 100$ to undertake a risky project at $t = 0$. The project will deliver at $t = 2$ a cash flow $X = \{0, 140\}$. However there are two possible states which affect the success of the project: in state G the project is always successful, that is it delivers 140 with certainty; while in state B only with probability 0.5 and it fails with probability 0.5.

If E exerts an effort (project H), state G occurs with probability 3/4, while in case he shirks (project L) state G occurs only with probability 1/4. Effort is privately costly to E, that is it costs $c = 10$. Given that the effort is not observable and the cost is entirely borne by E, there is moral hazard in the sense that E will try to avoid the effort.

At an interim date, $t = 1/2$, there is the possibility to terminate the project and sell the asset, that is to liquidate the project. In case of liquidation the project returns $L = 80$ with certainty. Finally notice that $X = 140 > I = 100 > L = 80 > p.X = 70$.

1. Compute the First Best choice in terms of Liquidation: assume that E has own funds I to undertake the project. Check his choice in terms of liquidation and effort.

2. Assume that the project is financed by an "arms' length" investor (bond-holder) who requires a face value D in exchange for the initial investment $I = 100$. Assume that $100 < D < 140$ and compute the optimal choice of effort for E given that he is in control of the liquidation choice. Check whether the bond-holder is willing to finance E, anticipating his choice of effort.

3. Assume now that the bank finances E with a loan with face value $100 < D < 140$. The bank can observe the realized state, either G or B: therefore the bank decides whether to liquidate the project, depending on the realized state. Check whether the bank is willing to finance E, anticipating his choice of effort.

Lecture Notes
Initial Public Offerings

- **Asymmetric information**: insiders (owners and managers) have better information than outsiders on:
  - value of assets
  - investment perspectives
  - collateral value
  - timing of future revenues
  - private benefits and costs of insiders
- Consequences $\Rightarrow$ in the worst cases: market breakdown (lemons market) or pooling equilibrium in which all type of projects are financed but best projects must cross-subsidize the worst.

Underpricing in IPOs

Three dates ($t = 0, 1, 2$). Gross alternative return $\gamma = 1$.

E has a pre-existent asset in place, without previous debt.

- At $t = 0$: pre-existent asset in place
  - Adverse selection: E can be of H-type or L-type (type is private information of E)
  - Probability that E is H-type is $q \in (0, 1)$, while probability of L-type is $(1-q)$.
- At $t = 1$: financing a new investment opportunity.
  - E needs initial liquidity $I > 0$
  - he has no internal funds $A = 0$.
  - capital markets are competitive.
- At $t = 2$: the project and pre-existent asset in place
  - H-type project returns X^+ with probability 1; L-type project returns X^- with probability 1, with $\Delta_X \equiv X^+ - X^- > 0$.
  - The new project returns Y with probability 1.

E sells $(1-\alpha)$ shares at price I; in exchange E promises a fraction $(1-\alpha)$ of future profits to investors.

Assumption: the new investment VAN is positive, $Y - I > 0$.

Symmetric information

When investors observe E's type, the share of equity would be type- contingent, that is:

$$1-\alpha_L = \frac{I}{X^- + Y}$$
$$1-\alpha_H = \frac{I}{X^+ + Y} < 1-\alpha_L$$

with a greater share for the worse type, in order to assure the financing. (Notice that the price of the two stocks is equal, as their expected return must be equal).

$$\frac{(1-\alpha_L)(X^- + Y)}{I} = \frac{(1-\alpha_H)(X^+ + Y)}{I}$$

Asymmetric information

When investors do not observe the type of E, they cannot price the equity share differently for each type of E. They require the same share $(1-\hat{\alpha})$ so that the expected return from the investment is zero:

$$(1-\hat{\alpha}) = \frac{I}{\left[\hat{X} + Y\right]} \tag{IR}$$

with $\hat{X} \equiv qX^+ + (1-q)X^-$.

Furthemore in order for E to undertake the investment it must be that:

$$\hat{\alpha}\left[X^+ + Y\right] \geq X^+ \tag{IC$_H$}$$
$$\hat{\alpha}\left[X^- + Y\right] \geq X^- \tag{IC$_L$}$$

from which

$$(1-\hat{\alpha}) \leq \frac{Y}{Y + X^+} < \frac{Y}{Y + X^-}$$

Pooling equilibrium:

The condition is :

$$1-\hat{\alpha} = \frac{I}{\left[\hat{X} + Y\right]} < \frac{Y}{Y + X^+}$$

N.B. When $q = 1$ this condition is always fulfilled, since $\hat{X} \in [X^-, X^+]$. When q diminishes it is less likely.

In the pooling eq. the H-type subsidizes the L-type E since

$$1-\hat{\alpha}=\frac{I}{\hat{X}+Y}>\frac{I}{X^{+}+Y}=1-\alpha_H$$

Good E sell their shares at a too low price, while bad E at a too high price. Thus the shares of good E are undervalued, while those of bad E are overvalued.

We must check that H-type E has the incentive to invest (type-L will always invest, given that he pretends to be type H). Substituting $(1-\hat{\alpha})Y=I-(1-\hat{\alpha})\hat{X}$ from the investor rationality condition into (IC_H), we derive:

$$Y-I>(1-\hat{\alpha})(X^{+}-\hat{X})>0 \qquad (*)$$

But this condition implies that not all investments with a $NPV>0$ will be financed.

In conclusion there will be two possible cases:

- Either condition (*) is fulfilled: then a pooling equilibrium occurs in which both types invest and issue shares that promise a share $(1-\hat{\alpha})$ of future profits. However: 1) shares of good E are undervalued and are issued at a low price; 2) some projects with not enough high NPV, although positive, will not be financed.
- Or condition (*) is not fulfilled: separating equilibrium arises where only L-type will invest. H-type finds too costly to signal their type. Firm's value pre-listing is :

  $$V_0=qX^{+}+(1-q)(X^{-}+Y-I)$$

  while post-listing :

  $$V_1=(X^{-}+Y-I)<V_0$$

  since condition (*) is not fulfilled by assumption $\Longrightarrow$ The price of shares falls after announcing a new issue of equity ("under-pricing").

Numerical example 6

Consider an entrepreneur E who owns an asset in place at $t=0$ that will return a cash flow at $t=2$: the cash flow will be $X^{+}=90$ if E is of type H, while $X^{-}=50$ if E is of type L. At $t=1$ there is a new investment opportunity: by investing $I=20$ at $t=1$ this project will return $Y=30$ at $t=2$ by sure. E has no funds to finance this new opportunity, hence he has to issue stocks on competitive financial markets.

1. Assume new investors observe the type of E: which fraction $(1-\alpha)\in(0,1)$ of the cash flow will investors request to finance a firm of type H? Which fraction to finance firm of type L?

2. Assume now that new investors do not observe the type of E. The probability that E is of type H is $q = 1/4$. If new investors expect that both type of E will invest, which fraction $(1 - \hat{\alpha})$ of cash flow must be promised to investors in order to convince them to finance the firm? Do you think investors' belief are correct at the equilibrium?

3. Assume now that investors expect that only type L will invest. Which fraction $1 - \hat{\alpha}$ must be promised to new investors in this case? Are the expectations correct at the equilibrium?

4. Compute the probability q (of type H) for which a pooling equilibrium exists in which both types E invest.

Pecking order in finance
(Myers & Majluf, JFE-1984)

- Given asymmetric information, good E will fund their investments in order to minimize the underpricing.
- Given that funding sources react differently to asymmetric information one can derive a pecking order in the sources of external finance.
- The funding sources can be ranked according to the following order starting from the least to the most sensitive source to asymmetric information:
  - Self-finance
  - Non-risky debt
  - Risky debt
  - Convertible debt
  - Equity

Implications

- Firms prefer to self-finance their investments;
- When asking for external finance, firms follow the pecking order.

Price reactions to new equity issues (event studies)

- Non positive reactions of equity issues on secondary market.

- The more the financial asset resembles equity the more negative the reaction:
  - New issue of debt: small effect
  - Convertible debt: $\simeq -2\%$
  - Equity: $\simeq -3\%$
- Positive reaction of stock prices to transactions that reduce the amount of outstanding shares.
  - Positive reaction to announcement of :
    - exchange of shares with bonds;
    - exchange of preferred shares with ordinary ones;
  - Negative reactions to news of :
    - exchange of bonds with shares;
    - ordinary shares with preferred ones.

$\Rightarrow$ "pecking order " within the sources of finance seems to be coherent with the empirical evidence.

Decision to go public (Zingales, JF 1995)

- The IPO serves the insider E to extract as much as possible from a future transfer of control to a Raider:
  - The greater the share of equity sold to outside dispersed investors $(1-\alpha)$, the smaller the block of control for which a Raider is willing to bid.
  - The greater the share sold to outside investors, the more the incumbent E will be able to extract from the future Raider when he bids for the shares in the hands of outside investors, also those in the hands of the incumbent E.

Model (no asymmetric information)

A firm's control is in the hands of an insider I: the value of shares is X_I under his control. In addition he enjoys a private benefit of control B_I.

At $t = 0$, the insider I might decide to go public, namely to sell a fraction of equity in the stock market (IPO):

- He keeps a fraction α big enough to retain control of his firm;
- Sells a fraction $(1-\alpha)$ of equity to dispersed investors;
- The insider's value of control is: $\alpha X_I + B_I$. That is, the monetary value of his shares αX_I and the private benefit of control B_I.

At $t = 1$ a Raider arrives who generates:

- A greater value (sum of monetary and private benefits): $X_R + B_R > X_I + B_I$
- For R the value of controlling the firm is: $\alpha X_R + B_R$
- $\Rightarrow$ the surplus from the transfer of control is

  $$\alpha \Delta X + \Delta B$$

  with $\Delta X = X_R - X_I > 0$ and $\Delta B = B_R - B_I$.
- I and R negotiate over the transfer of the control block and split the surplus $(50\%, 50\%)$ by considering their outside options.
- R bids $b^* = X_R$ to dispersed shareholders (due to the free-riding problem by Grossman-Hart they extract the maximum value from the Raider)

We solve the model by backward induction: first we solve the negotiation game at $t = 1$ for given fraction of inside equity α; then we go back to date $t = 0$ and we set the optimal share of equity $(1 - \alpha)$ to be sold at the IPO.

Second stage ($t = 1$)

Assume that R and I have already exchanged the control block and that R must bid in order to buy shares from dispersed investors: due to the free-riding problem by dispersed shareholders he must bid $b^* = X_R$ since $\Delta X > 0$. Hence he does not gain from dispersed investors.

Case 1: $\alpha\Delta X + \Delta B < 0 \Rightarrow$ Transfer of control does not take place.

- If I sells a too large fraction of equity during the IPO (hence he retains a too small fraction α), he looses any interest in the monetary transfer, while values his private benefit form control too much. Whenever
$$\frac{-\Delta B}{\Delta X} = \frac{B_I - B_R}{X_R - X_I} > \alpha > 0$$
his private benefits of control are too big, and he does not transfer the control.
- Dispersed shareholders receive in $t = 1 : (1 - \alpha)X_I$ (since R does not bid for their shares)

Case 2: $\alpha\Delta X + \Delta B > 0 \Rightarrow$ Transfer of control occurs.

- The insider I chooses his best action by comparing
  - if he sells the control block he earns P
  - if he keeps the controlling block, he values it: $\alpha X_I + B_I$ (outside option)
- The Raider chooses his best action by comparing
  - if he buys the control block his revenue is $\alpha X_R + B_R - P$
  - if he does not buy, his return is: 0 (outside option)

The two parties split the surplus $(50\%, 50\%)$ that is:
$$P - (\alpha X_I + B_I) = \alpha X_R + B_R - P$$
The price at which the control block is transferred is therefore:
$$P = \frac{1}{2}(\alpha X_I + B_I) + \frac{1}{2}(\alpha X_R + B_R)$$
Let's compute the revenue for each one of the three players at $t = 1$:

- Insider I earns P; adding and subtracting $\frac{1}{2}(\alpha X_R + B_R)$ to P we can rewrite it as:

$$P = (\alpha X_R + B_R) - \frac{1}{2}(\alpha \Delta X + \Delta B)$$

- The Raider earns in case he bids for the control block:

$$(\alpha X_R + B_R) - P = \frac{1}{2}(\alpha \Delta X + \Delta B) \geq 0$$

- Dispersed investors gain $(1-\alpha)X_R$ when the control block is in the hands of the Raider, since he will bid $b = X_R$ to buy the shares from dispersed shareholders.

First stage ($t = 0$)

Now we study the optimal fraction of equity $(1-\alpha)$ that the Insider I is willing to sell to dispersed investors in the IPO .

- In a perfectly competitive market the Insider extracts fully the future value of the firm.
- Thus, if he manages to transfer the control block to the Raider in the future, investors are willing to pay $(1-\alpha)X_R$, otherwise $(1-\alpha)X_I$.

$\Rightarrow$ The Insider I can exploit strategically dispersed investors to extract the maximum value out of the sale of the shares in the IPO.

Hence, assuming that there will be the transfer of the control block, the revenue of the insider at $t = 0$ is

$$\Pi_I = [(\alpha X_R + B_R) - \frac{1}{2}(\alpha \Delta X + \Delta B)] + [(1-\alpha)X_R]$$

given the constraint

$$\frac{1}{2}(\alpha \Delta X + \Delta B) > 0$$

otherwise the Raider does not bid for the control block. The optimal choice of α is therefore

$$\frac{d\Pi_I}{d\alpha} = -\frac{1}{2}\Delta X$$

Given that $\Delta X > 0$, the solution is the minimum α compatible with the constraint $(\alpha \Delta X + \Delta B) > 0$, hence

$$\alpha^* = -\frac{\Delta B}{\Delta X}$$

- Dispersed investors earn:

  - all the monetary benefit from the change of control
  - no private benefit of control

- Insider I earns (provided he keeps the incentive to sell the company):
  - a fraction of the monetary benefit from the change of control
  - a fraction of the private benefit of the change of control

Numerical example 7

Consider a company entirely in the hands of an incumbent entrepreneur I at $t = 0$. The shares of the company under his control are valued $X_I = 100$, while his private benefit from control is $B_I = 50$. Assume that at a future date $t = 1$ a good raider R will make an offer to buy all the shares of the company: the shares of the company if he manage to acquire the company are $X_R = 150$, while his benefit from control is $B_R = 40$.

1. Assume that I decide to make an IPO in order to sell a fraction $1-\alpha$ of the shares of the company at $t = 0$ to dispersed shareholders. What is the price P (as a function of α) that the raider has to pay to I in order to buy the controlling block? [Assume that I and R have the same bargaining power, hence they share the surplus of the transaction equally]
2. Compute the optimal fraction α that the insider I will keep as a controlling block to maximize his profit at $t = 0$.

Lecture Notes
Conglomerate premium

Diversification

Two-Project Model

Suppose E has now (simultaneously) two independent and identical projects in a model la Holmstrom-Tirole.

- each project generates $X \in \{0, X^+\}$
- succeeds with p_H if E works and with $p_L = p_H - \Delta_p$ if E shirks.
- Shirking results in (non-transferable) private benefits B to E.

E has wealth A per project and must borrow $I - A$ per project.

Single project financing

If each project is financed separately, E promises to repay R_1 per project at $t = 2$ when $X = X^+$ in exchange for $(I - A)$ per project at $t = 0$.

The incentive constraint for E to exert effort is defined for each project

$$p_H(X^+ - R_1) \geq p_L(X^+ - R_1) + B$$

The investor's participation constraint is given by

$$p_H R_1 \geq (I - A)$$

The (per-project) financing condition is as in the credit rationing model:

$$A \geq \bar{A}_1 = I - p_H[X^+ - \frac{B}{\Delta_p}]$$

Project financing ignores that E can pledge the proceeds from one project as collateral when raising funds for the other project.

Integration of the two projects

Under integration (joint liability), E can forgive her claim from a successful project when the other fails.

- Indeed, given E is risk-neutral, it is optimal to reward E only if both projects succeed.
- The financial contract goes as follows: in exchange for $2(I-A)$ at $t=0$, E promises to repay R_2 when both projects succeed, that is when he gains $2X^+$, 0 otherwise.
- The investor retains the cash flow in all other events, hence X^+ is cashed by the investor when only one of the two projects succeeds.

E has the choice between exerting effort on both projects or shirking on both projects:

$$p_H^2(2X^+ - R_2) \geq p_L^2(2X^+ - R_2) + 2B$$

thus

$$(2X^+ - R_2) \geq \frac{2B}{\Delta_p(p_H + p_L)} \tag{6}$$

in alternative, E can decide to shirk only on one project, instead of shirking on both. In this case the incentive constraint would be

$$p_H^2(2X^+ - R_2) \geq p_H p_L(2X^+ - R_2) + B$$

thus

$$(2X^+ - R_2) \geq \frac{B}{\Delta_p p_H} \tag{7}$$

Given that $p_H > p_L$ the incentive constraint in (6) is more stringent than (7). We can therefore concentrate on (6) and discard the second one. Hence the maximum pledgeable income is:

$$R_2 \leq 2[X^+ - \frac{B}{\Delta_p(p_H + p_L)}] \tag{8}$$

The investor requires:

$$p_H^2 R_2 + 2p_H(1 - p_H)X^+ \geq 2(I - A)$$

thus

$$R_2 \geq \frac{2[(I - A) - p_H(1 - p_H)X^+]}{p_H^2} \tag{9}$$

Combining the two conditions, (8) and (9), we derive

$$X^+ - \frac{B}{\Delta_p(p_H + p_L)} \geq \frac{[(I - A) - p_H(1 - p_H)X^+]}{p_H^2}$$

Finally, solving for A:

$$A \geq [I - p_H(X^+ - B/\Delta_p)] - \frac{B}{\Delta_p} p_H[1 - \frac{p_H}{(p_H + p_L)}]$$

Minimum per-project wealth of a 2 projects firm is lower than a single-project firm:

$$A \geq \bar{A}_2 = \bar{A}_1 - p_H \frac{B}{\Delta_p} \times \frac{p_L}{p_L + p_H}$$

It is evident that $\bar{A}_2 < \bar{A}_1$, namely *integrating two independent projects under the same roof improves the conditions for a company that has to raise external finance.*

Concluding remarks

- Under integration, E can lose more from project failure than zero, namely her proceeds from the other project (when successful).
  - Cross-pledging is equivalent to relaxing the limited liability constraint.
  - This reduces the agency rent, thereby increasing the pledgeable income.
- Mechanism relies on the diversification effect: one of the two projects serves as collateral, only to the extent that their returns are uncorrelated.
  - With perfectly correlated payoffs, integration has no advantage with respect to single project financing.
  - Diversification effect increases with number of (independent) projects.

While diversification can facilitate (per project) lending, there are various impediments.

- Limits of diversification:
  - Endogeneity of correlation (E cannot alter payoff correlation by selecting projects),
  - Expertise in limited areas,
  - Limits due to ”span of control”